Matrices and Determinants Rawalpindi Board G9 Mathematics notes

Exercise 1.1

Q.1) Find the order of the following matrices

$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$,

$D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$, $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$, $F = \begin{bmatrix} 2 \end{bmatrix}$

$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$, $H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Answer:

$$A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$$

N.B: Order of matrix is (number of rows)-by-(number of columns).
So, order of matrix A is 2-by-2 or 2×2.

$$B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

Order of matrix B is 2-by-2 or 2×2.

$$C = \begin{bmatrix} 2 & 4 \end{bmatrix}$$

Order of matrix C is 1-by-2 or 1×2.

$$D = \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix}$$

Order of matrix D is 3-by-1 or 3×1.

Q.2) Which of the following matrices are equal ?

$A = [3]$, $B = [3 \quad 5]$, $C = [5 - 2]$,

$D = [5 \quad 3]$, $E = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $F = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$,

$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix}$, $H = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$, $I = [3 \quad 3+2]$,

$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix}$

Answer:

In order to find equal matrices, first we simplify C, G, I and J.

$$C = \begin{bmatrix} 5-2 \end{bmatrix} = \begin{bmatrix} 3 \end{bmatrix}$$

$$G = \begin{bmatrix} 3-1 \\ 3+3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 3 & 3+2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \end{bmatrix}$$

$$J = \begin{bmatrix} 2+2 & 2-2 \\ 2+4 & 2+0 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix}$$

Now, observing A, B, C, D, E, F, G, H, I and J we find

$$A = C$$

$$B = I$$

$$E = J = H$$

$$F = G$$

Q.3) Find the values of a , b ,c and d which satisfy the matrix equation

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Answer:

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Two equal matrices have equal corresponding entries, keeping this condition of equal matrices in mind we have a system of four equations as below,

$$a + c = 0 \tag{1.1.1}$$

$$a + 2b = -7 \tag{1.1.2}$$

$$c - 1 = 3 \tag{1.1.3}$$

$$4d - 6 = 2d \tag{1.1.4}$$

Solving 1.1.3,

$$\begin{aligned} c &= 3+1 \\ \mathbf{c} &= \mathbf{4} \end{aligned}$$

Putting the value of "c" in 1.1.1

$$\begin{aligned} a+4 &= 0 \\ \mathbf{a} &= \mathbf{-4} \end{aligned}$$

Putting the value of "a" in 1.1.2,we get

$$\begin{aligned} -4+2b &= -7 \\ 2b &= -7+4 \\ 2b &= -3 \\ b &= \frac{-3}{2} \\ \mathbf{b} &= \mathbf{-1.5} \end{aligned}$$

Solving 1.1.4,

$$\begin{aligned} 4d-6 &= 2d \\ 4d-2d &= 6 \\ 2d &= 6 \\ \mathbf{d} &= \mathbf{3} \end{aligned}$$

Exercise 1.2

Q.1) From the following matrices , identify unit matrices, row matrices , column matrices and null matrices

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = [2 \quad 3 \quad 4]$, $C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$,

$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $E = [\,0\,]$, $F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$

Answer:

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

A is **Null matrix.**

$$B = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

B is **Row matrix.**

$$C = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$$

C is **Column matrix.**

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

D is **Identity matrix.**

$$E = \begin{bmatrix} 0 \end{bmatrix}$$

E is **Null matrix.**

$$F = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

F is **Column matrix.**

Q.2) From the following matrices, identify

(a) Square matrices (b) Rectangular matrices

(c) Row matrices (d) Column matrices

(e) Identity matrices (f) Null matrices

(i) $\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ (vi) $\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$

(vii) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ix) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Answer:

1.

$$\begin{bmatrix} -8 & 2 & 7 \\ 12 & 0 & 4 \end{bmatrix}$$

Rectangular matrix

2.

$$\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Column matrix/Rectangular matrix

3.

$$\begin{bmatrix} 6 & -4 \\ 3 & -2 \end{bmatrix}$$

Square matrix

4.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Identity matrix/Square matrix

5.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Rectangular matrix

6.

$$\begin{bmatrix} 3 & 10 & -1 \end{bmatrix}$$

Row matrix/Rectangular matrix

7.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Column matrix/Rectangular matrix

8.

