

Descriptive Questions and Answers

Q1. Discuss in brief the condition of South Asia before the advent of Islam and write in detail the influence of Islam on South Asian Society.

Ans. Introduction:

Pakistan, our dear mother land, came into existence vitally due to the advent of Islam in South Asian region. Islam came to this region through Arab traders. Regular preaching of Islam started after the invasion of Muhammad Bin Qasim, who defeated Raja Dahir in 712.

Condition of South Asia before the advent of Islam:

Before the advent of Islam South Asian society was divided into many small kingdoms who were always at daggers drawn with each other. The Hindus were in majority and they kept on torturing other minorities like Buddhist. The Hindu society was full of social injustice. The following were the discrepancies of this society.

- i. They were divided into four major castes
 - a) Brahmins (The priests)
 - b) The Kashatriyas (warrior class)
 - c) The Vashiya (the artists)
 - d) The Shudras (Lowest worker class)
- ii. The Barhamins and Kashatriyas enjoyed power and status and the other two classes were in miserable condition.
- iii. Narrow mindedness had plagued Hindu society.
- iv. Women were given the lowest status.
- v. Widows were treated badly, mostly they were burned with husband's dead body (sutti)

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Impact of Islam:

i. Civilizational Influence:

Muslim culture imbibed the culture characteristics of the entire civilized world so Muslim civilization held an upper status which changed the inferior and backward culture of South Asia and it brought many significant changes in the culture of South Asia.

ii. Political influence:

- a) Muslims laid the foundation of strong central government and thus brought an end to lawlessness.
- b) .With political unity, administrative equality was also introduced, which created Social equality.
- c) As Muslim conquerers were a part of bigger Islamic Society so they connected South Asia to the outer world.
- d) The new military strategies, disciplines, and ways of fighting were introduced. Naval forces were introduced for the first time.

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Preparation Paper 2019-2020

PHYSICS – X

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Chapter # 1 (Introduction To Physics)

- Q.1. Define Physics and its branches.
Q.2. Define scientific method with steps.
Q.3. Define Error. What are the different types of Error?
Q.4. Write down the contribution of:
(a) Yaqub-Al-Kindi (b) Ibn-Al Haitham
(c) Dr. Abdul Qadeer Khan (d) Al-Beruni

Chapter # 2 (Measurement)

- Q.1. Define standard of:
(a) Mass (b) Length (c) Time
Q.2. Define:
(a) Least count (b) Significant figure (c) Accuracy
Q.3. Write down some derived quantities with their units and symbols.
Q.4. Advantages of S.I units.
Q.5. Conversions of different units of mass, length and time.

Solved examples: 2.4**Important Problems:** 2.2, 2.3, 2.4

Chapter # 3 (Kinematics Of Linear Motion)

- Q.1. Difference between
(a) Scalar quantity & Vector quantity.
Q.2. Define:
(a) Kinematics (b) Mechanics (c) Dynamics
(d) Distance (e) Displacement (f) Speed
(g) Velocity (h) Acceleration
Q.3. Derive the second and third equation of motion.
Q.4. Define motion and its types.
Q.5. State Newton's First, Second and Third law of motion.

Solved examples: 38, 40, 41**Important Problems:** 3.1, 3.3, 3.6, 3.9, 3.11, 3.12 3.13

Chapter # 4 (Force And Motion)

- Q.1. State Newton's Laws of Motion (1st, 2nd, 3rd)
Q.2. Define tension. Motion of two bodies connected with string (case I and II) find acceleration and tension.
Q.3. What is Momentum? Also state law of Conservation of Momentum.
Q.4. Define friction. Write down its expression and on what factors it depends?
Q.5. Write down the different types of friction. Also write advantages and disadvantages of friction.
Q.6. Write some methods of reducing friction.

Solved examples: 4.2, 4.5**Important Problems:** 4.2, 4.3, 4.5, 4.6, 4.7

SCALARS:

Physical quantities, having magnitude only but no direction, are called Scalars. Scalars are completely described by

- i) a number
- ii) a suitable unit

Scalars can be added, subtracted, multiplied and divided by simple arithmetical rules. For example: mass, distance, time, speed, temperature, energy, work, volume and density etc.

VECTORS:

Physical quantities, having both magnitude and direction and also obeying commutative law of vector addition, are called vectors. Vectors are completely described by

- i) a number
- ii) a suitable unit
- iii) a certain direction

Vectors can not be added, subtracted, multiplied and divided by simple arithmetical rules. For example: displacement, velocity, acceleration, force, angular velocity, torque, weight, electric field strength etc.

Representation of a Vector:

A vector is represented graphically by a directed line segment or an arrowhead line segment. The length of the line, according to the scale chosen, represents the magnitude of the vector while the arrowhead indicates the direction. To represent a vector, we need:

1. A suitable scale.
2. Reference axes i.e. x, y and z-axis, or horizontal and vertical directions or the directions of north, south, east and west.

Types of Vectors:

- | | | |
|--------------------|----------------|--------------------|
| 1) Unit vector | 2) Free vector | 3) Position vector |
| 4) Negative vector | 5) Null vector | |

1) Unit Vector:

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A unit vector is that whose magnitude is unity i.e. equal to 1 and it represents the direction of a vector only.

A unit vector can be obtained by dividing the vector with its magnitude i.e.

$$\text{unit vector} = \frac{\text{Vector}}{\text{magnitude of vector}}$$

If unit vector of a vector $\vec{A}$ is $\hat{a}$, then

$$\hat{a} = \frac{|\vec{A}|}{|\vec{A}|} = \frac{\vec{A}}{A}$$

Unit vectors along x, y and z-axis are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

Rectangular Components of a Vector in Terms of Unit Vectors:

Consider a vector $\vec{A}$, starting from the origin "O" of a rectangular coordinate system, as shown in the figure.

