

NUMERICAL PROBLEMS:

Q.1. State which of the following are scalars and which are vectors.

- | | |
|------------------------------|--------------------------------|
| (1) Weight | (2) Calorie |
| (3) Specific heat | (4) Momentum |
| (5) Density | (6) Energy |
| (7) Volume | (8) Distance |
| (9) Speed | (10) Magnetic field intensity |
| (11) Entropy | (12) Work |
| (13) Centrifugal force | (14) Temperature |
| (15) Gravitational potential | (16) Charge |
| (17) Shearing stress | (18) Frequency |
| (19) Kinetic energy | (20) Electric field intensity. |

Solution:

SCALARS

- 1) Specific heat
- 2) Calorie
- 3) Density
- 4) Energy
- 5) Volume
- 6) Distance
- 7) Speed
- 8) Entropy
- 9) Work
- 10) Temperature
- 11) Gravitational potential
- 12) Charge
- 13) Frequency
- 14) Kinetic Energy

VECTORS

- 1) Weight
- 2) Momentum
- 3) Magnetic field intensity
- 4) Centrifugal force
- 5) Shearing stress
- 6) Electric field intensity

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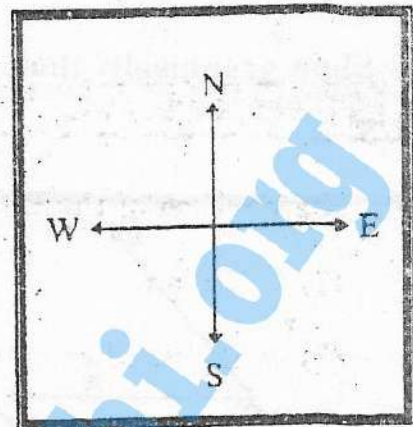
Q.2. Find the resultant of the following displacements:

$\vec{A} = 20\text{km } 30^\circ \text{ south of east.}$

$\vec{B} = 50\text{km due west.}$

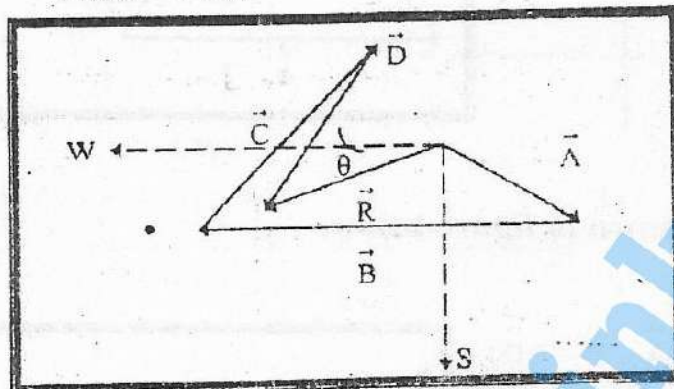
$\vec{C} = 40\text{km north east.}$

$\vec{D} = 30\text{km } 60^\circ \text{ south of west}$



Scale:

10km = 1 cm



$\theta = 21.65^\circ \text{ south of west}$

$R = 2.09\text{cm} \times 10$

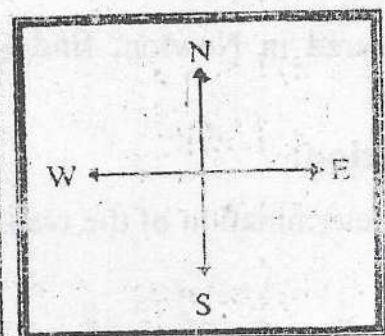
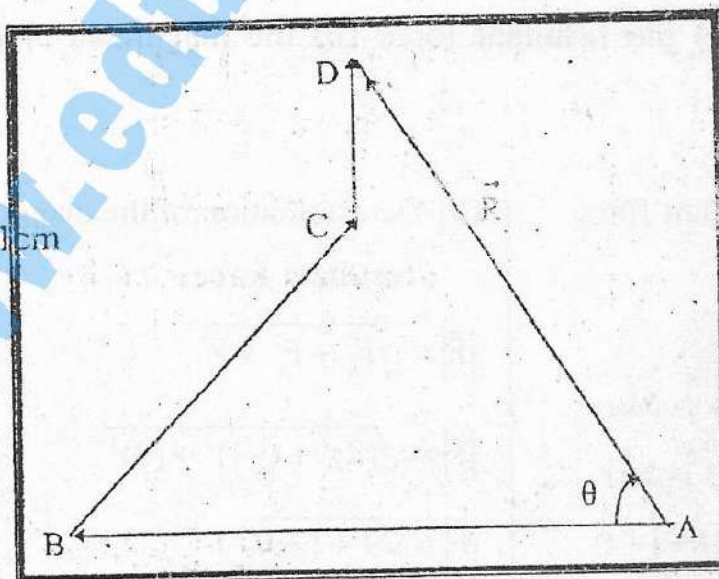
$= 20.9\text{km}$

Magnitude 20.9km. direction 21.65° south of west. Ans.

Q.3. An aeroplane flies 400 km due west from city A to city B, then 300km north east to city C, and finally 100 km north to city D. How far is it from city A to D? In what direction must the aeroplane had to return directly to city A from city D?

Scale:

50km = 1cm

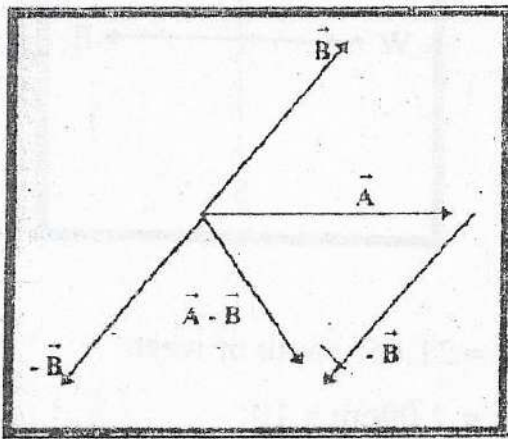


$\theta = 52^\circ \text{ North of West, and for return journey } \theta = 38^\circ \text{ East of South}$

