WAVE MOTION AND SOUND 8

This chapter is divided into three parts.

(a) Simple Harmonic Motion (b) Waves (c) Sound

SIMPLE HARMONIC MOTION

VIBRATORY MOTION:

If a body moves to and fro about its mean position, its motion is said to be "vibratory" e.g. motion of the prongs of a tuning fork when struck, motion of the string of sitar when plucked, motion of mass attached to an elastic spring, motion of pendulum, motion of the membrane of a drum etc.

(i) Periodic Motion:

The motion of a body, which repeats itself in equal intervals of time, is called periodic motion.

(ii) Vibration:

One complete round trip of a body, about its mean position, is called one vibration.

(iii) Time Period 'T'

The time, required to complete one vibration, is called time period. It is denoted by T.

(iv) Frequency 'υ'

The number of vibrations which are completed in one second, is called frequency. It is denoted by 'υ'.

(v) Displacement (x)

It is the distance of vibrating, body at any instant from the mean position. It is denoted by 'x'.

(vi) Amplitude x_0

The maximum displacement of a vibrating body from the mean position is called amplitude. It is denoted by 'x_0'.

Hook's Law:

The extension 'x' produced in the spring is directly proportional to the applied force 'F', provided the elastic limit is not violated.

$$F \propto x$$

$$\boxed{F = kx}$$

Where k is a constant of proportionality and is called spring constant.

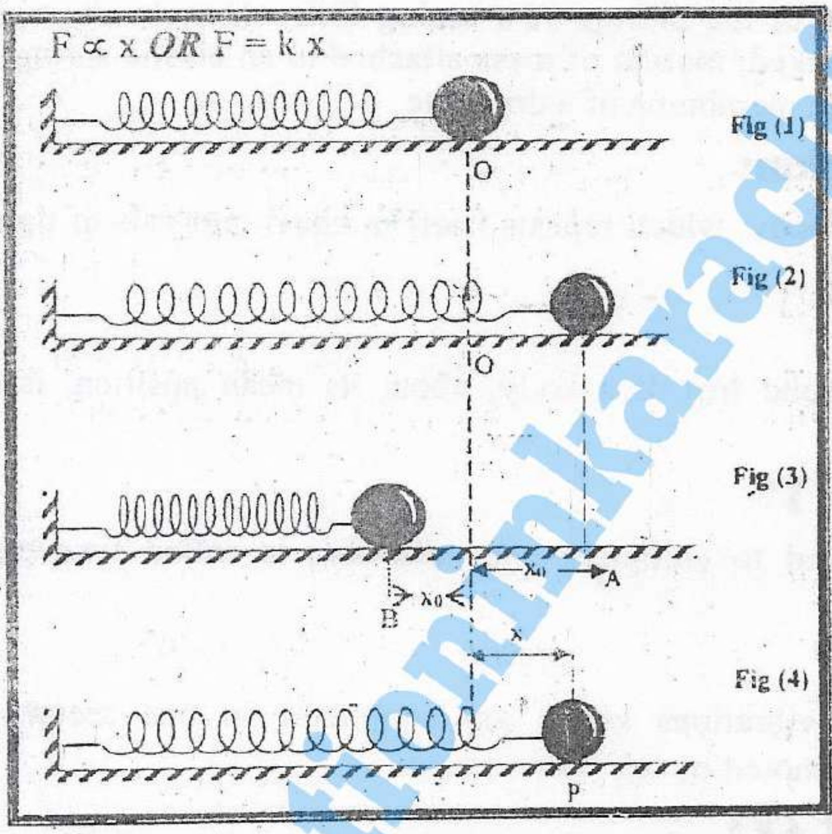
Restoring Force:

It is the force which brings the body back towards the mean position. This force is equal in magnitude but opposite to the applied force and given by

$$F = -kx$$

MOTION UNDER ELASTIC RESTORING FORCE OR MASS ATTACHED WITH SPRING:

Consider a body of mass "m" attached to a horizontal helical spring as shown in the fig (1). The whole system is placed on a horizontal frictionless surface. If the spring is stretched or compressed through a small displacement 'x' from the equilibrium position, then according to Hooke's law, force 'F' is proportional to the displacement x.



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Where k = Force constant of spring due to elasticity, the spring exerts force which is equal and opposite to the applied force. This force is called "RESTORING FORCE" and is given by.

$$F = -k\,x \longrightarrow \quad (1)$$

The negative sign indicates that the force exerted by the spring on the body is always directed opposite to the displacement.

The work done in displacing the body from O to A will be stored in the spring as P.E. If the body is now released, it will move towards O. At O, all the P.E is converted into K.E. and due to inertia, the body continues to move and reaches B, where all the K.E, is converted to P.E., the spring is now compressed and pushes the body towards O. In this way, the body continues to move between A & B thus its motion is vibratory.

ACCELERATION OF THE BODY:

Let at any instant "t", the body be at point "P" which is at a distance "x" from O, as shown in fig. Then restoring force on the body will be

$$F = -kx \rightarrow \quad (2)$$

If "a" be the acceleration of the body at P, then the Newton's second law of motion is: $F = ma \rightarrow (3)$

Comparing eq. (2) and (3)

$$ma = -kx$$

$$\boxed{a = -\frac{k}{m}x} \longrightarrow (4)$$

Eq. (4) gives the acceleration of the body.

Since $\frac{k}{m}$ is constant

$$\therefore a = -(\text{constant})x$$

$$\textbf{or } a \propto -x$$

i.e acceleration $\propto$ – displacement.

This shows that the acceleration of a vibrating body is directly proportional to the instantaneous displacement & is always directed towards equilibrium position. Such a motion is called "SIMPLE HARMONIC MOTION" (SHM).

SIMPLE HARMONIC MOTION:

If a body is performing vibratory motion so that acceleration of the vibrating body is directly proportional to –ve of its displacement from mean position or equilibrium position,

$$a \propto -x$$

i.e acceleration $\propto$ – displacement

CONDITIONS OF SIMPLE HARMONIC MOTION:

Following are the conditions of S.H.M.