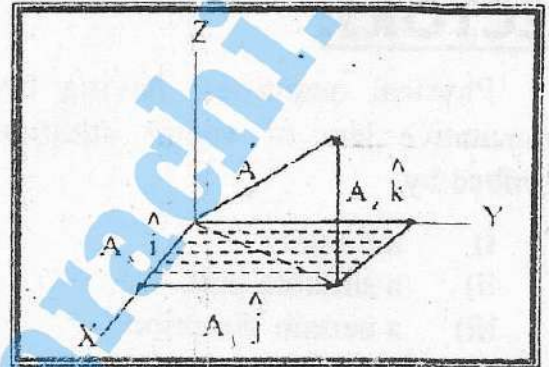
Its rectangular components, in the direction of positive x, y and z-axis, are $A_x \hat{i}$, $A_y \hat{j}$ and $A_z \hat{k}$ respectively.

Conversely, the sum of rectangular component vectors produces the original vector $\vec{A}$ i.e.

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

The magnitude of $\vec{A}$ is given by the formula:

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



2) Free Vector:

A vector, which can be displaced parallel to itself and applied at any point, is said to be a free vector.

3) Position Vector:

A vector, which represents the position of a point with reference to a fixed point is called **POSITION VECTOR**.

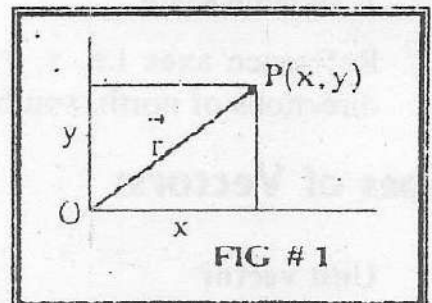
If the fixed point is taken as origin O and the coordinates of point P are x and y as shown in fig (1) then, by Pythagoras theorem, the magnitude of position vector $\vec{r}$ is given by:

$$r^2 = x^2 + y^2$$

$$|\vec{r}| = r = \sqrt{x^2 + y^2}$$

In three dimensional rectangular coordinate system,

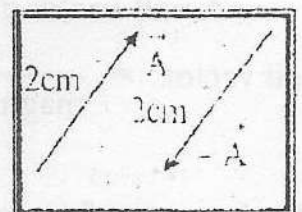
$$|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$$



The components of a position vector are known as "Co-ordinates".

4) Negative Vector:

A vector, having the same magnitude as that of the given vector but opposite in direction, is called a negative vector of the given vector. e.g. Negative vector of a vector is as shown in the figure.



5) Null Vector: www.educationinkarachi.org

A vector, whose magnitude is zero and has no direction or it may have all directions, is said to be a **NULL VECTOR**. It is denoted by $\vec{0}$.

A null vector can be obtained by adding two or more vectors.

Direction Cosines:

If $\vec{r}$ makes angle α , β & γ with x , y , and z axes respectively, then

$$\cos \alpha = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos \beta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos \gamma = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

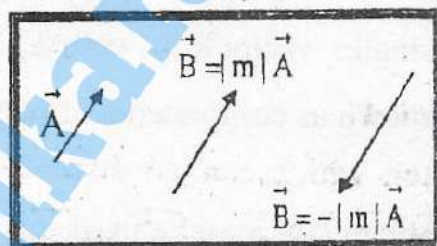
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Multiplication of a Vector by a Number:

When vector $\vec{A}$ is multiplied by a number "m", a new vector is generated whose magnitude is $|m|$ times the magnitude of $\vec{A}$. If $\vec{B}$ is a new vector, then its magnitude is given by

$$|\vec{B}| = |m| |\vec{A}|$$

The direction of $\vec{B}$ is the same as that of vector $\vec{A}$ if "m" is positive.



The direction of vector $\vec{B}$ is opposite to that of $\vec{A}$ if "m" is negative.

The multiplication of a vector, by one or more numbers (say m , n), is governed by the following rules:

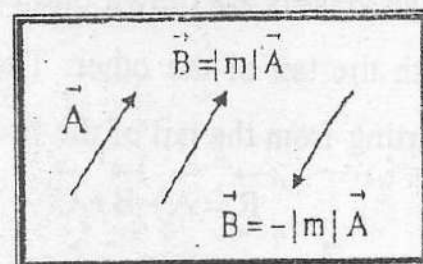
1. $\vec{A} m = m \vec{A}$ Commutative law for multiplication.
2. $m(n \vec{A}) = (mn) \vec{A}$ Associative law for multiplication.
3. $(m+n) \vec{A} = m \vec{A} + n \vec{A}$ Distributive law.
4. $m(\vec{A} + \vec{B}) = m \vec{A} + m \vec{B}$ Distributive law.

Division of a Vector by a Number (Non-Zero):

When a vector $\vec{A}$ is divided by a number "n", a new vector is generated whose magnitude is $\left|\frac{1}{n}\right|$ times the magnitude of $\vec{A}$. If $\vec{B}$ is a new vector then its magnitude is

given by $|\vec{B}| = \left|\frac{1}{n}\right| |\vec{A}|$. If we write $\left|\frac{1}{n}\right| = m$, then

$$|\vec{B}| = |m| |\vec{A}|$$



The direction of the new vector $\vec{B}$ is the same as that of $\vec{A}$ if "n" is positive.

The direction of the new vector $\vec{B}$ is opposite to that of $\vec{A}$ if "n" is negative.

Addition of Vectors by Geometrical Method:

There are three ways of addition of vectors by geometrical method.