$R = 7.6 \text{ cm} \times 50 = 380\text{km.}$

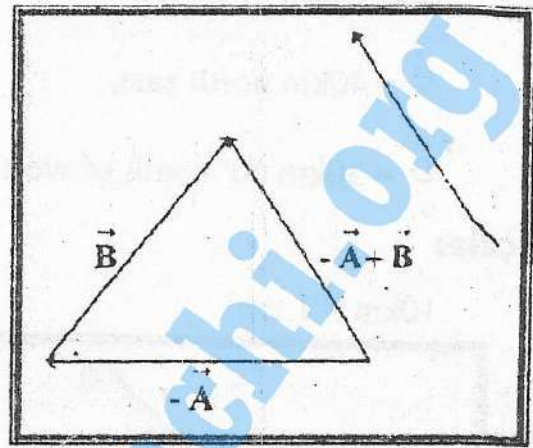
Q.4. Show graphically that: $-(\vec{A} - \vec{B}) = -\vec{A} + \vec{B}$

L. H. S.



R. H. S.

$-(\vec{A} - \vec{B})$

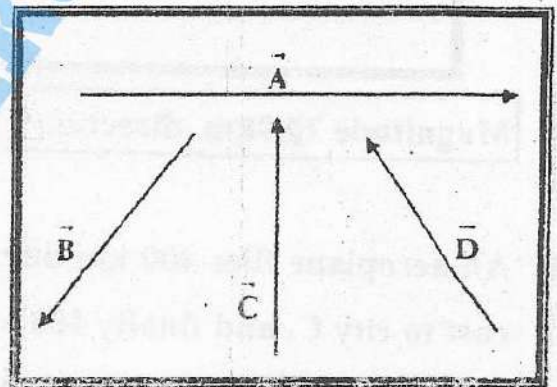


SELF TEST (1)

Q.5. Given vector $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ as shown in figure below.

Construct (a) $4\vec{A} - 3\vec{B} - (2\vec{C} + \vec{D})$

(b) $(1/2)(\vec{C}) + (1/3)(\vec{A} + \vec{B} + 2\vec{D})$



Q.6. The following forces act on particle P:

$$\vec{F}_1 = 2\hat{i} + 3\hat{j} - 5\hat{k}, \quad \vec{F}_2 = -5\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{F}_3 = \hat{i} - 2\hat{j} + 4\hat{k}, \quad \vec{F}_4 = 4\hat{i} - 3\hat{j} - 2\hat{k},$$

measured in Newton, find (a) the resultant force (b) the magnitude of the resultant force.

Solution:

(a) Determination of the resultant force.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\vec{F} = (2\hat{i} + 3\hat{j} - 5\hat{k}) + (-5\hat{i} + \hat{j} + 3\hat{k}) + (\hat{i} - 2\hat{j} + 4\hat{k}) + (4\hat{i} - 3\hat{j} - 2\hat{k})$$

$$\vec{F} = 2\hat{i} + 3\hat{j} - 5\hat{k} - 5\hat{i} + \hat{j} + 3\hat{k} + \hat{i} - 2\hat{j} + 4\hat{k} + 4\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\boxed{\vec{F} = 2\hat{i} - \hat{j}} \text{ Ans.}$$

(b) Determination of the magnitude of resultant force.

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$|\vec{F}| = \sqrt{(2)^2 + (-1)^2 + (0)^2}$$

$$|\vec{F}| = \sqrt{4 + 1 + 0}$$

$$\boxed{|\vec{F}| = \sqrt{5}} \text{ Ans.}$$

Q.7. If $\vec{A} = 3\hat{i} - \hat{j} - 4\hat{k}$, $\vec{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$, $\vec{C} = \hat{i} + 2\hat{j} - \hat{k}$, find.

(a) $2\vec{A} - \vec{B} + 3\vec{C}$ (b) $|\vec{A} + \vec{B} + \vec{C}|$ (c) $|3\vec{A} - 2\vec{B} + 4\vec{C}|$

(d) a unit vector parallel to $3\vec{A} - 2\vec{B} + 4\vec{C}$

Data:

$$\vec{A} = 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\vec{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

$$\vec{C} = \hat{i} + 2\hat{j} - \hat{k}$$

To Find:

(a) $2\vec{A} - \vec{B} + 3\vec{C} = ?$

(b) $|\vec{A} + \vec{B} + \vec{C}| = ?$

(c) $|3\vec{A} - 2\vec{B} + 4\vec{C}| = ?$

(d) a unit vector parallel to

$$3\vec{A} - 2\vec{B} + 4\vec{C} = ?$$

Solution:

(a) $2\vec{A} - \vec{B} + 3\vec{C}$

$$= 2(3\hat{i} - \hat{j} - 4\hat{k}) - (-2\hat{i} + 4\hat{j} - 3\hat{k}) + 3(\hat{i} + 2\hat{j} - \hat{k})$$

$$2\vec{A} - \vec{B} + 3\vec{C} = 6\hat{i} - 2\hat{j} - 8\hat{k} + 2\hat{i} - 4\hat{j} + 3\hat{k} + 3\hat{i} + 6\hat{j} - 3\hat{k}$$

$$2\vec{A} - \vec{B} + 3\vec{C} = 11\hat{i} - 8\hat{k} \quad \text{Ans.}$$

(b) $|\vec{A} + \vec{B} + \vec{C}|$

$$\vec{A} + \vec{B} + \vec{C} = 3\hat{i} - \hat{j} - 4\hat{k} + (-2\hat{i} + 4\hat{j} - 3\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})$$

$$\vec{A} + \vec{B} + \vec{C} = 3\hat{i} - \hat{j} - 4\hat{k} - 2\hat{i} + 4\hat{j} - 3\hat{k} + \hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{A} + \vec{B} + \vec{C} = 2\hat{i} + 5\hat{j} - 8\hat{k}$$

Magnitude of resultant vector

$$|\vec{A} + \vec{B} + \vec{C}| = \sqrt{(2)^2 + (5)^2 + (-8)^2}$$

$$= \sqrt{4 + 25 + 64}$$

$$|\vec{A} + \vec{B} + \vec{C}| = \sqrt{93}$$

(c) **SELF TEST (2)**

$$3\vec{A} - 2\vec{B} + 4\vec{C} = 17\hat{i} - 3\hat{j} - 10\hat{k}$$

$$\text{Ans: } \sqrt{398}$$

(d) Unit vector parallel to

$$\text{Let } 3\vec{A} - 2\vec{B} + 4\vec{C} = \vec{D}$$

$$3\vec{A} - 2\vec{B} + 4\vec{C} \text{ will be } = \frac{\vec{D}}{|\vec{D}|}$$

$$= \frac{17\hat{i} - 3\hat{j} - 10\hat{k}}{\sqrt{398}}$$

$$\vec{u} = \frac{17}{\sqrt{398}}\hat{i} - \frac{3}{\sqrt{398}}\hat{j} - \frac{10}{\sqrt{398}}\hat{k} \quad \text{Ans.}$$