1. It should be vibratory motion.
2. The acceleration of the body (vibrating) should be proportional to –ve of its displacement from mean position.
3. The acceleration should be directed towards mean position.
4. The system must be frictionless.

EXAMPLES OF S.H.M:

Following are a few examples of S.H.M.

1. Motion of the mass attached to a spring.
2. Motion of a string of sitar when plucked.
3. Motion of a pendulum.
4. Motion of the prongs of tuning fork when struck.
5. Motion of the membrane of a drum when beaten .

Q: A particle/body is moving in a circle, show that its projection performs S.H.M. Derive expression for its.

(1) Acceleration (2) Displacement (3) Time period and (4) Frequency.

Consider a point mass "m" moving in a circle of radius "x_o" with constant angular velocity "ω". As the point mass "m" rotates along the circumference of the circle, its projection Q moves back and forth along the diameter AB. Thus Q performs vibratory motion.

Let at any instance "t" the point mass be at P and its projection Q is at a distance "x" from O. The magnitude of centripetal acceleration of P is given by.

Here.

$$a_c = \frac{V^2}{r}$$

$$a_c = a_p$$

$$V = V_p$$

$$r = x_0$$

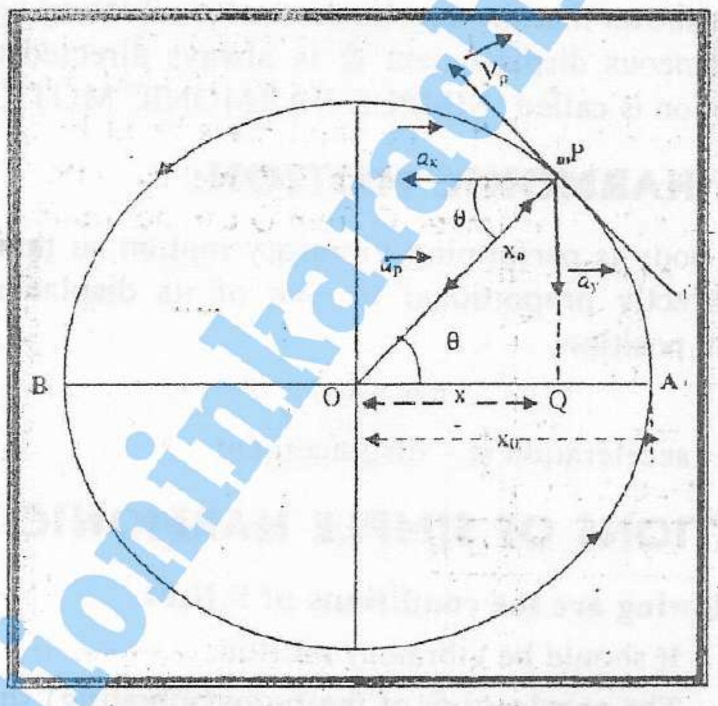
$$\therefore \quad a_p = \frac{V_p^2}{x_o}$$

But $\quad v_p = x_o \omega$

$$a_p = \frac{x_o^2 \omega^2}{x_o}$$

Or

$$\boxed{a_p = x_o \omega^2}$$



Resolve this acceleration into two components.

(i) $\quad a_p \cos\theta = x_o \omega^2 \cos\theta$ (Horizontal Component)

(ii) $\quad a_p \sin\theta = x_o \omega^2 \cos\theta$ (Vertical Component)

As the projection "Q" vibrates along the diameter AB about O, therefore acceleration will be equal to the horizontal component.

Acceleration of $Q = a_x = -x_o \omega^2 \cos\theta$

Negative sign is taken because it is directed towards the mean position.

$$\text{In } \Delta OQP, \quad \cos\theta = \frac{OQ}{OP} = \frac{x}{x_o}$$

$$\therefore \quad a_x = -x_o \omega^2 \frac{x}{x_o}$$

$$\boxed{a_x = -\omega^2 x} \longrightarrow (1)$$

As ω is constant, $\quad \therefore \omega^2 = \text{Constant}$

$$\therefore a_x = -(\text{Constant})x$$

$$\boxed{a_x \propto -x}$$

Thus acceleration of Q is proportional to displacement and is directed towards the mean position.

Hence motion of Q is S.H.M.

Equation (1) gives the "Instantaneous Acceleration" of Q.

MAXIMUM ACCELERATION:

The acceleration will be maximum "a_{max}" at extreme position where $x = x_o$ ∴ Eq (1) becomes

$$\boxed{a_{max} = -\omega^2 x_o}$$

MINIMUM ACCELERATION:

The acceleration will be minimum "a_{min}" at mean position where

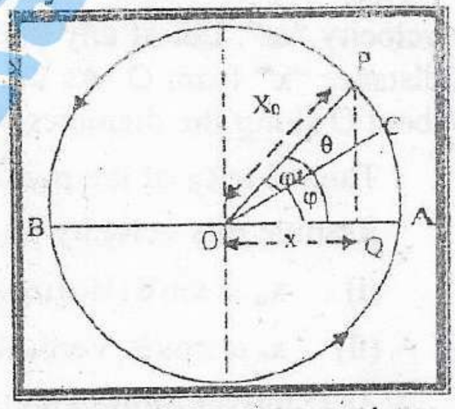
$x = o$ eq. (1) becomes $a_{min} = -(\omega^2)\, o$

$$\boxed{a_{min} = o}$$

DISPLACEMENT:

Suppose at time $t = 0$, the point mass is at C and the angle which the OC makes with the x-axis is "Φ" then it is known as "INTIAL PHASE ANGLE"

Let at any instant "t", the point mass be at P. The angle between OC & OP is "ωt" while the total angle which OP makes with x-axis is $\Phi + \omega t = \theta$. The displacement "x" of the image Q from O can be found by