1. Law of Parallelogram of vectors.
2. Law of Triangle of vectors.
3. Head-to-Tail Rule:

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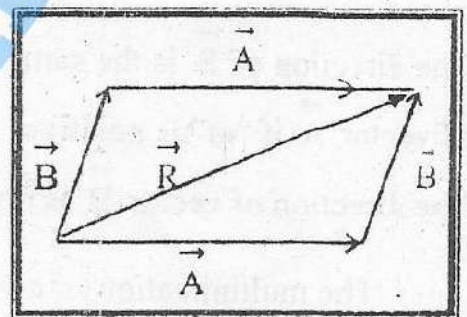
1. The Law of Parallelogram of Vectors:

According to this law, "if two vectors $\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by two sides of a parallelogram drawn from a point, their resultant $\vec{R}$ will be completely represented by the diagonal passing through the same point.

The single vector $\vec{R}$ is called the resultant of vector $\vec{A}$ and $\vec{B}$. The vectors $\vec{A}$ and $\vec{B}$ are known as components of resultant vector $\vec{R}$.

The parallelogram law may be used to find the resultant of any two vectors. In the above figure,

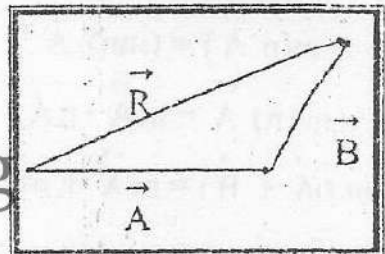
$$\vec{R} = \vec{A} + \vec{B}$$



Law of Triangle of Vectors:

If $\vec{A}$ and $\vec{B}$ are two sides of a triangle taken in order, then their resultant $\vec{R}$ will be represented by the third side taken in opposite order. i.e.

$$\vec{R} = \vec{A} + \vec{B}$$

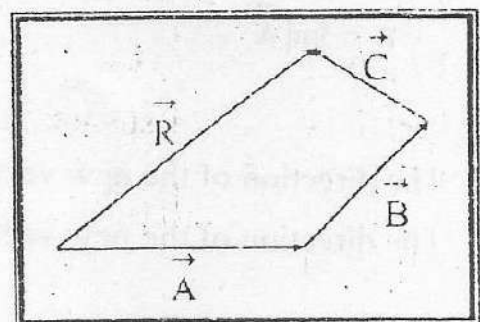


Head-to-Tail Rule:

Two or more vectors can be added by head-to-tail rule. According to the rule, "The vectors are drawn one by one in such a way that the head of one vector coincides with the tail of the other. The resultant is given in magnitude and direction by a line starting from the tail of the first vector to the head of the last vector". i.e.

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

Where $\vec{R}$ is the resultant of vector $\vec{A}$, $\vec{B}$ and $\vec{C}$.



The magnitude and the direction of the resultant vector can be found by scale & protector.

The magnitude and the direction of the resultant vector can also be found by using trigonometric formula i.e. law of cosine and law of sine.

LAW OF COSINE:

In a triangle ABC, the side opposite to $\angle A = a$, the side opposite to $\angle B = b$ and the side opposite to $\angle C = c$, as the shown in figure:

$$a^2 = b^2 + c^2 - 2bc \cos \angle A$$

$$b^2 = a^2 + c^2 - 2ac \cos \angle B$$

$$c^2 = a^2 + b^2 - 2ab \cos \angle C$$

These equations are known as "Law of Cosine".

If this law is applied on triangle OPS, we get:-

$$R^2 = A^2 + B^2 - 2AB \cos \angle OPS$$

OR $R = \sqrt{A^2 + B^2 - 2AB \cos \angle OPS}$

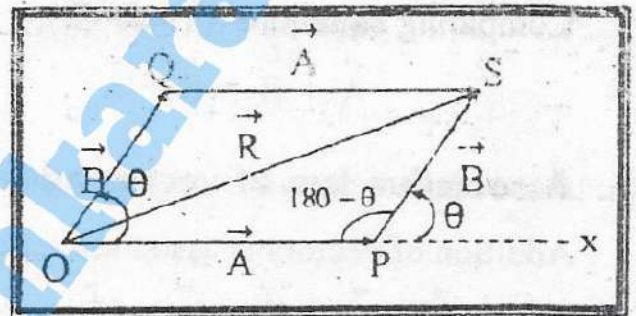
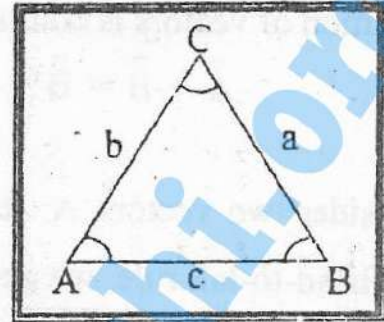
If $\angle SPX = \theta$ then

$$\angle OPS = 180 - \theta$$

$$\cos \angle OPS = \cos(180 - \theta) = -\cos \theta$$

Thus equation (1) becomes.

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$



LAW OF SINE:

In triangle ABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

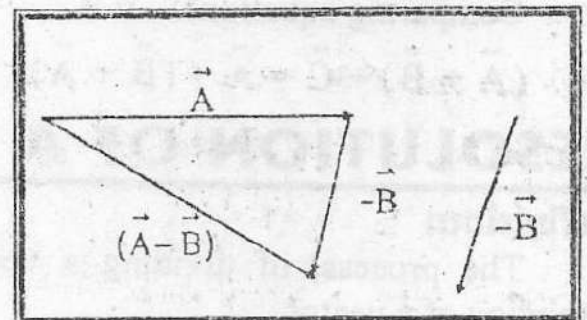
In triangle OPS,

$$\frac{R}{\sin \angle OPS} = \frac{A}{\sin \angle OSP} = \frac{B}{\sin \angle POS}$$



Subtraction of Vectors:

Consider two vectors $\vec{A}$ and $\vec{B}$. If we want to subtract vector $\vec{B}$ from vector $\vec{A}$, then it can be done by making vector (i.e. $-\vec{B}$) and adding it with vector $\vec{A}$. The resultant is $(\vec{A} - \vec{B})$, as shown in the figure.