SELF TEST (3)

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that

$$\vec{A} = 3\hat{i} + 4\hat{j} - \hat{k}$$

$$\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{C} = 3\hat{i} - 2\hat{j} + \hat{k} \quad \text{Find.}$$

(a) unit vector parallel to $\vec{A}$

(b) Unit vector parallel to $\vec{B}$

(c) unit vector parallel to $2\vec{A} - \vec{B}$

(d) $2\vec{A} - 3\vec{B} + \vec{C}$ (e) $|3\vec{A} - 2\vec{B} + 2\vec{C}|$

(f) $2\vec{A} - \vec{B} + \vec{C}$

(g) unit vector parallel to $3\vec{A} - 2\vec{B} + 2\vec{C}$

Ans. (a) $\frac{3}{\sqrt{26}}\hat{i} + \frac{4}{\sqrt{26}}\hat{j} - \frac{1}{\sqrt{26}}\hat{k}$ (b) $\frac{2}{\sqrt{6}}\hat{i} + \frac{4}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ (c) $4\hat{i} + 9\hat{j} - \hat{k}$

(d) $3\hat{i} + 9\hat{j} + 2\hat{k}$ (e) $11\hat{i} + 6\hat{j} + \hat{k}$ (f) $7\hat{i} + 5\hat{j}$ (g) $\frac{7}{74}\hat{i} + \frac{5}{74}\hat{j}$

Q.8. Two tugboats are towing a ship, each exerts a force of 6000 N and the angle between the ropes is 60° . Calculate the resultant force on the ship.

Given Data:

$$F_1 = 6000 \text{ N}$$

$$F_2 = 6000 \text{ N}$$

$$\theta = 60^\circ$$

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To Find:

$$|\vec{F}| = ?$$

Solution:

Magnitude of resultant force

$$|\vec{F}| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$|\vec{F}| = \sqrt{(6000)^2 + (6000)^2 + 2(6000)(6000)\cos 60^\circ}$$

$$|\vec{F}| = \sqrt{36000000 + 36000000 + 72000000 \times 0.5}$$

Ans: $\frac{1}{2}\sqrt{114}$

$F = 10392.305 \text{ N}$ Ans.

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SELF TEST (4)

Two tugboats are towing a ship, each exerts a force of 4.5KN, with an angle of 50° between the ropes. Calculate the resultant force.

Ans: 8.156KN

Q.9. The position vectors of points P and Q are given by $\vec{r}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{r}_2 = 4\hat{i} - 3\hat{j} + 2\hat{k}$. Determine $\overline{PQ}$ in terms of rectangular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ and find its magnitude.

Given Data:

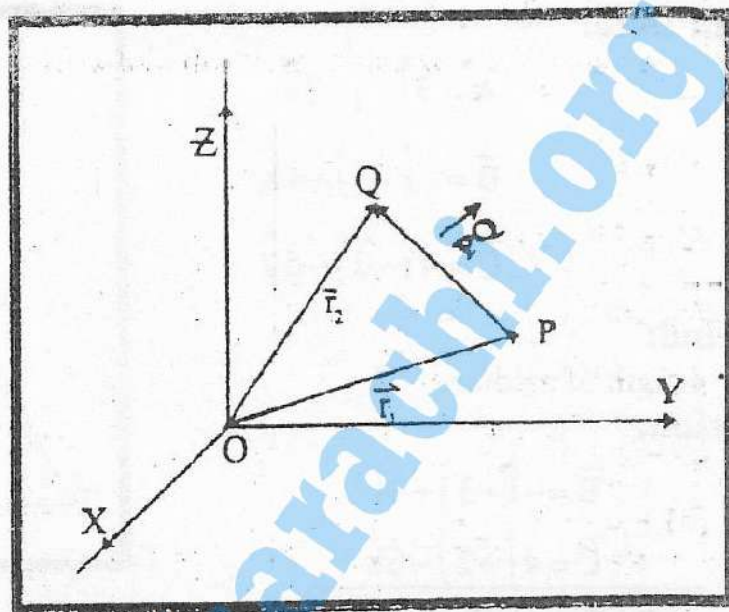
$$\overline{OP} = \vec{r}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\overline{OQ} = \vec{r}_2 = 4\hat{i} - 3\hat{j} + 2\hat{k}$$

To Find:

$$\overline{PQ} = ?$$

$$|\overline{PQ}| = ?$$



Solution:

According to head to tail rule

$$(i) \quad \overline{OP} + \overline{PQ} = \overline{OQ}$$

$$\overline{PQ} = \overline{OQ} - \overline{OP}$$

$$\overline{PQ} = \vec{r}_2 - \vec{r}_1$$

$$\overline{PQ} = 4\hat{i} - 3\hat{j} + 2\hat{k} - (2\hat{i} + 3\hat{j} - \hat{k})$$

$$\overline{PQ} = 4\hat{i} - 3\hat{j} + 2\hat{k} - 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\boxed{\overline{PQ} = 2\hat{i} - 6\hat{j} + 3\hat{k}} \text{ Ans.}$$

(ii) Magnitude of $\overline{PQ}$

$$|\overline{PQ}| = \sqrt{(2)^2 + (-6)^2 + (3)^2}$$

$$|\overline{PQ}| = \sqrt{49}$$

$$\boxed{|\overline{PQ}| = 7} \text{ Ans.}$$

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SELF TEST (5)

The position vectors of points P and Q are given by, $\vec{r}_1 = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{r}_2 = 4\hat{i} - 3\hat{j} + 2\hat{k}$. Determine $\overline{PQ}$ in terms of rectangular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, and find its magnitude.

$$[\text{Ans. } -2\hat{i} + 6\hat{j} - 3\hat{k}, 7]$$

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Q.10. Prove that the vectors $\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{C} = 4\hat{i} - 2\hat{j} - 6\hat{k}$, can form the sides of a triangle. Find the length of the medians of the triangle.

Given Data:

$$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{C} = 4\hat{i} - 2\hat{j} - 6\hat{k}$$

To Find:

length of medians = ?