Considering △OQP, here

$$\text{Cos } \theta = \frac{OQ}{OP} = \frac{x}{x_o}$$

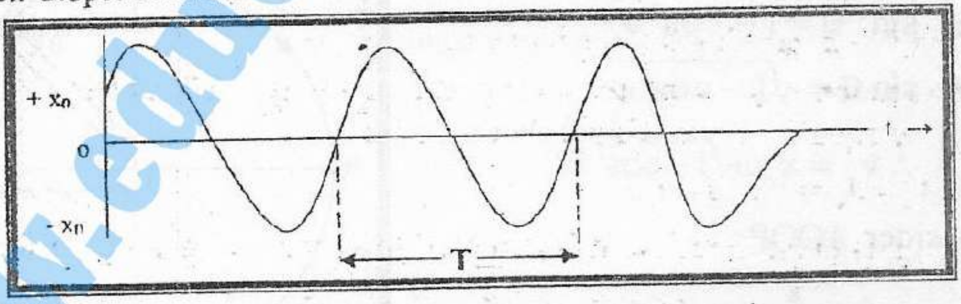
Or $\quad x = x_o \cos\theta$

But $\quad \theta = \Phi + \omega t$

∴ $\quad \boxed{x = x_o \cos(\Phi + \omega t)}$

"x" will be the position when displacement is to the right & negative when displacement is to the left of the mean position.

As the point mass circulates, displacement changes its direction, thus if a graph is plotted between displacement "x" & time "t" it will be a sinusoidal curve as shown in the figure.



TIME PERIOD:

The time in which the point mass "P" completes its one round of the circle, its projection "Q" completes one vibration.

The value of x at time t is equal to the value of x at time (t + T) and we know that the phase increase by 2π radians as t increases by T, then

$$\omega t + \phi + 2\pi = (t + T) + \phi$$

$$\omega T = 2\pi$$

$$\text{or} \quad \boxed{T = \frac{2\pi}{\omega}}$$

FREQUENCY:

The number of vibrations or rotations completed by a body in one second is its frequency. It is denoted by "υ". Frequency and time period are related by the formula $\upsilon = \frac{1}{T}$

Putting the value of "T" from eq. (2)

$$\upsilon = \frac{1}{\frac{2\pi}{\omega}}$$

$$\boxed{\upsilon = \frac{\omega}{2\pi}} \longrightarrow (3)$$

VELOCITY OF PROJECTION / PARTICLE / BODY EXECUTING S.H.M.

Consider a point mass "m" moving in a circle of radius "x_o" with constant angular velocity "ω". Let at any instant "t" the point mass be at P and its projection Q be at distance "x" from O. As the point mass circulates, its projection Q moves to and fro about O along the diameter AB. Hence, Q performs vibratory motion.

The velocity of the point mass at P is given by $v_p = x_o\,\omega$ $[v = r\omega]$

Resolve this velocity into two components.

(i) $x_o\,\omega\sin\theta$ (Horizontal Component parallel to the diameter)

(ii) $x_o\,\omega\cos\theta$ (Vertical Component perpendicular to the diameter)

As Q moves along x-axis therefore its velocity will be.

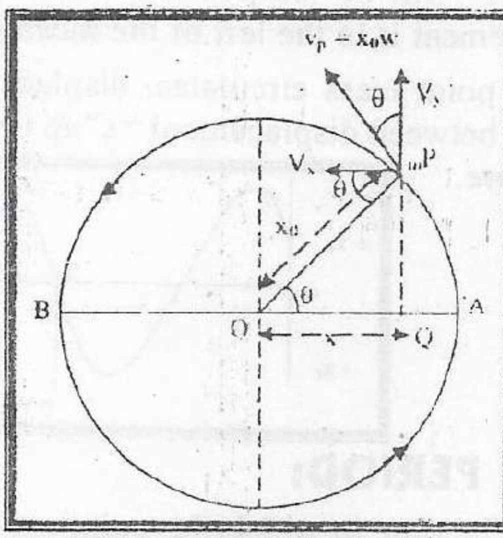
$$V_x = x_o\omega\sin\theta$$

As $\sin^2\theta + \cos^2\theta = 1$

Or $\sin^2\theta = 1 - \cos^2\theta$

Or $\sin\theta = \sqrt{1-\cos^2\theta}$

$$\therefore v_x = x_o\omega\sqrt{1-\cos^2\theta}$$



Consider ΔOQP

$$\cos\theta = \frac{x}{x_o}$$

$$v_x = x_o\omega\sqrt{1-\left(\frac{x}{x_o}\right)^2}$$

$$= x_o\omega\sqrt{1-\frac{x^2}{x_o^2}}$$

$$v_x = x_o\omega\sqrt{\frac{x_o^2 - x^2}{x_o^2}} = x_o\,\omega\frac{\sqrt{x_o^2 - x^2}}{x_o}$$

$$\boxed{v_x = \omega\sqrt{x_o^2 - x^2}} \longrightarrow (i)$$

Equation (i) gives the "INSTANTANEOUS VELOCITY" of Q

MAXIMUM VELOCITY:

The velocity of Q will be maximum at mean position where $x = o$

$\therefore$ Eq(i)becomes

$$v_{max} = \omega \sqrt{x_o^2 - o} = \omega \sqrt{x_o^2}$$

$$\boxed{v_{max} = x_o \omega}$$

MINIMUM VELOCITY:

The velocity of Q will be minimum at extreme position where $x = x_o$

$\therefore$ Eq (i) becomes.

$$V_{min} = \omega (x_o - x_o)^2 = \omega^2 \times o$$

$$\boxed{V_{min} = 0}$$

TIME PERIOD OF A MASS ATTACHED WITH SPRING:

If a body of mass "m" is attached to one end of an elastic spring and performs S.H.M. then.

$$a = -\frac{k}{m} x \longrightarrow (a)$$

Acceleration on the diameter of the projection of a point mass moving in a circle is given by. $a = -\omega^2 x \longrightarrow (b)$

Comparing eq. (a) & (b)

$$-\omega^2 x = -\frac{k}{m} x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}} \rightarrow (c)$$

The time period of a body performing S.H.M. is given by. $T = \frac{2\pi}{\omega}$