PROPERTIES OF VECTOR ADDITION:

1. Commutative law of vector addition:

Addition of vectors is commutative i.e.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Proof:-

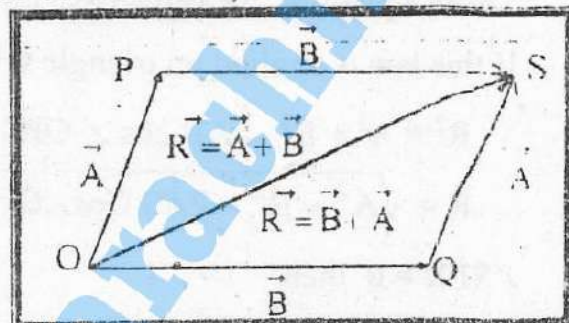
Consider two vectors $\vec{A}$ & $\vec{B}$ which can form a parallelogram. If we add these vectors by head-to-tail rule, we get a resultant vector $\vec{R}$. In ΔOPS ,

$$\vec{R} = \vec{A} + \vec{B} \text{ -----(1)}$$

$$\text{In } \Delta OQS \quad \vec{R} = \vec{B} + \vec{A} \text{ -----(2)}$$

Comparing equations (1) & (2),

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



2. Associative law of vector addition:

Addition of vectors is associative i.e.

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

Proof:

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. If we add $\vec{A}$ & $\vec{B}$ vectors by head-to-tail rule, we get a resultant vector $(\vec{A} + \vec{B})$.

Now, on adding vector $\vec{C}$ to vector $(\vec{A} + \vec{B})$ by head-to-tail rule, we get a resultant vector

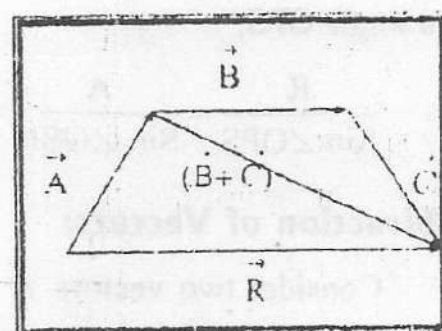
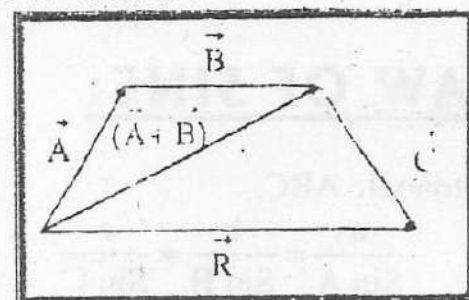
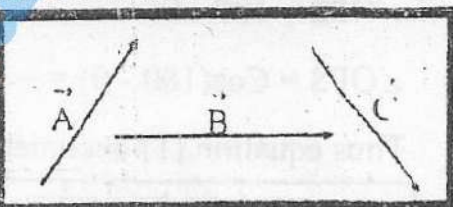
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} \text{ ----- (1)}$$

On the other hand, if we add $\vec{B}$ and $\vec{C}$ first, we get a resultant vector $(\vec{B} + \vec{C})$. Now, on adding vector $(\vec{B} + \vec{C})$ to vector $\vec{A}$ by head-to-tail rule, we get a resultant vector $\vec{R}$

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) \text{ ----- (2)}$$

Comparing equations 1 & 2

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C}) \text{ Hence proved.}$$



RESOLUTION OF A VECTOR:

Definition:

The process, of dividing a vector into its rectangular components, is called resolution of a vector.

Explanation:

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Consider a vector $\vec{A}$ represented by line $\overline{OP}$ which makes an angle θ with the x-axis. To resolve this vector, draw perpendiculars $\overline{PQ}$ and $\overline{PS}$ on x and y axis respectively, as shown in the figure. The line $\overline{OQ}$ is along the x-axis and is denoted by $\vec{A}_x$ and the line $\overline{OS}$ is along the y-axis and is denoted by $\vec{A}_y$.

It can be seen by head-to-tail rule that:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Since $\vec{A}_x$ and $\vec{A}_y$ are mutually perpendicular, they are referred to as **RECTANGULAR COMPONENTS** of vector $\vec{A}$. The magnitudes of $\vec{A}_x$ and $\vec{A}_y$ can be found by considering ΔOPQ . Here

$$\cos\theta = \frac{OQ}{OP} = \frac{A_x}{A}$$

OR

$$A_x = A \cos\theta$$

Similarly,

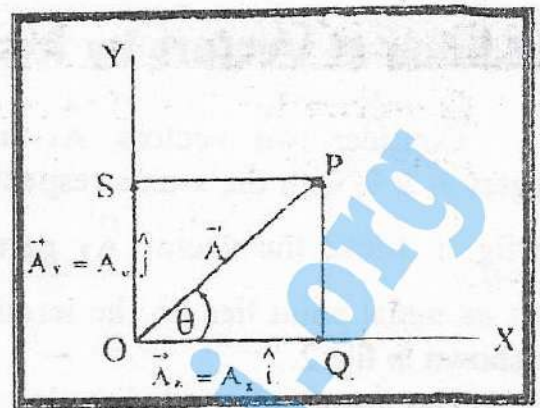
$$\sin\theta = \frac{QP}{OP} = \frac{A_y}{A}$$

$$\sin\theta = \frac{A_y}{A}$$

$$A_y = A \sin\theta$$

$$\cos = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\sin = \frac{\text{Perp}}{\text{Hypotenuse}}$$



Where A , A_x and A_y are the magnitudes of vectors $\vec{A}$, $\vec{A}_x$ and $\vec{A}_y$ respectively. The vector $\vec{A}$ can be written in terms of its components and rectangular unit vectors as,

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Composition of Vector by Its Rectangular Components:

Definition:

The process by which a vector can be made from its rectangular components is called composition of a vector.