Solution:

$$\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{C} = 4\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{B} + \vec{C} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{B} + \vec{C} = \vec{A}$$

$\Rightarrow$ These vectors can form a Δ (Triangle)

There are three medians i.e. $\vec{PM}_1$, $\vec{QM}_2$ and $\vec{RM}_3$

For 1st median:

From fig

$$\vec{PM}_1, \vec{PQ} + \vec{QM}_1$$

$$\vec{PM}_1, \vec{B} + \frac{1}{2}\vec{C}$$

$$= (-\hat{i} + 3\hat{j} + 4\hat{k}) + \frac{1}{2}(4\hat{i} - 2\hat{j} - 6\hat{k})$$

$$\begin{aligned}\vec{PM}_1 &= -\hat{i} + 3\hat{j} + 4\hat{k} + 2\hat{i} - \hat{j} - 3\hat{k} \\ &= \hat{i} + 2\hat{j} + \hat{k}\end{aligned}$$

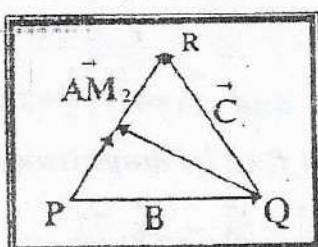
$$|\vec{PM}_1| = \sqrt{(1)^2 + (2)^2 + (1)^2}$$

$$|\vec{PM}_1| = \sqrt{1+4+1}$$

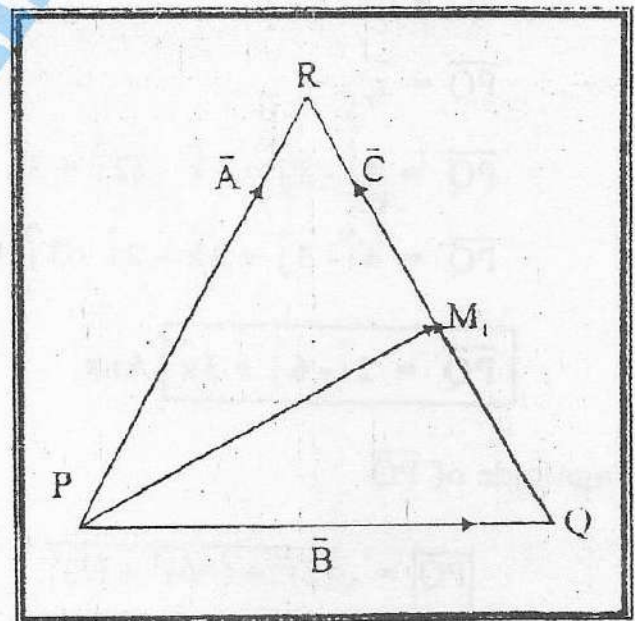
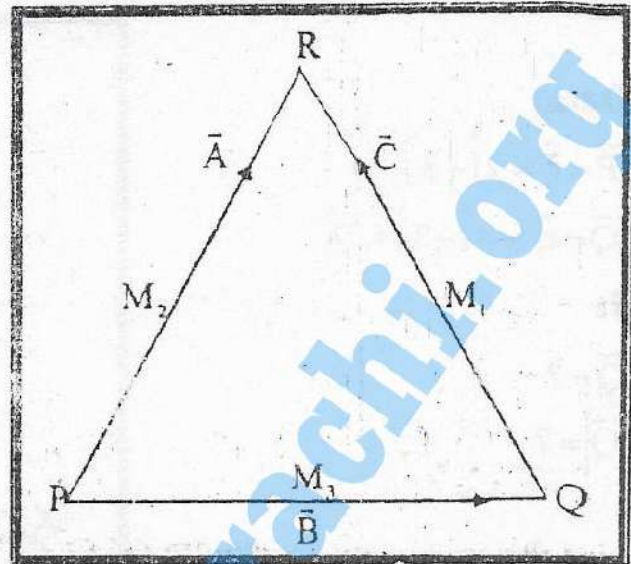
$$|\vec{PM}_1| = \sqrt{6}$$

SELF TEST (6)

For 2nd median



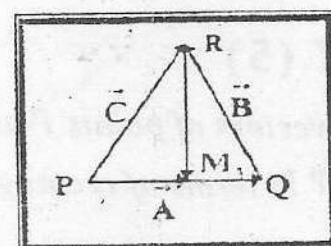
$$\text{Ans: } \frac{1}{2}\sqrt{114}$$



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SELF TEST (7)

For 3rd median



$$\text{Ans: } \frac{1}{2}\sqrt{150}$$

Q.11. Find the rectangular components of a vector $\vec{A}$, 15 unit long when it forms an angle with respect to +ve x-axis of 50° .

Given Data:

$$|\vec{A}| = 15 \text{ unit}$$

$$\theta = 50^\circ$$

To Find:

$$A_x = ?$$

$$A_y = ?$$

Solution:

$$A_x = A \cos \theta$$

$$A_x = 15 \cos 50^\circ$$

$$A_x = 9.641 \text{ Unit} \quad \text{Ans.}$$

$$A_y = A \sin \theta$$

$$A_y = 15 \sin 50^\circ$$

$$A_y = 15 \times 0.766$$

$$A_y = 11.490 \text{ Unit} \quad \text{Ans.}$$

SELF TEST (8)

Find the rectangular components of a vector $\vec{A}$, 15 unit long when it forms angles of 130° , 230° and 310° with respect to +ve x-axis.

$$\text{Ans: } -9.11, 5$$

$$-9.6, -11.5$$

$$-9.6, -11.5$$

Q.12. Two vectors 10cm and 8 cm long form an angle of 60° . Find the magnitude of difference and the angle with respect to the larger vector.