Putting the value of "ω" from equation (c) $T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$ $\boxed{\therefore T = 2\pi \sqrt{\frac{m}{k}}} \longrightarrow (d)$

FREQUENCY:

The frequency of the body is the reciprocal of its time period.

i.e. $\nu = \frac{1}{T}$

Putting the value of "T" from eq. (d)

$$\nu = \frac{1}{2\pi \sqrt{\frac{m}{k}}}$$

$$\boxed{\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}} \longrightarrow (e)$$

INSTANTANEOUS VELOCITY:

The instantaneous velocity of the body performing S.H.M. is given by.

$$V_x = \omega\sqrt{x_0^2 - x^2}$$

Putting the value of "ω" from eq. (c)

$$\boxed{V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}} \longrightarrow (e)$$

MAXIMUM VELOCITY:

The velocity will be maximum at mean position where $x = 0$

∴ eq. (e) becomes

$$V_{max} = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - (0)^2} = \sqrt{\frac{k}{m}}\sqrt{x_0^2}$$

$$\boxed{V_{max} = x_0\sqrt{\frac{k}{m}}} \longrightarrow (f)$$

MINIMUM VELOCITY:

The velocity will be minimum at extreme position where $x = x_0$

∴ eq (e) becomes

$$V_{min} = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x_0^2} = \sqrt{\frac{k}{m}} \times 0$$

$$\boxed{V_{min} = 0}$$

RELATIONSHIP BETWEEN INSTANTANEOUS AND MAXIMUM VELOCITY:

Equation (e) can be written as.

$$V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2\left(1 - \frac{x^2}{x_0^2}\right)}$$

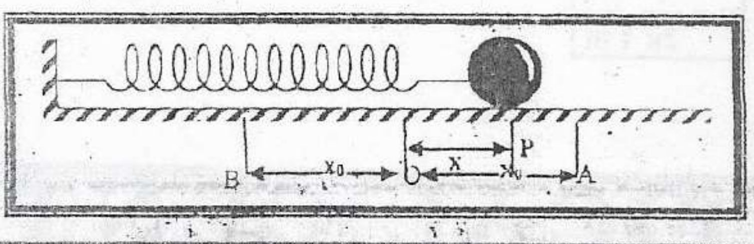
$$V_x = x_0\sqrt{\frac{k}{m}}\sqrt{1 - \frac{x^2}{x_0^2}}$$

But from equation (f) $x_0\sqrt{\frac{k}{m}} = V_{max}$

$$\therefore V_x = V_{max}\sqrt{1 - \frac{x^2}{x_0^2}}$$

ENERGY OF A BODY EXECUTING S.H.M:

Consider a body of mass "m" attached to one end of a helical spring whose other end is fixed. The whole system is placed on a horizontal frictionless surface. When the body is at equilibrium position O, the force acting on it is zero. When the body is moved to an extreme position A or B and then released, it moves to and fro around mean position. Let at any instant "t" the body be at P which is at a distance "x" from O.



INSTANTANEOUS KINETIC ENERGY:

The instantaneous speed of the body at distance "x" from the mean position is given by:

$$V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}$$

As $$K{\cdot}E = \frac{1}{2}mV_x^2$$

$$\therefore \quad K{\cdot}E = \frac{1}{2}m\left[\sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2}\right]^2$$

$$K{\cdot}E = \frac{1}{2}m \times \frac{k}{m} \times (x_0^2 - x^2)$$

$$\boxed{K{\cdot}E = \frac{1}{2}k\,(x_0^2 - x^2)}$$

MAXIMUM K·E:

The K·E of the body is maximum at mean position where x = 0

As $$K{\cdot}E. = \frac{1}{2}k(x_0^2 - x^2)$$

$$(K{\cdot}E.)_{max} = \frac{1}{2}k[x_0^2 - (o)^2]$$

$$\boxed{(K{\cdot}E.)_{max} = \frac{1}{2}kx_0^2} \longrightarrow (1)$$

MINIMUM K·E:

The K·E. of the body is minimum at two extreme position were $x = x_o$

As $$K{\cdot}E. = \frac{1}{2}k(x_0^2 - x^2)$$

$$(K{\cdot}E.)_{min} = \frac{1}{2}k\left[x_0^2 - x_0^2\right]$$

$$(K{\cdot}E.)_{min} = \frac{1}{2}k \times 0$$

$$\boxed{(K{\cdot}E.)_{min} = 0}$$

INSTANTANEOUS P·E:

When the body is at mean position where x = 0

$$F_1 = k\,x$$

$$F_1 = k \times 0$$

$$\boxed{F_1 = 0}$$

When the body is at displacement 'x' from the mean position $F_2 = Kx$

Average force on the body

$$Fav = \frac{F_1 + F_2}{2}$$

$$Fav = \frac{0 + kx}{2}$$

$$Fav = \frac{1}{2}kx$$

Work done in moving the body through a displacement 'x' is given by

$$\text{Work} = (\text{Force})(\text{Displacement})$$

$$\text{Work} = \left(\frac{1}{2}kx\right)(x)$$

$$\text{Work} = \frac{1}{2}kx^2$$

This work is stored in the body as its P·E

$$\boxed{P \cdot E = \frac{1}{2}kx^2}$$

MAXIMUM P·E:

The P·E of the body is maximum at two extreme position where $x = x_o$

$$\text{As} \quad P \cdot E \ \frac{1}{2}kx^2$$

$$\boxed{(P \cdot E)_{max} \ \frac{1}{2}kx_o} \longrightarrow \quad (2)$$

MINIMUM P·E:

The P·E of the body is minimum at mean position where $x = o$

$$\text{As} \quad P \cdot E \ \frac{1}{2}kx^2$$

$$(P \cdot E)_{min} \ \frac{1}{2} K \times 0$$

$$\boxed{(P \cdot E)_{min} = 0}$$

The total energy of the body at any point can be found by adding its P.E & K.E at that point.