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Square matrix

9.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Null matrix/Rectangular matrix

Q.3) From the following matrices , identify diagonal , scalar and unit (identity) matrices

$$A=\begin{bmatrix}4&0\\0&4\end{bmatrix},\quad B=\begin{bmatrix}2&0\\0&-1\end{bmatrix},\quad C=\begin{bmatrix}1&0\\0&1\end{bmatrix},$$

$$D=\begin{bmatrix}3&0\\0&0\end{bmatrix},\quad E=\begin{bmatrix}5-3&0\\0&1+1\end{bmatrix}$$

Answer:

$$A=\begin{bmatrix}4&0\\0&4\end{bmatrix}$$

A is **Diagonal matrix/Scalar matrix**.

$$B=\begin{bmatrix}2&0\\0&-1\end{bmatrix}$$

B is **Diagonal matrix**.

$$C=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

C is **Unit matrix/Diagonal matrix**.

$$D=\begin{bmatrix}3&0\\0&0\end{bmatrix}$$

D is **Diagonal matrix**.

$$E=\begin{bmatrix}5-3&0\\0&1+1\end{bmatrix}$$

E is **Diagonal matrix/Scalar matrix**.

Q.4) Find negative of matrices A, B, C, D and E when

$$A=\begin{bmatrix}1\\0\\-1\end{bmatrix},\quad B=\begin{bmatrix}3&-1\\2&1\end{bmatrix},\quad C=\begin{bmatrix}2&6\\3&2\end{bmatrix},$$

$$D=\begin{bmatrix}-3&2\\-4&5\end{bmatrix},\quad E=\begin{bmatrix}1&-5\\2&3\end{bmatrix}$$

Answer:

$$A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \text{then} \quad -A = \begin{bmatrix} -1 \\ 0 \\ +1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \quad \text{then} \quad -B = \begin{bmatrix} -3 & +1 \\ -2 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix} \quad \text{then} \quad -C = \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$

$$D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix} \quad \text{then} \quad -D = \begin{bmatrix} +3 & -2 \\ +4 & -5 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix} \quad \text{then} \quad -E = \begin{bmatrix} -1 & +5 \\ -2 & -3 \end{bmatrix}$$

Q.5) Find the transpose of each of the following matrices:

$$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \quad B = [5 \quad 1 \quad -6], \quad C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer:

$$A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \quad \text{then} \quad A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix} \quad \text{then} \quad B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix} \quad \text{then} \quad C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \quad \text{then} \quad D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

$$E = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} \quad \text{then} \quad E^t = \begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{then} \quad F^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Q.6)

$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

i) $(A^t)^t = A$

ii) $(B^t)^t = B$

Answer:

1.

$$(A^t)^t = A$$

Given

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \text{then} \quad A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Considering

$$L.H.S = (A^t)^t$$

putting the value of A^t

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= A$$

$$= R.H.S$$

2.

$$(B^t)^t = B$$

Given

$$B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \quad \text{then} \quad B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

Considering

$$L.H.S = (B^t)^t$$

putting the value of B^t

$$= \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$= B$$

$$= R.H.S$$

Exercise 1.6

Q.1) Use matrices, if possible, to solve the following systems of linear equations by:

i) the matrix inversion method

ii) the Cramer's rule.

i) $2x - 2y = 4$
$3x + 2y = 6$

ii) $2x + y = 3$
$6x + 5y = 1$

iii) $4x + 2y = 8$
$3x - y = -1$

iv) $3x - 2y = -6$
$5x - 2y = -10$

v) $3x - 2y = 4$
$-6x + 4y = 7$

vi) $4x + y = 9$
$-3x - y = -5$

vii) $2x - 2y = 4$
$-5x - 2y = -10$

viii) $3x - 4y = 4$
$x + 2y = 8$

Answer:

1.

$$2x - 2y = 4 \quad (1.1.1)$$

$$3x + 2y = 6 \quad (1.1.2)$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \quad (1.1.3)$$

$$|A| = 4 + 6 = 10 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{10} \times (2) & \frac{1}{10} \times (2) \\ \frac{1}{10} \times (-3) & \frac{1}{10} \times (2) \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.3 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{10} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5} \times 4 + \frac{1}{5} \times 6 \\ -\frac{3}{10} \times 4 + \frac{1}{5} \times 6 \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{5} + \frac{6}{5} \\ -\frac{6}{5} + \frac{6}{5} \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