Data:

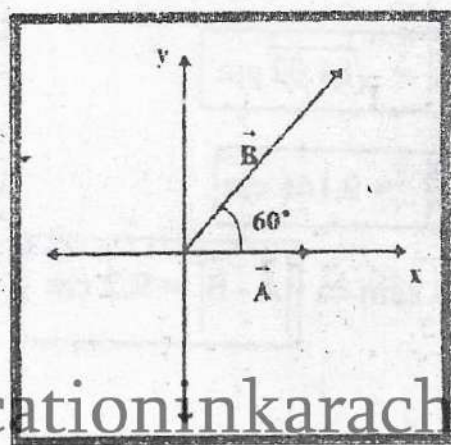
$$|\vec{A}| = 10 \text{ cm}$$

$$|\vec{B}| = 8 \text{ cm}$$

$$\theta_1 = 0^\circ$$

$$\theta_1 = 60^\circ$$

$$\theta_R = ?$$



Solution:

Taking vector $\vec{A}$ along X-axis

Resolving vector $\vec{A}$ into rectangular components:

$$A_x = A \cos \theta_1, \quad A_y = A \sin \theta_1$$

$$A_x = 10 \cos 0^\circ, \quad A_y = 10 \sin 0^\circ$$

$$A_x = 10 (1), \quad A_y = 10 (0)$$

$$A_x = 10 \text{ cm}, \quad A_y = 0$$

Resolving vector $\vec{B}$ into rectangular components:

$$B_x = B \cos \theta_2, \quad B_y = B \sin \theta_2$$

$$B_x = 8 \cos 60^\circ, \quad B_y = 8 \sin 60^\circ$$

$$B_x = 8 \times 0.5, \quad B_y = 8 \times 0.866$$

$$B_x = 4 \text{ cm}, \quad B_y = 6.928 \text{ cm.}$$

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Resultant along x-axis:

$$R_x = A_x - B_x$$

$$R_x = 10 - 4$$

$$R_x = 6 \text{ cm}$$

Resultant along y-axis:

$$R_y = A_y - B_y$$

$$R_y = 0 - 6.92$$

$$R_y = -6.93 \text{ cm}$$

Magnitude of resultant vector:

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$|\vec{R}| = \sqrt{(6)^2 + (-6.93)^2}$$

$$|\vec{R}| = \sqrt{36 + 48.02}$$

$$|\vec{R}| = \sqrt{84.02} \text{ cm}$$

$$|\vec{R}| = 9.166 \text{ cm}$$

$$\text{OR } R = 9.2 \text{ cm} \Rightarrow |\vec{A} - \vec{B}| = 9.2 \text{ cm} \quad \text{Ans.}$$

Direction of resultant vector:

$$\theta_R = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$$\theta_R = \tan^{-1} \left(\frac{-6.93}{6} \right)$$

$$\theta_R = 49^\circ \quad \text{Ans.}$$

OR

$$A = 10 \text{ cm}, B = 8 \text{ cm}, \theta = 60^\circ$$

$$|\vec{A} - \vec{B}| = ? \quad \theta_R = ?$$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$= \sqrt{10^2 + 8^2 - 2(10)(8) \cos 60^\circ}$$

$$= \sqrt{100 + 64 - 160(0.5)}$$

$$= \sqrt{164 - 80}$$

$$= \sqrt{84} = 9.16 \text{ cm.}$$

$$\Rightarrow |\vec{A} - \vec{B}| = 9.2 \text{ cm.}$$

$$\theta_R = \tan^{-1} \frac{B \sin \theta}{A - B \cos \theta}$$

$$\theta_R = \tan^{-1} \left(\frac{8 \sin 60^\circ}{10 - 8 \cos 60^\circ} \right) = 49^\circ$$

$$\theta_R = 49^\circ$$

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SELF TEST (9)

Two vectors, 10cm and 8 cm long, form angles of 90° and 120° . Find the magnitude of difference and the angle with respect to the larger vector.

$$\text{Ans: } 12.8 \text{ cm}, 38.68^\circ$$

$$15.6 \text{ cm}, 26.3^\circ$$

Q.13 The angle between the vectors $\vec{A}$ and $\vec{B}$ is 60° . Given that $|\vec{A}| = |\vec{B}| = 1$,

Calculate (a) $|\vec{B} - \vec{A}|$ (b) $|\vec{B} + \vec{A}|$.

Given Data:

$$|\vec{A}| = 1$$

$$|\vec{B}| = 1$$

$$\theta = 60^\circ$$

To Find:

(a) $|\vec{B} - \vec{A}| = ?$

(b) $|\vec{B} + \vec{A}| = ?$

Solution:

$$|\vec{B} - \vec{A}| = \sqrt{(\vec{B} - \vec{A}) \cdot (\vec{B} - \vec{A})}$$

$$= \sqrt{\vec{B} \cdot \vec{B} - \vec{B} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{A}}$$

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2$$

$$\vec{B} \cdot \vec{B} = |\vec{B}|^2$$

$$|\vec{B} - \vec{A}| = \sqrt{|\vec{B}|^2 - 2(\vec{B} \cdot \vec{A}) + |\vec{A}|^2}$$

$$\vec{B} \cdot \vec{A} = |\vec{B}| |\vec{A}| \cos \theta$$

(a)

$$|\vec{B} - \vec{A}| = \sqrt{(|\vec{B}|)^2 + (|\vec{A}|)^2 - 2|\vec{B}||\vec{A}|\cos \theta}$$

$$|\vec{B} - \vec{A}| = \sqrt{(1)^2 + (1)^2 - 2(1)(1)\cos 60^\circ}$$

$$|\vec{B} - \vec{A}| = \sqrt{1+1-2 \times 0.5}$$

$$|\vec{B} - \vec{A}| = \sqrt{2-1}$$

$$|\vec{B} - \vec{A}| = \sqrt{1}$$

$$|\vec{B} - \vec{A}| = 1 \text{ Ans.}$$

(b)

$$|\vec{B} + \vec{A}| = \sqrt{(\vec{B} + \vec{A}) \cdot (\vec{B} + \vec{A})}$$

$$= \sqrt{\vec{B} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{A}}$$

$$= \sqrt{|\vec{B}|^2 + |\vec{A}|^2 + 2(\vec{B} \cdot \vec{A})}$$

$$\vec{B} \cdot \vec{A} = |\vec{B}| |\vec{A}| \cos \theta$$

$$|\vec{B} + \vec{A}| = \sqrt{(|\vec{A}|)^2 + (|\vec{B}|)^2 + 2|\vec{A}||\vec{B}|\cos \theta}$$

$$|\vec{B} + \vec{A}| = \sqrt{(1)^2 + (1)^2 + 2(1)(1)\cos 60^\circ}$$

$$|\vec{B} + \vec{A}| = \sqrt{1+1+2 \times 0.5}$$

$$|\vec{B} + \vec{A}| = \sqrt{2+1}$$

$$|\vec{B} + \vec{A}| = \sqrt{3} \text{ Ans.}$$

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SELF TEST (10)

The angle between the vectors $\vec{A}$ and $\vec{B}$ is 30° , given that $|\vec{A}| = 2$, and $|\vec{B}| = 3$,

Calculate (a) $|\vec{A} + \vec{B}|$ (b) $|\vec{A} - \vec{B}|$

Ans. 4.83, 1.615

Q.14. A car weighing 10,000N on a hill, makes an angle of 20° with the horizontal. Find the components of car's weight parallel and perpendicular to the road?