Thus total energy at P= P.E at P + K.E at P

$$= \frac{kx^2}{2} + \frac{kx_o^2}{2} - \frac{kx^2}{2}$$

$$= \frac{1}{2}kx_o^2 \longrightarrow \quad (3)$$

From equations (1), (2), & (3) it can be seen that the total energy of the body executing S.H.M remains constant.

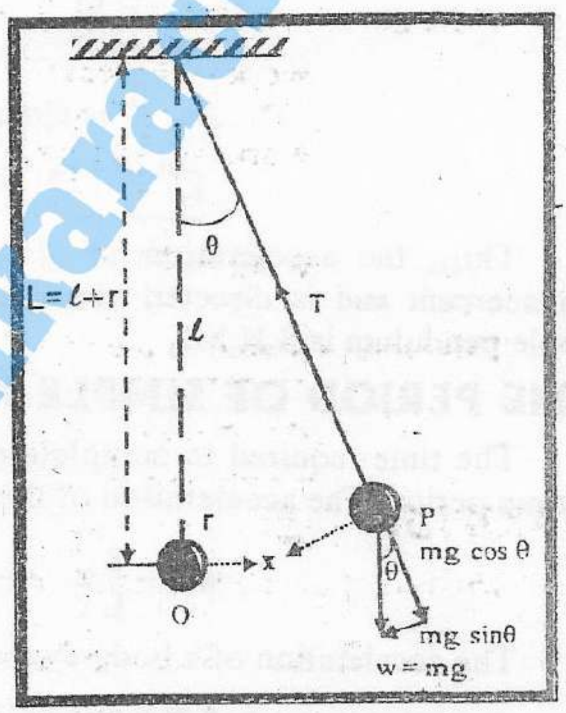
SIMPLE PENDULUM:

An ideal simple pendulum consists of a spherical bob suspended from a light, flexible and inextensible string tied to a rigid and frictionless support. When the bob is displaced from its equilibrium position, it begins to perform oscillatory motion.

Consider a simple pendulum of length "L" consisting of a bob of mass "m". If the pendulum is disturbed from its mean position it starts vibrating between two extreme positions. Let at any instant "t" the bob be at point P and at this point two forces are acting on the bob.

1. The force of gravity = $F_g = mg$

2. Tension in the string = T



the force of gravity is the weight of the body and can be resolved into two components.

(i) $(F_g)_{11} = mg \cos\theta$, along the string

(ii) $(F_g)_\perp = mg \sin\theta$, perpendicular to the string.

Since there is no motion along the string

$\therefore \; T = mg \cos\theta$

The only force responsible for the motion of the bob is mg sin θ as this force is directed towards the mean position i.e acts as a restoring force, therefore we shall assign a negative sign to it.

$\therefore \quad F = -mg \sin\theta \quad (1)$

If "a" be the acceleration of the bob at P, then by Newton's second law of motion.

$F = ma \longrightarrow \quad (2)$

Comparing eq. (1) & (2)

$ma = -mg \sin\theta$

$a = -g \sin\theta$

If "θ" is small then $\sin\theta \cong \theta$

$\therefore \quad a = -g\,\theta$

But

$$\theta \text{ (in radian)} = \frac{\text{length of arc}}{\text{radius}}$$

$$\theta = \frac{x}{L}$$

Then $$a = -g\frac{x}{L}$$

$$\boxed{a = -\frac{g}{L}x}$$

As $\frac{g}{L}$ is Constant

$$\therefore \boxed{a \propto -x}$$

Thus, the acceleration of simple pendulum is directly proportional to its displacement and is directed towards its mean position. Hence the motion of the simple pendulum is S.H.M.

TIME PERIOD OF SIMPLE PENDULUM:

The time required to complete one oscillation by a simple pendulum is called its time period. The acceleration of the simple pendulum is given by.

$$a = -\frac{g}{L}x \longrightarrow \text{(i)}$$

The acceleration of a body executing S.H.M is given by.

$$a = -\omega^2 x \longrightarrow \text{(ii)}$$

Comparing equation (i) & (ii) we get

$$\omega^2 = \frac{g}{L}$$

$$\omega = \sqrt{\frac{g}{L}} \longrightarrow \text{(iii)}$$

The time period of a body executing S.H.M is given by.

$$T = \frac{2\pi}{\omega}$$

Putting the value of "ω" from e.g. (iii)

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}}$$

$$\boxed{\therefore T = 2\pi\sqrt{\frac{L}{g}}}$$

This shows that the time period of a simple pendulum depends upon its length and acceleration due to gravity and it is independent of the mass of the bob.

FREQUENCY OF SIMPLE PENDULUM:

Frequency is reciprocal of time period i.e.

$$\nu = \frac{1}{T}$$

But $$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\therefore \nu = \frac{1}{2\pi\sqrt{\frac{L}{g}}}$$

$$\boxed{\upsilon = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$$

GENERAL FORMULA FOR THE TIME PERIOD:

The general formula for time period of a body performing S.H.M is

$$\boxed{T = \frac{2\pi}{\text{Constant}}} \;\ldots\ldots$$

1. In the case of circle the constant is ω^2

$$\therefore T = \frac{2\pi}{\sqrt{\omega^2}} \qquad \left\{\begin{array}{l} a = -\omega^2 x \\ a = -(\text{Constant})x \end{array}\right\}$$

$$T = \frac{2\pi}{\omega}$$

2. In the case of the body attached to an elastic spring, the constant is $\frac{k}{m}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{k}{m}}} \qquad \left\{\begin{array}{l} a = -\frac{k}{m}x \\ a = -(\text{Constant})x \end{array}\right\}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In the case of simple pendulum, the constant is $\frac{g}{L}$

$$\therefore T = \frac{2\pi}{\sqrt{\frac{g}{L}}} \qquad \left\{\begin{array}{l} a = -\frac{g}{L}x \\ a = -(\text{constant})x \end{array}\right\}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Second's Pendulum:

A pendulum, which completes one vibration in two seconds, is called second's pendulum.