So, we have $x = 2$ and $y = 0$

Using Cramer's Rule

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{8+12}{10} \\ &= \frac{20}{2} = 2 \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{12-12}{10} \\ &= \frac{0}{10} = 0 \end{aligned}$$

2.

$$2x + y = 3 \tag{1.1.4}$$

$$6x + 5y = 1 \tag{1.1.5}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.6}$$

$$|A| = 10 - 6 = 4 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= (\frac{1}{4}) \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \times 5 & \frac{1}{4} \times (-1) \\ \frac{1}{4} \times (-6) & \frac{1}{4} \times 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.6 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{5}{4}\times 3 - \frac{1}{4}\times 1 \\ -\frac{3}{2}\times 3\frac{1}{2}\times 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{15}{4} - \frac{1}{4} \\ -\frac{9}{2} + \frac{1}{2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix} \end{aligned}$$

So, we have $x = \frac{7}{2}$ and $y = -4$

Using Cramer's Rule

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 3 \\ 6 & 1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{15-1}{4} \\ &= \frac{14}{4} = \frac{7}{2} \end{aligned}$$

$$\mathbf{y} = \frac{|A_y|}{|A|} = \frac{2-18}{4} = \frac{-16}{4} = -4$$

3.

$$4x + 2y = 8 \tag{1.1.7}$$

$$3x - y = -1 \tag{1.1.8}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.9}$$

$$|A| = -4 - 6 = -10 \neq 0$$

Since A is non singular, so

$$\begin{aligned}
A^{-1} &= \frac{Adj\,A}{|A|} \\
&= (-\frac{1}{10})\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix} \\
&= \begin{bmatrix} -\frac{1}{10}\times(-1) & -\frac{1}{10}\times(-2) \\ -\frac{1}{10}\times(-3) & -\frac{1}{10}\times 4 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{bmatrix}
\end{aligned}$$

Putting the values in 1.1.39 and simplifying, we get

$$\begin{aligned}
X &= A^{-1}B \\
&= \begin{bmatrix} \frac{1}{10} & \frac{1}{5} \\ \frac{3}{10} & -\frac{2}{5} \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{10}\times 8+\frac{1}{5}\times(-1) \\ \frac{3}{10}\times 8-\frac{2}{5}\times(-1) \end{bmatrix} \\
&= \begin{bmatrix} \frac{4}{5}-\frac{1}{5} \\ \frac{12}{5}+\frac{2}{5} \end{bmatrix} \\
&= \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}
\end{aligned}$$

So, we have $x = \frac{3}{5}$ and $y = \frac{14}{5}$

Using Cramer's Rule

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= -\frac{-8+2}{10} \\ &= -\frac{-6}{10} = \frac{3}{5} \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= -\frac{-4-24}{10} \\ &= -\frac{-28}{10} = \frac{14}{5} \end{aligned}$$

4.

$$3x - 2y = -6 \quad (1.1.10)$$

$$5x - 2y = -10 \quad (1.1.11)$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \qquad (1.1.12)$$

$$|A| = -6 + 10 = 4 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj A}{|A|} \\ &= (\frac{1}{4})\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \times (-2) & \frac{1}{4} \times 2 \\ \frac{1}{4} \times (-5) & \frac{1}{4} \times 3 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{5}{4} & \frac{3}{4} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.12 and simplifying, we get

$$
\begin{aligned}
X &= A^{-1}B \\
&= \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{5}{4} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} -6 \\ -10 \end{bmatrix} \\
&= \begin{bmatrix} -\frac{1}{2} \times (-6) + \frac{1}{2} \times (-10) \\ -\frac{5}{4} \times (-6) + \frac{3}{4} \times (-10) \end{bmatrix} \\
&= \begin{bmatrix} 3 - 5 \\ \frac{15}{2} - \frac{15}{2} \end{bmatrix} \\
&= \begin{bmatrix} -2 \\ 0 \end{bmatrix}
\end{aligned}
$$

So, we have $x = -2$ and $y = 0$

Using Cramer's Rule

$$A = \begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} -6 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 3 & -6 \\ 5 & -10 \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{x} &= \frac{|A_x|}{|A|} \\
&= \frac{12 - 20}{4} \\
&= \frac{-8}{4} = -2
\end{aligned}
$$

$$\mathbf{y} = \frac{|A_y|}{|A|} = \frac{-30+30}{4} = \frac{0}{4} = 0$$

5.

$$3x - 2y = -1 \tag{1.1.13}$$

$$-6x + 4y = 7 \tag{1.1.14}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 3 & -2 \\ -6 & +4 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.15}$$

$$|A| = 12 - 12 = 0$$

Since A is singular, so solution is not possible.