Given Data:

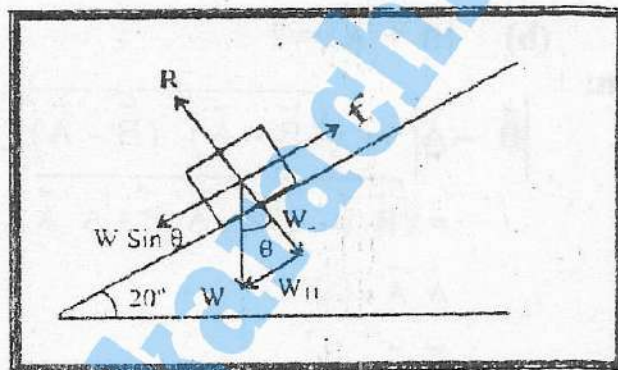
$$W = 10,000 \text{ N}$$

$$\theta = 20^\circ$$

To Find:

$$(a) \quad W_{||} = ?$$

$$(b) \quad W_{\perp} = ?$$



Solution:

(a) The component of the weight of car parallel to the road is

$$W_{||} = W \sin \theta$$

$$W_{||} = 10,000 \sin 20^\circ$$

$$W_{||} = 10,000 \times 0.342$$

$$W_{||} = 3420 \text{ N} \quad \text{Ans.}$$

(b) The component of the weight of car perpendicular to the road is

$$W_{\perp} = W \cos \theta$$

$$W_{\perp} = 10,000 \cos 20^\circ$$

$$W_{\perp} = 10,000 \times 0.939$$

$$W_{\perp} = 9390 \text{ N} \quad \text{Ans.}$$

Q.15. Find the angle between $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$. (1992)

Given Data:

$$\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\vec{B} = 6\hat{i} + 3\hat{j} + 2\hat{k}$$

To Find:

$$\theta = ?$$

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Solution:

We know that dot product

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Where ' θ ' is the required angle between $\vec{A}$ and $\vec{B}$

Therefore

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} \quad (1)$$

$$\vec{A} \cdot \vec{B} = (2\hat{i} + 2\hat{j} - \hat{k}) \cdot (6\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{A} \cdot \vec{B} = 12(\hat{i} \cdot \hat{i}) - 6(\hat{j} \cdot \hat{j}) - 2(\hat{k} \cdot \hat{k})$$

$$\vec{A} \cdot \vec{B} = 12 - 6 - 2$$

$$\vec{A} \cdot \vec{B} = 4$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

And

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{(2)^2 + (2)^2 + (-1)^2}$$

$$|\vec{A}| = \sqrt{4 + 4 + 1}$$

$$|\vec{A}| = 3$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$|\vec{B}| = \sqrt{(6)^2 + (-3)^2 + (2)^2}$$

$$|\vec{B}| = \sqrt{36 + 9 + 4}$$

$$|\vec{B}| = \sqrt{49}$$

$$|\vec{B}| = 7$$

Putting the value in Equation (1)

$$\cos \theta = \frac{4}{(3)(7)}$$

$$\cos \theta = \frac{4}{(3)(7)}$$

$$\cos \theta = 0.190$$

$$\theta = \cos^{-1}(0.190)$$

$$\theta = 79^\circ \quad \text{Ans.}$$

Q.16. Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ onto the direction of vector

$$\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}.$$

(2013)

Given Data:

$$\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

To Find:

Projection of $\vec{A}$ onto $\vec{B} = ?$

Solution:

Projection of $\vec{A}$ onto $\vec{B} = \vec{A} \cdot \hat{b}$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$|\vec{B}| = \sqrt{(4)^2 + (-4)^2 + (7)^2}$$

$$|\vec{B}| = \sqrt{16 + 16 + 49}$$

$$|\vec{B}| = \sqrt{81}$$

$$|\vec{B}| = 9$$

To find unit vector of $\vec{B}$,

$$\hat{b} = \frac{\vec{B}}{|\vec{B}|}$$

$$\hat{b} = \frac{4\hat{i} - 4\hat{j} + 7\hat{k}}{9}$$

$$\hat{b} = \frac{4\hat{i}}{9} - \frac{4\hat{j}}{9} + \frac{7\hat{k}}{9}$$

$$\hat{b} = \frac{4}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}$$

Projection of $\vec{A}$ onto $\vec{B} = (\hat{i} - 2\hat{j} + \hat{k}) \cdot \left(\frac{4}{9}\hat{i} - \frac{4}{9}\hat{j} + \frac{7}{9}\hat{k}\right)$

$$\therefore \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{and } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$= \frac{4}{9}(\hat{i} \cdot \hat{i}) + \frac{8}{9}(\hat{j} \cdot \hat{j}) + \frac{7}{9}(\hat{k} \cdot \hat{k})$$

$$= \frac{4}{9} + \frac{8}{9} + \frac{7}{9} = \frac{4+8+7}{9}$$

$$\text{Projection of } \vec{A} \text{ onto } \vec{B} = \frac{19}{9} \text{ Ans.}$$

SELF TEST (11)

Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that,

$$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{B} = -\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{C} = 4\hat{i} - 2\hat{j} - 6\hat{k}$$

Find

(a) $\vec{A} \cdot \vec{B}$

(b) $\vec{B} \cdot \vec{C}$

(c) $\vec{A} \cdot \vec{C}$

(d) Angle between $\vec{A}$ and $\vec{B}$

(e) Angle between $\vec{B}$ and $\vec{C}$

(f) Angle between $\vec{A}$ and $\vec{C}$

(g) projection of $\vec{A}$ on to $\vec{B}$

(h) projection of $\vec{B}$ on to $\vec{C}$

(i) projection of $\vec{A}$ on to $\vec{C}$

Q.17 Find the angles α , β and γ which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the positive x, y and z axis respectively.