Explanation:

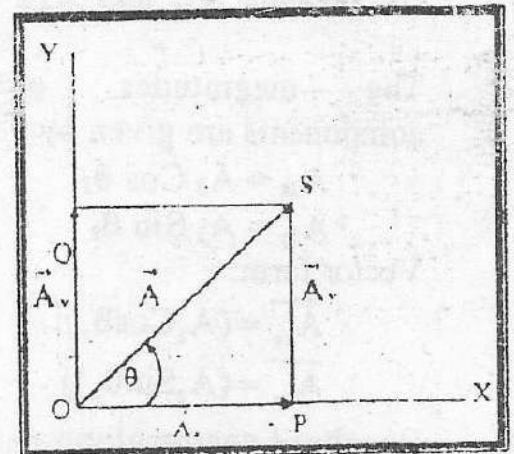
Consider two rectangular components $\vec{A}_x$ and $\vec{A}_y$ of a vector $\vec{A}$. If we add these components by head-to-tail rule, we get a resultant vector $\vec{A}$ of which $\vec{A}_x$ and $\vec{A}_y$ are the components, as shown in the figure.

The magnitude of $\vec{A}$ can be found by considering ΔPOS . By Pythagoras theorem

$$(OS)^2 = (OP)^2 + (PS)^2$$

$$A^2 = (A_x)^2 + (A_y)^2$$

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$



The direction of $\vec{A}$ with respect to + x-axis can be found by the formula:

$$\tan\theta = \frac{PS}{OP}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

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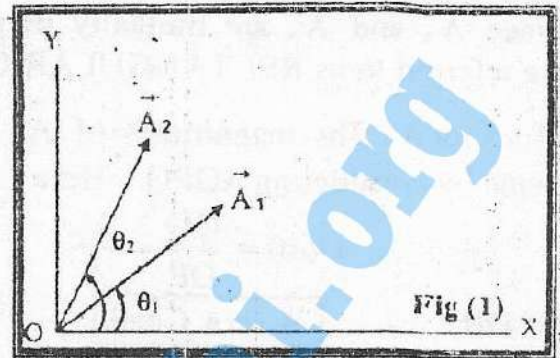
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Addition of Vectors by Rectangular Components Method:

Consider two vectors $\vec{A}_1$ and $\vec{A}_2$, making angles θ_1 & θ_2 with the x-axis respectively, as shown in fig. 1. Move the vector $\vec{A}_2$ parallel to itself so that its initial point lies on the terminal point of $\vec{A}_1$ as shown in fig. 2.



The magnitude and direction of the resultant vector $\vec{A}$ can be found with the help of following steps.

1. Rectangular Components of $\vec{A}_1$:

Resolve $\vec{A}_1$ into its rectangular components $\vec{A}_{1x}$ and $\vec{A}_{1y}$, the magnitudes of these components are given by:

$$A_{1x} = A_1 \cos \theta_1$$

$$A_{1y} = A_1 \sin \theta_1$$

Vector form:

$$\vec{A}_{1x} = (A_1 \cos \theta_1) \hat{i}$$

$$\vec{A}_{1y} = (A_1 \sin \theta_1) \hat{j}$$

2. Rectangular Components of $\vec{A}_2$:

Resolve $\vec{A}_2$ into its rectangular components $\vec{A}_{2x}$ and $\vec{A}_{2y}$.

The magnitudes of these components are given by:

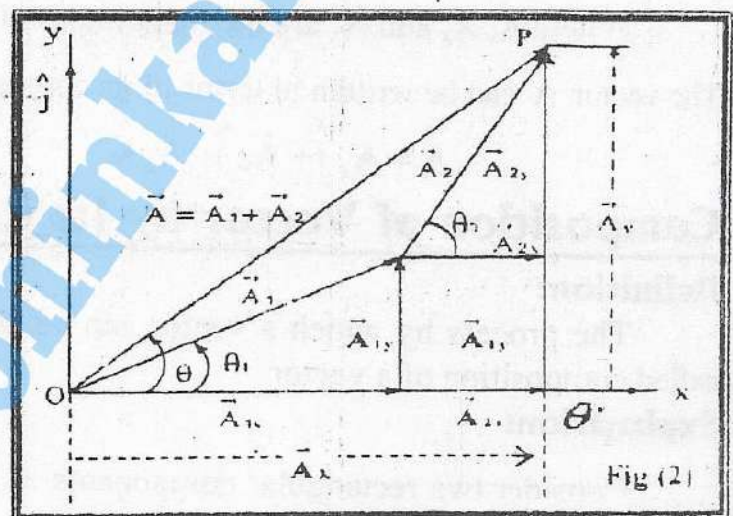
$$A_{2x} = A_2 \cos \theta_2$$

$$A_{2y} = A_2 \sin \theta_2$$

Vector form:

$$\vec{A}_{2x} = (A_2 \cos \theta_2) \hat{i}$$

$$\vec{A}_{2y} = (A_2 \sin \theta_2) \hat{j}$$



3. Resultant vector along x-axis:

$$\vec{A}_x = \vec{A}_{1x} + \vec{A}_{2x}$$

$$A_{x\hat{i}} = A_{1x\hat{i}} + A_{2x\hat{i}}$$

$$A_{x\hat{i}} = (A_{1x} + A_{2x}) \hat{i}$$

$$\text{OR } A_x = A_{1x} + A_{2x} \quad \text{--- (1)}$$

$$\text{OR } A_x = A_1 \cos \theta_1 + A_2 \cos \theta_2$$

4. Resultant vector along y-axis:

$$\vec{A}_y = \vec{A}_{1y} + \vec{A}_{2y}$$

$$A_{y\hat{j}} = A_{1y\hat{j}} + A_{2y\hat{j}}$$

$$\text{OR } A_{y\hat{j}} = (A_{1y} + A_{2y}) \hat{j}$$

$$A_y = A_{1y} + A_{2y} \quad \text{--- (2)}$$

$$\text{OR } A_y = A_1 \sin \theta_1 + A_2 \sin \theta_2$$

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5. For 'n' vector:

If there are "n" vectors $\vec{A}_1, \vec{A}_2, \dots, \vec{A}_n$ making angles $\theta_1, \theta_2, \dots, \theta_n$ with the x-axis respectively, then we shall resolve these rectangular components.

