# Gravitation Class 11 handwritten notes



## Gravitation

The force of attraction between any two bodies in the universe is called gravitational force or gravitation

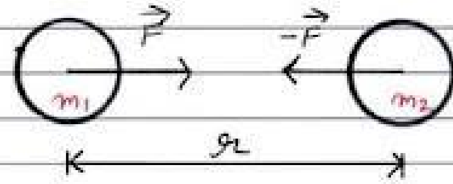
The force of attraction exerted by earth on a body lying on or near the surface of earth has given the special name gravity or gravitational pull.

## Newton's law of Gravitation

Newton's law of gravitation states that every body in the universe attract every other body with a force which is directly propotional to product of their masses and inversely propotional to the square of distance between centres

This force act along the line joining the centres of two bodies.

let us assume two bodies having mass $m_1$ and $m_2$ are seprated by distance $r$ as shown in fig




Acc to Newton's law of Gravitation

$$F \propto m_1 m_2$$
$$F \propto \frac{1}{r^2}$$

Combining above two

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

where G is constant of proportionality and called universal gravitational constant

The value of $G = 6.67 \times 10^{-11}\ N\,m^2\,kg^{-2}$

The force on mass ($m_1$) and mass ($m_2$) are equal in magnitude but opposite in direction

Dimensional formula of $G = [M^{-1} L^3 T^{-2}]$

Gravitational constant (G) is defined as the force of attraction between two masses each of one kg and seprated by one meter of distance

Points to be noted

1. Gravitational force between two bodies are always attractive

2. The value of G does not depend upon nature, size and shape of masses G is truely universal throughout the universe

3. The gravitational force of attraction between two bodies is not altered by presence of other body

4. G is measured experimentally and small value of G tells us that force of attraction between ordinary sized object is very small

Q. Calculate the force of attraction between two bodies each of mass 100 kg and 1 m apart on the surface of earth.

Will the force of attraction be different if the bodies are taken on moon while their seperation remain constant.

Sol As per Newton's law of gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 100 \times 100}{1^2}$$

$$F = 6.67 \times 10^{-7} N$$

No, the force of attraction will remain the same if the bodies are taken on the surface of moon.

Ques An apple of mass 0.25 kg falls from a tree. What is the acceleration of the apple toward the earth? Also calculate the acceleration of the earth toward the apple

Given Mass of Earth → $5.983 \times 10^{24}$ kg
Radius of Earth → $6.378 \times 10^{6}$ m
G → $6.67 \times 10^{-11} Nm^2 kg^{-2}$

Sol As per Newton's law of gravitation both apple and the earth are attracting each other with same force and which is given by

$$F = \frac{G m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 0.25 \times 5.983 \times 10^{24}}{(6.378 \times 10^{6})^2}$$

The height of tree is very very small in comparison with radius of earth so neglecting the height of tree

$$F = 2.452\ N$$

Acc to Newton's second law

$$F = ma$$

Acceleration of apple toward earth

$$a = \frac{F}{m_1}$$

$$a = \frac{2.452}{.25}$$

$$a = 9.81\ m/s^2$$

Acceleration of earth toward apple

$$a = \frac{F}{m_2}$$

$$a = \frac{2.452}{5.983 \times 10^{24}}$$

$$a = 4.099 \times 10^{-25}\ m/s^2$$





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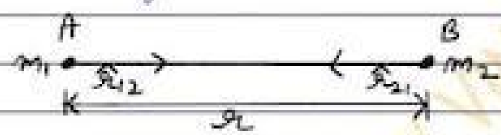
PH: 7015250795, 8930908558

## Vector form of Newton's law of Gravitation

Consider two bodies A and B of masses $m_1$ and $m_2$ placed at a distance $r$ apart.

let $\hat{r}_{12}$ is unit vector from A to B
$\hat{r}_{21}$ is unit vector from B to A