6.

$$4x + y = 9 \tag{1.1.16}$$

$$-3x - y = -5 \tag{1.1.17}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.18}$$

$$|A| = -4 + 3 = -1 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj A}{|A|} \\ &= -\begin{bmatrix} -1 & -1 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 \\ -3 & -4 \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.18 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} 1 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} 9 \\ -5 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 9 + 1 \times (-5) \\ (-3) \times 9 + (-4) \times (-5) \end{bmatrix} \\ &= \begin{bmatrix} 9 - 5 \\ -27 + 20 \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ -7 \end{bmatrix} \end{aligned}$$

So, we have $x = 4$ and $y = -7$

Using Cramer's Rule

$$A = \begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 9 & 1 \\ -5 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 9 \\ -3 & -5 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= -(-9 + 5) \\ &= -(-4) = 4 \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= -(-20+27) \\ &= -(7) = -7 \end{aligned}$$

7.

$$2x - 2y = 4 \tag{1.1.19}$$

$$-5x - 2y = -10 \tag{1.1.20}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.21}$$

$$|A| = -4 - 10 = -14 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= -\frac{1}{14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{14} \times (-2) & -\frac{1}{14} \times (2) \\ -\frac{1}{14} \times (5) & -\frac{1}{14} \times (2) \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ -\frac{5}{14} & -\frac{1}{7} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.21 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{7} & -\frac{1}{7} \\ -\frac{5}{14} & -\frac{1}{7} \end{bmatrix} \begin{bmatrix} 4 \\ -10 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{7} \times 4 - \frac{1}{7} \times (-10) \\ -\frac{5}{14} \times 4 - \frac{1}{7} \times (-10) \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{7} + \frac{10}{7} \\ -\frac{10}{7} + \frac{10}{7} \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

So, we have $x = 2$ and $y = 0$

Using Cramer's Rule

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= -\frac{-8-20}{14} \\ &= -\frac{-28}{14} = 2 \end{aligned}$$

$$
\begin{aligned}
\mathbf{y} &= \frac{|A_y|}{|A|} \\
&= -\frac{-20+20}{14} \\
&= -\frac{0}{14} = 0
\end{aligned}
$$

8.

$$3x - 4y = 4 \tag{1.1.22}$$

$$x + 2y = 8 \tag{1.1.23}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.24}$$

$$|A| = 4 + 6 = 10 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{10}\times(2) & \frac{1}{10}\times(4) \\ \frac{1}{10}\times(-1) & \frac{1}{10}\times(3) \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.24 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5}\times 4+\frac{2}{5}\times 8 \\ -\frac{1}{10}\times 4+\frac{3}{10}\times 8 \end{bmatrix} \\ &= \begin{bmatrix} \frac{4}{5}+\frac{16}{5} \\ -\frac{2}{5}+\frac{12}{5} \end{bmatrix} \\ &= \begin{bmatrix} 4 \\ 2 \end{bmatrix} \end{aligned}$$

So, we have $x = 4$ and $y = 2$

Using Cramer's Rule

$$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{8+32}{10} \\ &= \frac{40}{2} = 4 \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{24-4}{10} \\ &= \frac{20}{10} = 2 \end{aligned}$$

Q.2) Solve the follwoing word problems by using

i) Matrix inversion

ii) Crammer's rule

The length of a rectangle is 4 times its width. The perimeter of the rectangle is 150 cm. Find the dimensions of the rectangle.