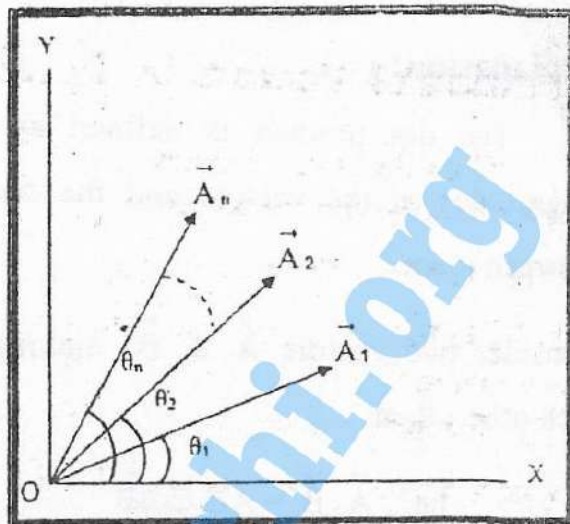
$A_{1x}, A_{2x}, \dots, A_{nx}$ and $A_{1y}, A_{2y}, \dots, A_{ny}$.

Thus equation (1) becomes

$$A_x = A_{1x} + A_{2x} + \dots + A_{nx}$$

Similarly, eq (2) become

$$A_y = A_{1y} + A_{2y} + \dots + A_{ny}$$



6. Magnitude of $\vec{A}$:

The magnitude of resultant vector $\vec{A}$ can be found by Pythagoras theorem i.e.

$$A^2 = (A_x)^2 + (A_y)^2$$

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

7. Direction of $\vec{A}$:

The direction of resultant vector can be found by the formula $\tan\theta = \frac{A_y}{A_x}$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

Product of Two Vectors:

When two vectors are multiplied with each other, the answer may be a scalar or a vector quantity.

Thus, the product of two vectors is divided into two categories.

- 1) Scalar product or dot product.
- 2) Vector product or cross product.

SCALAR PRODUCT OR DOT PRODUCT:

When two vectors are multiplied with each other and the answer is a scalar quantity, then such a product is called "Scalar product".

A dot (.) is placed between the vectors which are multiplied with each other. That is why it is also called "dot product". i.e. Scalar = vector . vector

Examples:

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1. The product of force $\vec{F}$ and displacement $\vec{S}$ is work "W".

$$\text{i.e. } W = \vec{F} \cdot \vec{S}$$

2. The product of force $\vec{F}$ and velocity $\vec{V}$ is power "P".

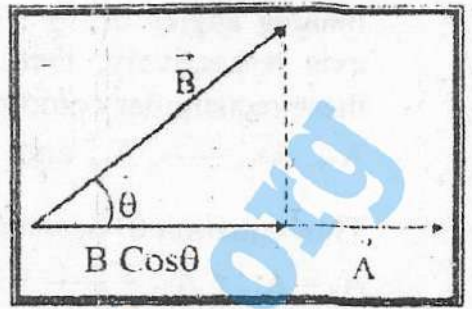
$$\text{i.e. } P = \vec{F} \cdot \vec{V}$$

3. The product of electric intensity $\vec{E}$ and area vector $\vec{A}$ is electric flux ϕ_e .

$$\text{i.e. } \phi_e = \vec{E} \cdot \vec{A}$$

Explanation:-

The dot product is defined as "The product of magnitudes of the vectors and the cosine of the angle between them."



Consider two vectors $\vec{A}$ & $\vec{B}$, making an angle θ with each other, then

$$\text{i.e. } \vec{A} \cdot \vec{B} = AB \cos \theta$$

Where $B \cos \theta$ is the component of $\vec{B}$ along vector $\vec{A}$ and

$$0 \leq \theta \leq \pi$$

Properties or Special Cases:

(i) If vector $\vec{A}$ is parallel to $\vec{B}$, then their scalar product is maximum.

$$\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB \cdot (1) = AB$$

(ii) Scalar product of same vectors is equal to square of their magnitude.

$$\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{A} = AA \cos 0^\circ = A^2 \times 1 = A^2$$

(iii) If two vectors are opposite to each other, then their scalar product will be negative.

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB \cos 180^\circ = AB (-1) = -AB$$

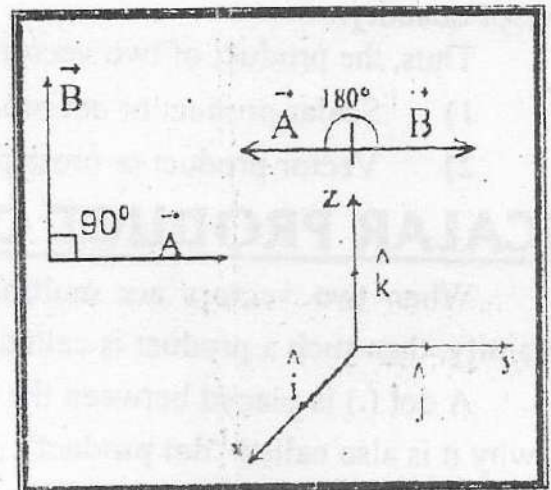
(iv) If $\vec{A}$ is perpendicular to $\vec{B}$, then their scalar product is minimum.