Given Data:

$$\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

To Find:

$$\alpha = ?$$

$$\beta = ?$$

$$\gamma = ?$$

Solution:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{(3)^2 + (-6)^2 + (2)^2}$$

$$|\vec{A}| = \sqrt{9 + 36 + 4}$$

$$|\vec{A}| = \sqrt{49}$$

$$|\vec{A}| = 7$$

According to direction cosines of vector $\vec{A}$

$$\cos \alpha = \frac{A_x}{|\vec{A}|} = \frac{3}{7}$$

$$\alpha = \cos^{-1}(0.428)$$

$$\alpha = 64.62^\circ \text{ Ans.}$$

$$\cos \beta = \frac{A_y}{|\vec{A}|} = \frac{-6}{7}$$

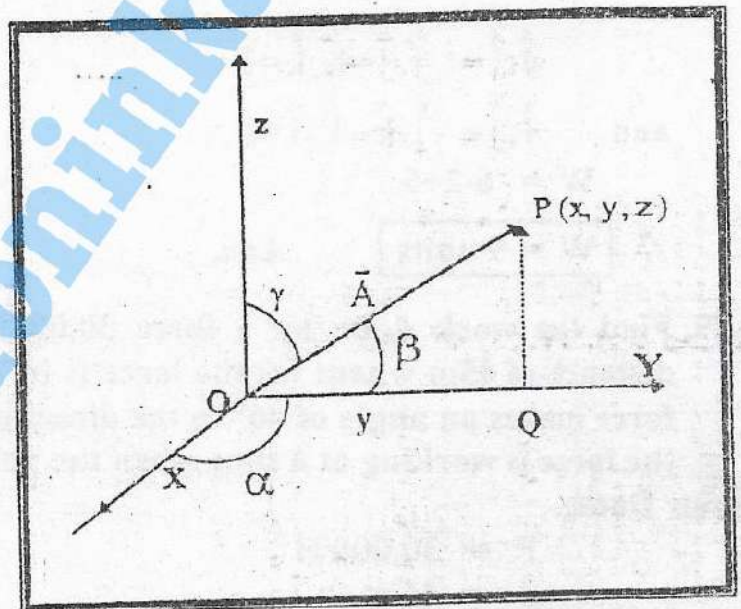
$$\beta = \cos^{-1}(-0.857)$$

$$\beta = 149^\circ \text{ Ans.}$$

$$\cos \gamma = \frac{A_z}{|\vec{A}|} = \frac{2}{7}$$

$$\gamma = \cos^{-1}(0.285)$$

$$\gamma = 73.39^\circ \text{ Ans.}$$



Q.18 Find the work done in moving an object along a vector $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ if the applied force is $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$.

Given Data:

$$\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

$$\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$$

To Find:

$$W = ?$$

Solution:

According to the definition of dot product.

$$W = \vec{F} \cdot \vec{r}$$

$$W = (2\hat{i} - \hat{j} - \hat{k}) \cdot (3\hat{i} + 2\hat{j} - 5\hat{k})$$

$$W = 6(\hat{i} \cdot \hat{i}) - 2(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k})$$

$$\therefore \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\text{and } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$W = 6 - 2 - 5$$

$$\therefore \boxed{W = 9 \text{ units}} \quad \text{Ans.}$$

Q.19. Find the work done by a force 30,000N in moving an object through a distance of 45m when: (a) the force is in the direction of motion; and (b) the force makes an angle of 40° to the direction of motion. Find the rate at which the force is working at a time when the velocity is 2m/sec.

Given Data:

$$F = 30,000 \text{ N}$$

$$d = 45 \text{ m}$$

$$V = 2 \text{ m/Sec.}$$

$$(a) \quad \theta = 0^\circ$$

$$(b) \quad \theta = 40^\circ$$

To Find:

$$(a) \quad P = ? \quad (b) \quad P = ?$$

$$W = ? \quad W = ?$$

Solution:

$$(a) \quad W = \vec{F} \cdot \vec{d}$$

$$W = Fd \cos \theta$$

$$W = (30,000)(45) \cos 0^\circ \quad \because \cos 0^\circ = 1$$

$$\boxed{W = 1,35,0000 \text{ joules}} \quad \text{Ans.}$$

$$P = \vec{F} \cdot \vec{V}$$

$$P = FV \cos \theta$$

$$P = (30,000)(2) \cos 0^\circ$$

$$P = 60,000 \times 1$$

$$\boxed{P = 60000 \text{ watts}} \quad \text{Ans.}$$

(b) $W = \vec{F} \cdot \vec{d}$
 $W = Fd \cos \theta$
 $W = (30,000)(45) \cos 40^\circ$
 $W = 1350000 \times 0.766$
 $W = 1034100 \text{ joules.} \quad \text{Ans.}$

$P = \vec{F} \cdot \vec{V}$
 $P = FV \cos \theta$
 $P = (30,000)(2) \cos 40^\circ$
 $P = 60,000 \times 0.766$
 $P = 60,000 \times 0.766$
 $P = 45960 \text{ watts.} \quad \text{Ans.}$

Q.20. Two vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}|=3$, $|\vec{B}|=4$, and $\vec{A} \cdot \vec{B} = -5$, find;

- (a) the angle between $\vec{A}$ and $\vec{B}$
 (b) the length $|\vec{A} + \vec{B}|$ and $|\vec{A} - \vec{B}|$ (2011 F)
 (c) the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ (2008)

Given Data:

$|\vec{A}| = 3$
 $|\vec{B}| = 4$
 $\vec{A} \cdot \vec{B} = -5$

To Find:

- (a) Angle between $\vec{A}$ and $\vec{B} = ?$
 (b) $|\vec{A} + \vec{B}|$ and $|\vec{A} - \vec{B}| = ?$
 (c) Angle between $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B}) = ?$

Solution:

We know, in dot product

(a) $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

where ' θ ' is the required angle between $\vec{A}$ and $\vec{B}$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\cos \theta = \frac{-5}{(3)(4)}$$

$$\theta = \cos^{-1}(-0.416)$$

$$\theta = 114.6^\circ \quad \text{Ans.}$$

(b) $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}}$

then $|\vec{A} + \vec{B}| = \sqrt{(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})}$

$$= \sqrt{\vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}}$$

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$$\vec{B} \cdot \vec{B} = |\vec{B}|^2 \text{ or } \vec{A} \cdot \vec{A} = |\vec{A}|^2$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$|\vec{A} + \vec{B}| = \sqrt{|\vec{A}|^2 + 2\vec{A} \cdot \vec{B} + |\vec{B}|^2}$$

$$|\vec{A} + \vec{B}| = \sqrt{(3)^2 + 2(-5) + (4)^2}$$

$$|\vec{A} + \vec{B}| = \sqrt{9 - 10 + 16}$$

$$|\vec{A} + \vec{B}| = \sqrt{15} \text{ Ans.}$$

And $|\vec{A} - \vec{B}| = \sqrt{(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})}$