Its time period = T = 2 Seconds

Frequency = $\upsilon = \frac{1}{T} = \frac{1}{2}$ = 0.5Hz

WAVES

WAVE PULSE

A single isolated disturbance which travels through a medium or space is known as a "WAVE PULSE"

TRAVELLING OR PROGRESSIVE WAVES:

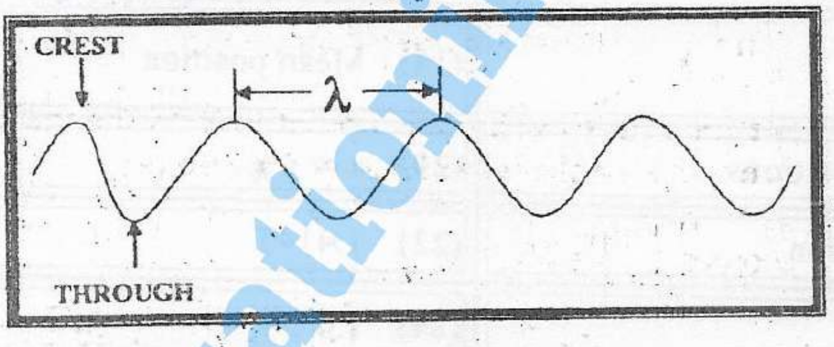
The waves which changes their position with the passage of time are called "travelling or progressive waves" e.g. water waves, sound waves, heat waves, light waves", etc.

TRANSVERSE WAVES:

Transverse waves are those in which particles of a medium vibrate perpendicular to the direction of propagation of waves. These waves consist of crests and troughs e.g. water waves, waves in string, electromagnetic waves etc. are all transverse waves.

CREST:

The portion of the transverse wave which is higher than the equilibrium position of the medium is called a crest.



TROUGH:

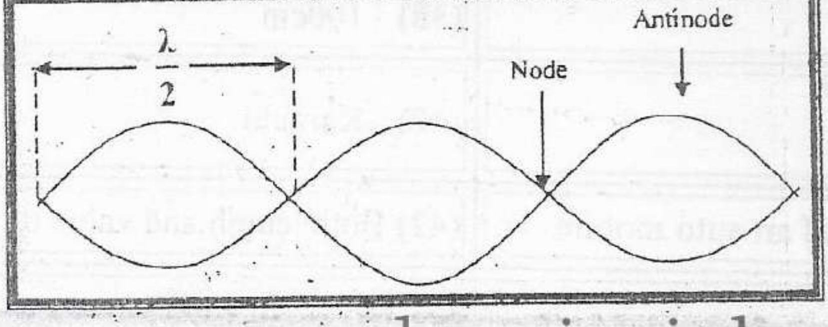
The position of the transverse wave which is lower than equilibrium position of the medium is called a trough.

STATIONARY WAVES OR STANDING WAVES:

When two similar waves move along the same line in a medium but in opposite direction these waves will combine according to the principle of super position and special kind of waves are formed, called stationary waves or standing waves.

NODE:

The point of a standing wave where displacement is minimum or zero is called a "node"



ANTINODE:

The point of a standing wave where displacement is maximum is called an "Antinode"

The distance between two consecutive nodes or antinodes is equal to half of the wavelength. $\left(\lambda/_2\right)$

The distance between a node and an antinode is equal to quarter of the wave length. $\left(\lambda/_4\right)$

WAVE LENGTH:

The distance between any two consecutive points on a wave that behave identically is called wavelength. Wave length can also be defined as the distance between two consecutive crests or troughs, compressions or rarefactions. It is denoted by "λ"

TIME PERIOD:

The time required to complete one vibration or one rotation is called "time period". It is denoted by T.

FREQUENCY:

The number of vibrations or rotations completed by a body in one second is called its frequency. It is denoted by "ν"

The frequency and time period are related by $\nu = \frac{1}{T}$

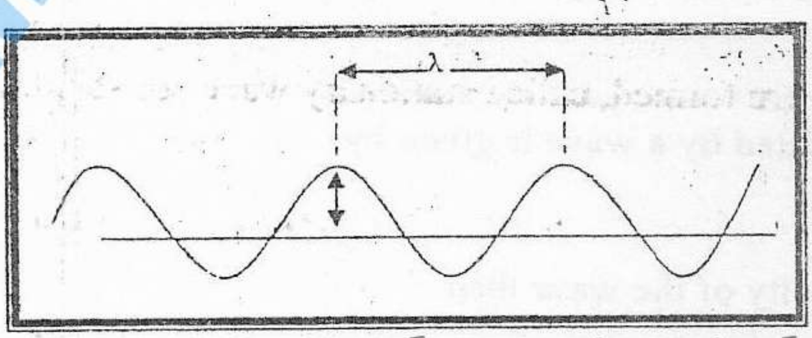
Consider a wave train passing through a point P. If "ν" be the frequency and V be the velocity of wave and λ be the wave length of wave then.

$S = Vt$ Becomes $\lambda = VT$

$$V = \frac{\lambda}{T}$$

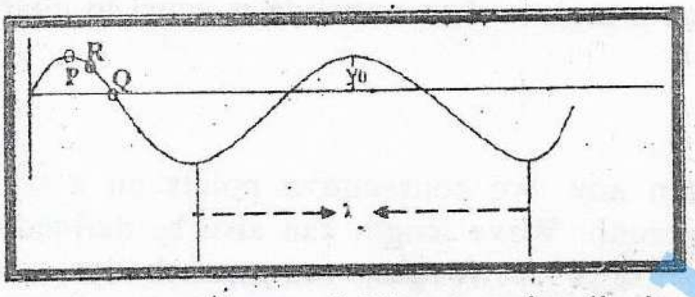
$$V = \frac{1}{T}\lambda$$

$$V = \nu\lambda$$



ENERGY IN WAVES:

A wave travelling through a medium carries energy with it which can be found by considering a wave produced in a string as shown in the figure. The points P, Q& R represent various segments of the wave.



The energy of P is entirely potential because its displacement from the mean position is maximum. The energy of the segment Q is entirely kinetic because its velocity is maximum. The energy of the segment R is partially potential and partially kinetic.