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ = AB \times 0 = 0$$

(v) For unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, the scalar product of same unit vectors is 1 and for different unit vectors, is zero.

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$



(vii) Commutative Law for Dot Product:

Dot product is commutative i.e. $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

Proof:-

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Consider two vectors $\vec{A}$ & $\vec{B}$, making an angle θ with each other. Their dot product is given by.

$$\vec{A} \cdot \vec{B} = AB_A = AB \cos\theta \quad \text{----- (1)}$$

Where $B_A = B \cos\theta$ is the projection of $\vec{B}$ onto the direction of $\vec{A}$. On the other hand,

$$\vec{B} \cdot \vec{A} = BA_B = BA \cos\theta \quad \text{----- (2)}$$

Where $A_B = A \cos\theta$ is the projection of $\vec{A}$ on to the direction of $\vec{B}$.

Comparing equations (1) & (2).

$$AB \cos\theta = BA \cos\theta$$

OR $AB_A = BA_B$

OR $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Thus, scalar product of two vectors does not change by changing the order of the vectors.

(viii) Distributive Law for Dot Product:

Dot product is distributive i.e.

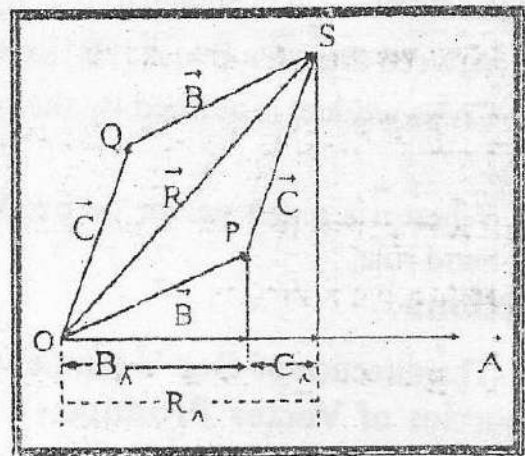
$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Proof:

Consider three vectors $\vec{A}$, $\vec{B}$ & $\vec{C}$. By head-to-tail rule, we get:-

$$\vec{R} = \vec{B} + \vec{C}$$

OR $\vec{R} = \vec{C} + \vec{B}$



Now draw perpendiculars from P and S on the direction of $\vec{A}$. Dot product of $\vec{A}$ and $\vec{R}$ is given by

$$\vec{A} \cdot \vec{R} = AR_A$$

But $\vec{R} = \vec{B} + \vec{C}$ and $R_A = B_A + C_A$

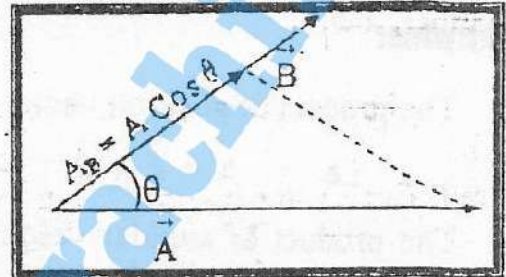
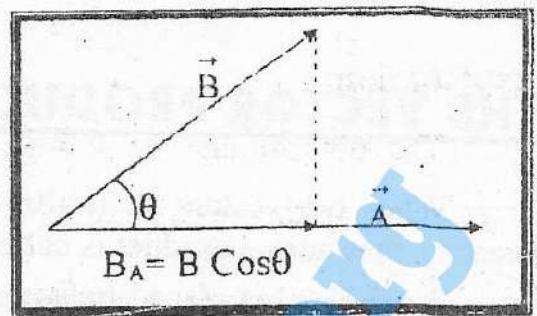
$$\therefore \vec{A} \cdot (\vec{B} + \vec{C}) = A(B_A + C_A)$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = AB_A + AC_A$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\therefore \vec{A} \cdot \vec{B} = AB_A, \vec{A} \cdot \vec{C} = AC_A$$

Where B_A & C_A are the projections of $\vec{B}$ and $\vec{C}$ onto the direction of $\vec{A}$.



THE VECTOR PRODUCT OR THE CROSS PRODUCT:

When two vectors are multiplied with each other and the answer is also a vector quantity, then such a product is called "**VECTOR PRODUCT**".

A cross (x) is placed between the vectors which are multiplied with each other. That is why it is also known as "cross product" i.e.

$$\text{Vector} = \text{Vector} \times \text{Vector}$$

Examples:

1. The product of position vector " $\vec{r}$ " of force and force " $\vec{F}$ " is Torque " $\vec{\tau}$ ".
i.e. $\vec{\tau} = \vec{r} \times \vec{F}$
2. The product of angular velocity $\vec{\omega}$ and radius vector $\vec{r}$ is tangential velocity $\vec{v}_t$.
i.e. $\vec{v}_t = \vec{\omega} \times \vec{r}$
3. The product of angular acceleration $\vec{\alpha}$ and radius vector $\vec{r}$ is tangential acceleration $\vec{a}_t$, i.e. $\vec{a}_t = \vec{\alpha} \times \vec{r}$
4. An electric charge moving with velocity $\vec{v}$ in a magnetic field of induction $\vec{B}$ experiences a force $\vec{F}$ which is given by:

$$\vec{F} = q (\vec{v} \times \vec{B})$$

Explanation:- It is equals to the product of magnitude of two vectors and sine of the angle between them: $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{n}$

Cross product is defined by the relation

Where $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ and it can be find out by Right hand rule.