Answer:

Rectangle is quadrilateral and has length and width.
Suppose,

$$\text{length of rectangle} = x$$

$$\text{width of rectangle} = y$$

According to given conditions, we have two equations

$$4x - y = 0 \tag{1.1.25}$$

$$2x + 2y = 150 \tag{1.1.26}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.27}$$

$$|A| = 8 + 2 = 10 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= \frac{1}{10}\begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{10} \times 2 & \frac{1}{10} \times 1 \\ \frac{1}{10} \times (-2) & \frac{1}{10} \times 4 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5} & \frac{1}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.27 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{5} & \frac{1}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 0 \\ 150 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{5} \times 0 + \frac{1}{10} \times 150 \\ -\frac{1}{5} \times 0 + \frac{2}{5} \times 150 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 15 \\ 0 + 60 \end{bmatrix} \\ &= \begin{bmatrix} 15 \\ 60 \end{bmatrix} \end{aligned}$$

So, we have $x = width = 15cm$ and $y = length = 60cm$

Using Cramer's Rule

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 0 & -1 \\ 150 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 4 & 0 \\ 2 & 150 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{0-(-150)}{10} \\ &= \frac{150}{10} = 15 \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{4(150)-0}{10} \\ &= \frac{600}{10} = 60 \end{aligned}$$

Q.3) Solve the follwoing word problems by using
(i) Matrix inversion
(ii) Crammer's rule
Two sides of a rectangle differ by 3.5cm. Find the dimensions of the rectangle if its perimeter is 67cm.
Answer:

Rectangle is quadrilateral and has length and width.
Suppose,

$$\text{length of rectangle} = x$$

$$\text{width of rectangle} = y$$

According to given conditions, we have two equations

$$x - y = 3.5 \tag{1.1.28}$$

$$2x + 2y = 67 \tag{1.1.29}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3.5 \\ 67 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.30}$$

$$|A| = 2 + 2 = 4 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj A}{|A|} \\ &= \frac{1}{4}\begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4}\times 2 & \frac{1}{4}\times 1 \\ \frac{1}{4}\times(-2) & \frac{1}{4}\times 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.30 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 3.5 \\ 67 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2}\times(3.5)+\frac{1}{4}\times 67 \\ -\frac{1}{2}\times(3.5)+\frac{1}{4}\times 67 \end{bmatrix} \\ &= \begin{bmatrix} \frac{3.5}{2}+\frac{67}{4} \\ -\frac{3.5}{2}+\frac{67}{4} \end{bmatrix} \\ &= \begin{bmatrix} 18.5 \\ 15 \end{bmatrix} \end{aligned}$$

So, we have $x = width = 18.5cm$ and $y = length = 15cm$

Using Cramer's Rule

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 3.5 & -1 \\ 67 & 2 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 2 & 67 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{7+67}{4} \\ &= \frac{74}{4} = 18.5 \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{67-7}{4} \\ &= \frac{60}{4} = 15 \end{aligned}$$

Q.4) Solve the follwoing word problems by using
i) Matrix inversion
ii) Crammer's rule
The third angle of an isosceles triangle is 16° less than the sum of the two equal angles. Find three angles of the triangle.
Answer:

Suppose,

$$\text{measure of each equal angle} = x$$

$$\text{Third angle} = y$$

According to given conditions, we have two equations

$$2x - y = 16 \quad (1.1.31)$$

$$2x + y = 180 \quad (1.1.32)$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 16 \\ 180 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \quad (1.1.33)$$

$$|A| = 2 + 2 = 4 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= \frac{1}{4}\begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \times 1 & \frac{1}{4} \times 1 \\ \frac{1}{4} \times (-2) & \frac{1}{4} \times 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.33 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 16 \\ 180 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{4} \times 16 + \frac{1}{4} \times 180 \\ -\frac{1}{2} \times 16 + \frac{1}{2} \times 180 \end{bmatrix} \\ &= \begin{bmatrix} 4 + 45 \\ -8 + 90 \end{bmatrix} \\ &= \begin{bmatrix} 49 \\ 82 \end{bmatrix} \end{aligned}$$

So, we have $x = 49°$ and $y = 82°$

Using Cramer's Rule

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 16 & -1 \\ 180 & 1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & 16 \\ 2 & 180 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{16 + 180}{4} \\ &= \frac{196}{4} = 49° \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{360 - 32}{4} \\ &= \frac{328}{4} = 82° \end{aligned}$$

Q.5) Solve the follwoing word problems by using

i) Matrix inversion

ii) Crammer's rule

One acute angle of a right triangle is 12° more than twice the other acute angle. Find the acute angles of the right triangle.