$\vec{F}_{12}$ is force on A by B
$\vec{F}_{21}$ is force on B by A




Acc to Newton's law of Gravitation

$$\vec{F}_{12} = \frac{G m_1 m_2}{r^2} \hat{r}_{12}$$

or $$\vec{F}_{12} = \frac{G m_1 m_2}{r^3} \vec{r}_{12} \quad \text{as } r\hat{r}_{12} = \vec{r}_{12}$$

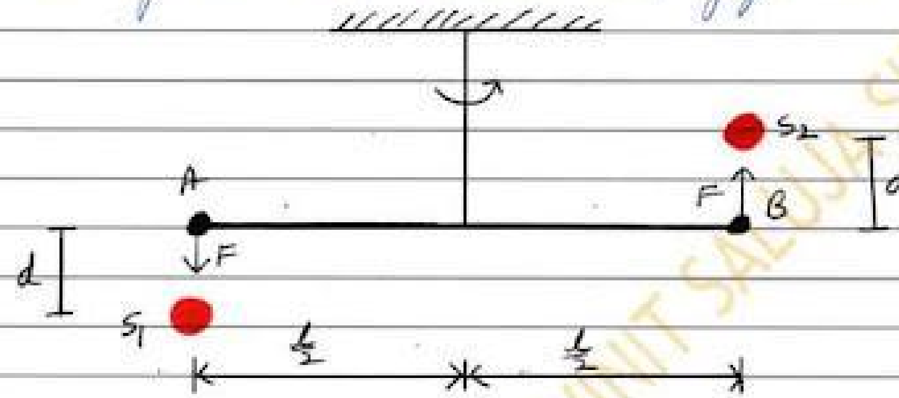
Similarly

$$\vec{F}_{21} = \frac{G m_1 m_2}{r^2} \hat{r}_{21}$$

or $$\vec{F}_{21} = \frac{G m_1 m_2}{r^3} \vec{r}_{21} \quad \text{as } r\hat{r}_{21} = \vec{r}_{21}$$

## Cavendish Experiment or Experimental determination of Gravitational constant

The appratus used for calulating the value of G is shown in fig




In the fig A and B are two small lead sphere each of mass m attached at the end of a bar of length l

The bar is suspended horizontally with thin wire. S₁ and S₂ are two big lead spheres each of mass M placed at a distance d from small sphere

The gravitational force on small lead sphere due to big sphere

$$F = \frac{GmM}{d^2}$$

As shown in fig the two forces are acting in opposite direction so net force is zero but some torque act on it

$$\tau = F \times \frac{l}{2} + F \times \frac{l}{2}$$

$$\tau = F \times l$$

capital ← $\tau = \frac{GmM\,l}{d^2}$

let us assume due to this torque bar is rotated by angle $\theta$ and a twist is produced in wire

when twist is produced, a restoring torque gets developed in wire

When equilibrium is obtained restoring torque become equal to applied torque

let

small ← $\tau$ → restoring torque per unit angle of twist

Total restoring torque $= \tau\theta$

At equilibrium

$$\tau\theta = \frac{GmM\,l}{d^2}$$

$$G = \frac{\tau\theta d^2 l}{mM}$$

By observing the value of $\theta$ and by knowing the values of other variable we can find the value of G

The value of G found out by cavendish experiment is $6.75 \times 10^{-11}\ Nm^2 kg^{-2}$

later on the value of G is improved by other scientists and presently accepted value is $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

## Acceleration due to gravity

When a body is allowed to fall freely toward earth then due to gravitational force it undergoes some acceleration and this acceleration is called acceleration due to gravity

It is represented by 'g'

Acc to Newton's $2^{nd}$ law

$$F = ma$$

here $F = F_g$ = Gravity force

$a = g$ = Acc due to gravity.

$$F_g = mg$$

if $m = 1$, then $g = F_g$

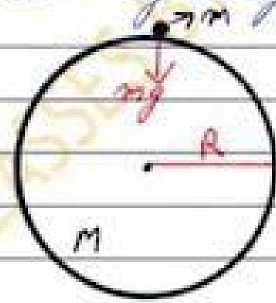
Thus acceleration due to gravity is defined as the force of gravity acting on unit mass placed near or on the surface of earth

SI unit → $m/s^2$

Dimensional formula → $[M^0 L^1 T^{-2}]$

**Relation between g and G**

Consider earth to be a spherical body of mass M, radius R with centre O. Suppose a body of mass m is placed on the surface of earth where acceleration due to gravity is g.