Direction:-

The direction of $\vec{C}$ or $\vec{V}$ can be found by "Right Hand Rule"

Properties of Vector Product:-

- (i) Vector product does not obey commutative law
 $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ or $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$
- (ii) Vector product obeys distributive law
 $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
- (iii) If vector $\vec{A}$ is parallel to $\vec{B}$, then their vector product is zero

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin 0^\circ$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \times 0$$

$$|\vec{A} \times \vec{B}| = 0$$

$$\begin{array}{c} \vec{A} \\ \theta = 0^\circ \\ \vec{B} \end{array}$$

- (iv) In case of $\hat{i}$, $\hat{j}$ and $\hat{k}$,

$$\hat{i} \times \hat{i} = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{k} = 0$$

(v) If vector $\vec{A}$ is perpendicular to $\vec{B}$, then their vector product is maximum

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

$$\theta = 90^\circ$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin 90^\circ$$

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}|$$

(vi) In case of $\hat{i}$, $\hat{j}$ and $\hat{k}$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \quad \therefore \sin 90^\circ = 1$$

(vii) vector product is not commutative.

Consider two vectors $\vec{A}$ and $\vec{B}$ in a certain plane. Their vector product is given by:

$$\vec{C} = \vec{A} \times \vec{B}$$

Where $\vec{C}$ is a new vector. Its magnitude is given by $C = AB \sin \theta$.

The direction of $\vec{C}$ can be found by right hand rule for cross product or right handed screw turning rule.

The cross product of $\vec{A}$ & $\vec{B}$ can also be given by:

$$\vec{C} = \vec{A} \times \vec{B} = (AB \sin \theta) \hat{u} \quad \text{----- (1)}$$

Where $\hat{u}$ is a unit vector in the direction of $\vec{C}$.

Similarly, a right-handed screw turning from $\vec{B}$ to $\vec{A}$ gives the direction of unit vector $\vec{D}$.

$$\vec{D} = \vec{B} \times \vec{A} = (BA \sin \theta) (-\hat{u})$$

OR

$$\vec{D} = \vec{B} \times \vec{A} = -(BA \sin \theta) \hat{u}$$

OR

$$-\vec{D} = -\vec{B} \times \vec{A} = (BA \sin \theta) \hat{u} \quad \text{----- (2)}$$

Comparing equations (1) & (2)

$$(AB \sin \theta) \hat{u} = (BA \sin \theta) \hat{u}$$

OR

$$\vec{C} = -\vec{D}$$

OR

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$$

$$\text{Thus } \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

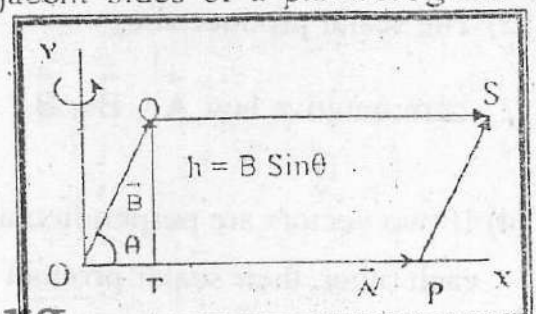
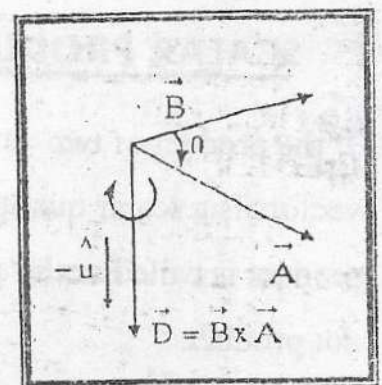
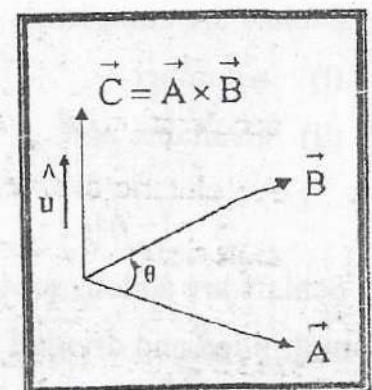
This shows that vector product is not commutative.

(viii) The area of parallelogram formed by two vectors is equal to the magnitude of vector product OR Physical interpretation.

Consider two vectors $\vec{A}$ & $\vec{B}$ which are the adjacent sides of a parallelogram OPSQ, as shown in the figure. From the figure, the area of the parallelogram = Ah

$$\text{But } h = B \sin \theta$$

Area of parallelogram = $AB \sin \theta$ where θ is the angle between $\vec{A}$ & $\vec{B}$.



$$\text{area of parallelogram} = |\vec{A} \times \vec{B}|$$

The direction of the vector area can be found by right-handed screw turning rule.

DIFFERENCES:

SCALARS

- 1) Physical quantities, having magnitude only but no direction, are called Scalars.
- 2) Scalars are completely described by
 - (i) a number
 - (ii) a suitable unit
- 3) Scalars are added, subtracted, multiplied and divided by simple arithmetical rules.

SCALAR PRODUCT

- 1) If the product of two vectors is a scalar quantity, the product is called scalar product or dot product.
- 2) The dot product is defined by the relation $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$
- 3) The scalar product obeys commutative law $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
- 4) If two vectors are perpendicular to each other, their scalar product is zero.

$$\vec{A} \cdot \vec{B} = 0$$

VECTORS

- 1) Physical quantities, having both magnitude and direction, are called Vectors.
- 2) Vectors are completely described by
 - (i) a number
 - (ii) a suitable unit
 - (iii) a certain direction
- 3) Vectors can not be added, subtracted, multiplied nor divided by simple arithmetical rules.

VECTOR PRODUCT

- 1) If the product of two vectors is a vector quantity, the product is called vector product or cross product.
- 2) The cross product is defined by the relation $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta \hat{u}$
- 3) The vector product does not obey commutative law $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
- 4) If two vectors are parallel to each other, their vector product is zero.

$$\vec{A} \times \vec{B} = 0$$