Consider the segment Q, if its velocity is V_{max} then its energy "ΔE" is given by. $\Delta E = \frac{1}{2} \Delta m\, V^2_{max}$

Where Δm = mas of the segment. Q

If "y_o be amplitude & "ω" be the frequency then $V_{max} = y_o\omega \quad \because V = r\omega$

$$\therefore \Delta E = \frac{1}{2} \Delta m \ (y_o\omega)^2$$

If "m" be the mass of whole wave then its energy "E" is given by.

$$E = \frac{1}{2} m \ y_o^2\omega^2$$

If "μ" be the linear density of the wave of string then $\mu = \frac{m}{\lambda}$

Or $m = \mu\lambda$

$$\therefore \boxed{E = \frac{1}{2}\mu\lambda y_o^2\omega^2}$$

where λ = wave length.

POWER TRANSMITED BY WAVE:

Energy transmitted in one second is called power i.e.

$$\text{Power} = \frac{\text{Energy}}{\text{Time}}$$

Power "P" transmitted by a wave is given by

$$P = \frac{E}{T} = \frac{1}{2}\frac{\mu\lambda y_o^2\omega^2}{T}$$

If "V" be the velocity of the wave then.

$$\frac{\lambda}{T} = V$$

$$\therefore \boxed{P = \frac{1}{2}\mu \ \ V \ \ y_o^2\omega^2}$$

The above relation shows that the power transmitted by any harmonic wave mechanical or electromagnetic) is proportional to

1. The square of the frequency
2. The square of the amplitude

ANALYTICAL TREATMENT OF STANDING WAVES:

Consider two sinusoidal waves with the same amplitude, frequency and wave length but travelling in opposite directions. These waves can be written as.

$y_1 = A_o \sin(kx - \omega t)$

$y_2 = A_o \sin(kx + \omega t)$

Where

y_1 = Displacement of wave traveling to the Right.

y_2 = Displacement of wave traveling to the Left (Reflected wave).

If

y = Resultant wave function then

$y = y_1 + y_2$

$y = A_o \sin(kx - \omega t) + A_o \sin(kx + \omega t)$

$y = A_o \{\sin(kx - \omega t) + \sin(kx + \omega t)\}$

As we know that

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

So

$$y = A_o 2\sin\left(\frac{kx - \omega t + kx + \omega t}{2}\right)\cos\left(\frac{kx - \omega t - kx - \omega t}{2}\right)$$

$y = (2A_o \sin kx)\cos\omega t$

Where $2A_o \sin kx$ = Amplitude of standing wave for any value of x.

POSITIONS OF ANTI NODES:

There will be Anti nodes when maximum amplitude is equals to 2A.

It happens when

$\sin kx = 1$

i.e. $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots, \frac{n\lambda}{4}$

$n = 1, 3, 5, \ldots\ldots$

POSITIONS OF NODES:

There will be Nodes when maximum amplitude is equals to zero. It happens when.

$\sin kx = 0$

i.e. $kx = \pi, 2\pi, 3\pi, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots, \frac{n\lambda}{2}$

Where $n = 1, 2, 3, \ldots\ldots$

This shows that two successive Nodes are separated by $\frac{\lambda}{2}$ and Anti nodes and Nodes are separated by $\frac{\lambda}{4}$.

ANALYTICAL TREATMENT OF STANDING WAVES:

Consider two sinusoidal waves with the same amplitude, frequency and wave length but travelling in opposite directions. These waves can be written as.

$y_1 = A_o \sin(kx - \omega t)$

$y_2 = A_o \sin(kx + \omega t)$

Where

y_1 = Displacement of wave traveling to the Right.

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If

y = Resultant wave function then

$y = y_1 + y_2$

$y = A_o \sin(kx - \omega t) + A_o \sin(kx + \omega t)$

$y = A_o \{\sin(kx - \omega t) + \sin(kx + \omega t)\}$

As we know that

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

So

$$y = A_o 2\sin\left(\frac{kx - \omega t + kx + \omega t}{2}\right)\cos\left(\frac{kx - \omega t - kx - \omega t}{2}\right)$$

$y = (2A_o \sin kx) \cos \omega t$

Where $2A_o \sin kx$ = Amplitude of standing wave for any value of x.

POSITIONS OF ANTI NODES:

There will be Anti nodes when maximum amplitude is equals to 2A.

It happens when

$\sin kx = 1$

i.e. $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots, \frac{n\lambda}{4}$

$n = 1, 3, 5, \ldots\ldots$

POSITIONS OF NODES:

There will be Nodes when maximum amplitude is equals to zero. It happens when.

$\sin kx = 0$

i.e. $kx = \pi, 2\pi, 3\pi, \ldots\ldots$

Since $k = \frac{2\pi}{\lambda}$

So $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots\ldots, \frac{n\lambda}{2}$

Where $n = 1, 2, 3, \ldots\ldots$

This shows that two successive Nodes are separated by $\frac{\lambda}{2}$ and Anti nodes and Nodes are separated by $\frac{\lambda}{4}$.

TRANSVERSE STATIONARY WAVES IN ASTRETCHED STRING

Consider a stretched string of length "L" fixed at both ends as shown in fig (1). If the string is plucked upward or downward then stationary waves will be produced in it. The string has a number of natural patterns of vibrations called "natural modes" each of these modes has a characteristic frequency called "FUNDAMENTAL FREQUENCY" or the FIRST HARMONIC. The higher frequencies are called OVERTONES.