Acc to Newton's law of Gravitation

$$F = \frac{GMm}{R^2} \quad \text{—— 1}$$

Acc to Newton's 2nd law

$$F = ma$$

as $a = g$, $F = mg$ —— 2

from eq 1 and eq 2



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PH: 7015250795, 8930908558

$$mg = \frac{GMm}{R^2}$$

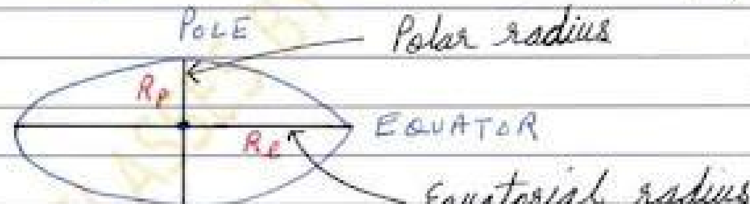
$$g = \frac{GM}{R^2}$$

above eq shows acceleration due to gravity does not depend upon mass of the body.

## Variation of Acceleration due to gravity

### 1. Effect of shape of Earth

Earth is not a perfect sphere it is egg shaped as shown in the fig




Equatorial radius ($R_e$) of earth is about 21 Km greater than the polar radius ($R_p$)

We know that $g = \frac{GM}{R^2}$

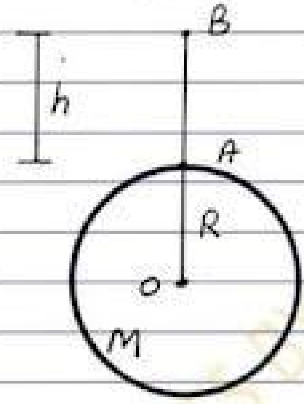
as $R_e > R_p$

So $g_p > g_e$

thus the value g at the pole is more than that at the equator

## Effect of Altitude or height

Consider earth to be a homogenous sphere of mass M, radius R with centre O. let g be the value of acceleration due to gravity at a point A on the surface of earth




$$g = \frac{GM}{R^2} \quad \text{——— } 1$$

If $g'$ is the acceleration due to gravity at a point B, at a height h above the surface of earth

$$g' = \frac{GM}{(R+h)^2} \quad \text{——— } 2$$

dividing eq 2 by eq 1

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \frac{R^2}{GM}$$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$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} \quad \text{——— 3}$$

Note → use this eq when $h <<< R$ is not true

$$\frac{g'}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

→ We use above formula when h is more than 5% of Radius of earth (320 km)

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

Expanding the above equation by binomial theorem

$$(1+x)^{-n} = 1 + \frac{(-n)x^1}{1!} + \frac{(-n)(-n-1)x^2}{2!} + \frac{(-n)(-n-1)(-n-2)x^3}{3!} + \cdots$$

↑ General form

$$g' = g\left[1 + \frac{(-2)\left(\frac{h}{R}\right)}{1!} + \frac{(-2)(-2-1)\left(\frac{h}{R}\right)^2}{2!} + \frac{(-2)(-2-1)(-2-2)\left(\frac{h}{R}\right)^3}{3!} + \cdots\right]$$

If $h <<< R$ neglecting the square and higher power of $\frac{h}{R}$

$$g' = g\left(1 - \frac{2h}{R}\right) \quad \text{——— 4}$$

Note → use this eq when $h <<< R$ is true

From Eq 3 and Eq 4 it is clear that value of g decreases with height.

→ we use above formula when h is upto 5% of radius of earth (320 km)