$$= \sqrt{\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}}$$

$$\vec{A} \cdot \vec{A} = |\vec{A}|^2 \quad \vec{B} \cdot \vec{B} = |\vec{B}|^2 \quad \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$|\vec{A} - \vec{B}| = \sqrt{|\vec{A}|^2 - 2\vec{A} \cdot \vec{B} + |\vec{B}|^2}$$

$$|\vec{A} - \vec{B}| = \sqrt{|\vec{A}|^2 + 2\vec{A} \cdot \vec{B} + |\vec{B}|^2}$$

$$|\vec{A} - \vec{B}| = \sqrt{(3)^2 - 2(-5) + (4)^2}$$

$$|\vec{A} - \vec{B}| = \sqrt{9 + 10 + 16}$$

$$|\vec{A} - \vec{B}| = \sqrt{35} \text{ Ans.}$$

(c) Let $\vec{A} + \vec{B} = \vec{C}$

$$\vec{A} - \vec{B} = \vec{D}$$

Then $\vec{C} \cdot \vec{D} = |\vec{C}| |\vec{D}| \cos \theta$

$$\cos \theta = \frac{\vec{C} \cdot \vec{D}}{|\vec{C}| |\vec{D}|}$$

$$\cos \theta = \frac{(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B})}{|\vec{A} + \vec{B}| |\vec{A} - \vec{B}|}$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B}}{|\vec{A} + \vec{B}| |\vec{A} - \vec{B}|}$$

$$\therefore \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad \vec{A} \cdot \vec{A} = |\vec{A}|^2 = \vec{B} \cdot \vec{B} |\vec{B}|^2$$

$$\cos \theta = \frac{|\vec{A}|^2 - \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{B} - |\vec{B}|^2}{|\vec{A} + \vec{B}| |\vec{A} - \vec{B}|}$$

$$\cos \theta = \frac{|\vec{A}|^2 - |\vec{B}|^2}{|\vec{A} + \vec{B}| |\vec{A} - \vec{B}|}$$

$$\cos \theta = \frac{(3)^2 - (4)^2}{(\sqrt{15})(\sqrt{35})}$$

SELF TEST (12)

Example 2.10

Two vectors $\vec{A}$ and $\vec{B}$ are such that

$$|\vec{A}| = 4 \text{ and } |\vec{B}| = 6 \quad \vec{A} \cdot \vec{B} = 13.5$$

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$$\cos \theta = \frac{9-16}{(3.87)(5.91)}$$

$$\cos \theta = \frac{-7}{22.8717}$$

$$\theta = \cos^{-1}(-0.306)$$

$$\theta = 107.8^\circ \text{ Ans.}$$

Find the magnitude of $|\vec{A} - \vec{B}|$ and $|\vec{A} + \vec{B}|$

$$\sqrt{25}, \sqrt{79} \text{ Ans.}$$

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Q.21. If $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$

$$\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k}$$

Find (a) $\vec{A} \times \vec{B}$ (b) $\vec{B} \times \vec{A}$ (c) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$

Solution:

$$(a) \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} -3 & -1 \\ 4 & -2 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix}$$

$$= \hat{i} \{(-3)(-2) - (-1)(4)\} - \hat{j} \{(2)(-2) - (-1)(1)\} + \hat{k} \{(2)(4) - (-3)(1)\}$$

$$= \hat{i} (+6+4) - \hat{j} (-4+1) + \hat{k} (8+3)$$

$$\vec{A} \times \vec{B} = 10\hat{i} + 3\hat{j} + 11\hat{k} \text{ Ans.}$$

$$(b) \quad \vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -2 \\ 2 & -3 & -1 \end{vmatrix}$$

$$\vec{B} \times \vec{A} = \hat{i} \begin{vmatrix} 4 & -2 \\ -3 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix}$$

$$= \hat{i} (-4-6) - \hat{j} (-1+4) + \hat{k} (-3-8)$$

$$\vec{B} \times \vec{A} = -10\hat{i} - 3\hat{j} - 11\hat{k} \text{ Ans.}$$

(c) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$

$$\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$$

$$\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{A} + \vec{B} = 3\hat{i} + \hat{j} - 3\hat{k}$$

And $\vec{A} = 2\hat{i} - 3\hat{j} - \hat{k}$

$$\vec{B} = \hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{A} - \vec{B} = \hat{i} - 7\hat{j} + \hat{k}$$

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$$\begin{aligned}
 \text{(a)} \quad (\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -7 & 1 \end{vmatrix} \\
 &= \hat{i} \begin{vmatrix} 1 & -3 \\ -7 & 1 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & -3 \\ 1 & 1 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & 1 \\ 1 & -7 \end{vmatrix} \\
 &= \hat{i}(1-21) - \hat{j}(3+3) + \hat{k}(-21-1)
 \end{aligned}$$

$$(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = -20\hat{i} - 6\hat{j} - 22\hat{k} \quad \text{Ans.}$$

Q.22. Determine the unit vector perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. (2011)

Given Data:

$$\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$$

$$\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$$

To Find:

Unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B} = ?$

Solution:

Let $\vec{C}$ be a vector which is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ then,

$$\text{then } \vec{C} = \vec{A} \times \vec{B}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i} \begin{vmatrix} -6 & -3 \\ 3 & -1 \end{vmatrix} - \hat{j} \begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & -6 \\ 4 & 3 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{i}(6+9) - \hat{j}(-2+12) + \hat{k}(6+24)$$

$$\vec{C} = 15\hat{i} - 10\hat{j} + 30\hat{k}$$

Now,

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

$$|\vec{C}| = \sqrt{(15)^2 + (-10)^2 + (30)^2}$$

$$|\vec{C}| = \sqrt{225 + 100 + 900}$$

$$|\vec{C}| = \sqrt{1225}$$

$$|\vec{C}| = 35$$

Unit vector perpendicular to the plane of $\vec{A}$ and $\vec{B} = \hat{C} = \frac{\vec{C}}{|\vec{C}|}$

$$\hat{C} = \frac{15\hat{i} - 10\hat{j} + 30\hat{k}}{35}$$

$$\hat{C} = \frac{15\hat{i}}{35} - \frac{10\hat{j}}{35} + \frac{30\hat{k}}{35}$$

$$\hat{C} = \frac{3\hat{i}}{7} - \frac{2\hat{j}}{7} + \frac{6\hat{k}}{7} \quad \text{Ans.}$$

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