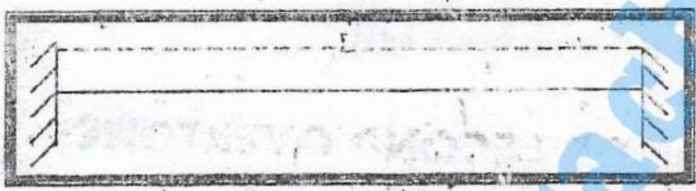

Fig (1)

FUNDAMENTAL FREQUENCY OR FIRST HARMONIC:

If the string is displaced at its middle point and released, it vibrates in the one loop, as shown in fig (2). In this case an antinode is formed at the middle while node at the ends of the string. If λ_1 is the wave length of wave formed then it is given by.

$$\frac{\lambda_1}{2} = L \text{ Or } \lambda_1 = 2L \quad (1)$$

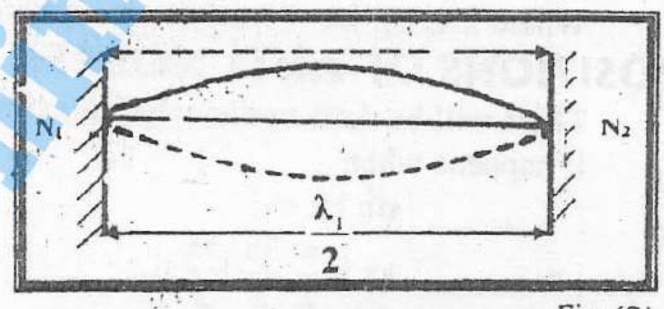

Fig (2)

If v_1 be the fundamental frequency then

$$V = v_1\lambda_1$$

$$\upsilon_1 = \frac{V}{2L} \longrightarrow (2)$$

The velocity "V" of the wave depends upon the tension in the string and linear density of the string.

If 'm' is the total mass of the string, then the velocity of the wave along the string is given by

$$V = \sqrt{\frac{TL}{m}}$$

$$\frac{m}{L} = \mu = \text{Linear density}$$

$$V = \sqrt{T/\mu}$$

$$\upsilon_1 = \frac{\sqrt{T/\mu}}{2L}$$

$$\upsilon_1 = \frac{1}{2L}\sqrt{T/\mu}$$

SECOND HARMONIC OR FIRST OVERTONE:

If the string is plucked at one quarter of its length it will vibrate in two loops, as shown in the figure (3). In this case there are two antinodes and three nodes. If λ_2 be the length of the wave formed then.

$$\frac{\lambda_2}{2} + \frac{\lambda_2}{2} = L$$

$$\frac{2\lambda_2}{2} = L$$

$$\lambda_2 = \frac{2L}{2} \longrightarrow (3)$$

If "ν_2" be the frequency of the vibrating string then. $\nu_2 = \dfrac{V}{\lambda_2}$

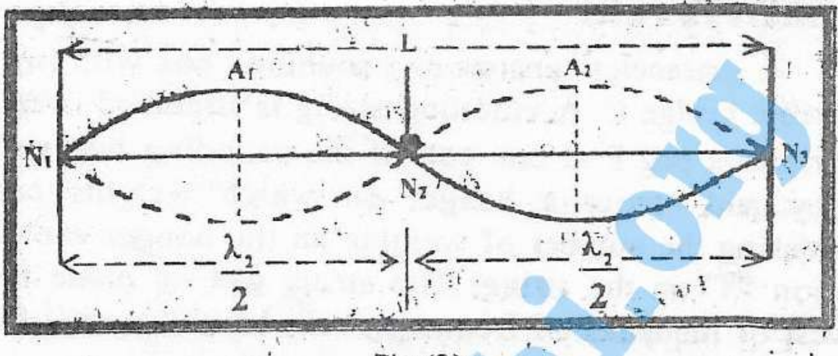

Fig (3)

Or $\quad \upsilon_2 = \dfrac{V}{\dfrac{2L}{2}}$

$$\upsilon_2 = \frac{2V}{2L}$$

But from eq. (2) $\upsilon_1 = \dfrac{V}{2L}$

$$\boxed{\therefore\ \upsilon_2 = 2\upsilon_1 \rightarrow (4)}$$

THIRD HARMONIC OR SECOND OVERTONE:

If the same string is plucked from a suitable place, it can vibrate in three loops as shown in fig (4). In this case there are three antinodes and four nodes. The length of the string will be.

$$\frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} = L$$

$$\frac{3\lambda_3}{2} = L$$

$$\lambda_3 = \frac{2L}{3} \rightarrow (5)$$

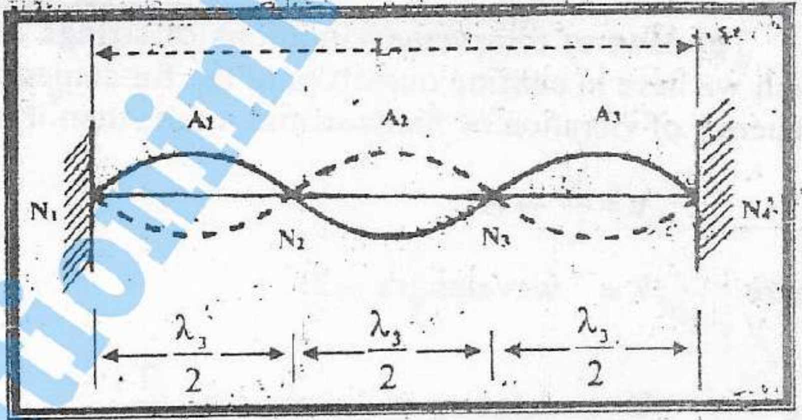

Fig (4)

If "υ_3" be the frequency of the vibrating string then.

$$\upsilon_3 = \frac{V}{\lambda_3}$$

$$\upsilon_3 = \frac{V}{\dfrac{2L}{3}}$$

$$\upsilon_3 = \frac{3V}{2L}$$

But from eq (2) $\quad \dfrac{V}{2L} = \upsilon_1$

$$\therefore \quad \boxed{\upsilon_3 = 3\upsilon_1} \longrightarrow \mathbf{(6)}$$

$$\upsilon_4 = 4\upsilon_1$$

$$\upsilon_5 = 5\upsilon_1$$

FOR n LOOPS:

If the string vibrates in n loops then like equation (1), (3) & (5) we can write

$$\upsilon_n = n\upsilon_1$$

Where n = 1, 2, 3, 4,..........