Answer:

Suppose,

$$\text{One acute angle} = x$$

$$\text{other acute angle} = y$$

According to given conditions, we have two equations

$$2x - y = -12 \tag{1.1.34}$$

$$x + y = 90 \tag{1.1.35}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -12 \\ 90 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.36}$$

$$|A| = 2 + 1 = 3 \neq 0$$

Since A is non singular, so

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{Adj\,A}{|A|} \\ &= \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} \times 1 & \frac{1}{3} \times 1 \\ \frac{1}{3} \times (-1) & \frac{1}{3} \times 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.36 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -12 \\ 90 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} \times (-12) + \frac{1}{3} \times 90 \\ -\frac{1}{3} \times (-12) + \frac{2}{3} \times 90 \end{bmatrix} \\ &= \begin{bmatrix} -4+30 \\ 4+60 \end{bmatrix} \\ &= \begin{bmatrix} 26 \\ 64 \end{bmatrix} \end{aligned}$$

So, we have $x = 26°$ and $y = 64°$

Using Cramer's Rule

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} -12 & -1 \\ 90 & 1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 2 & -12 \\ 1 & 90 \end{bmatrix}$$

$$\begin{aligned} \mathbf{x} &= \frac{|A_x|}{|A|} \\ &= \frac{-12+90}{3} \\ &= \frac{78}{3} = 26° \end{aligned}$$

$$\begin{aligned} \mathbf{y} &= \frac{|A_y|}{|A|} \\ &= \frac{180+12}{3} \\ &= \frac{192}{3} = 64° \end{aligned}$$

Q.6) Solve the follwoing word problems by using

i) Matrix inversion

ii) Crammer's rule

Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after 4.5 hours. Find the speed of each car.

Answer:

Suppose,

$$\text{Speed of car A} = x$$

$$\text{Speed of car B} = y$$

$$\text{Distance covered by Car A } 4\frac{1}{2} \text{ hours} = \frac{9}{2}x$$

$$\text{Distance covered by Car A } 4\frac{1}{2} \text{ hours} = \frac{9}{2}y$$

According to given conditions, we have two equations

$$\begin{aligned} \frac{9}{2}x + \frac{9}{2}y &= 600 - 123 \\ &= 477 \end{aligned}$$

$$x + y = 106 \tag{1.1.37}$$

$$x - y = 6 \tag{1.1.38}$$

Writing the system in matrix form, we have

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 106 \\ 6 \end{bmatrix}$$

where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 106 \\ 6 \end{bmatrix}$$

In order to find values of X, we have equation to solve

$$X = A^{-1}B \tag{1.1.39}$$

$$|A| = -1 - 1 = -2 \neq 0$$

Since A is non singular, so

$$\begin{aligned} A^{-1} &= \frac{AdjA}{|A|} \\ &= (-\frac{1}{2})\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2}\times(-1) & -\frac{1}{2}\times(-1) \\ -\frac{1}{2}\times(-1) & -\frac{1}{2}\times 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \end{aligned}$$

Putting the values in 1.1.39 and simplifying, we get

$$\begin{aligned} X &= A^{-1}B \\ &= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}\begin{bmatrix} 106 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2}\times 106+\frac{1}{2}\times 6 \\ \frac{1}{2}\times 106-\frac{1}{2}\times 6 \end{bmatrix} \\ &= \begin{bmatrix} 53+3 \\ 53-3 \end{bmatrix} \\ &= \begin{bmatrix} 56 \\ 50 \end{bmatrix} \end{aligned}$$

So, we have $x = 56km/hr$ and $y = 50km/hr$

Using Cramer's Rule

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$A_x = \begin{bmatrix} 106 & 1 \\ 6 & -1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 106 \\ 1 & 6 \end{bmatrix}$$

$$
\begin{aligned}
\mathbf{x} &= \frac{|A_x|}{|A|} \\
&= -\frac{-106-6}{2} \\
&= -\frac{-112}{2} = 56
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{y} &= \frac{|A_y|}{|A|} \\
&= -\frac{6-106}{2} \\
&= -\frac{-100}{2} = 50
\end{aligned}
$$