

UNIT
8
SETS AND FUNCTIONS
SHORT QUESTIONS

Q.1- Define a set and write some well-known sets of numbers.

Ans:

Set:- A collection of well defined distinct objects is called a "Set". For example a collection of students of 9th class, members of a cricket team etc.

Sets of Numbers:-

Set of Natural Numbers = $N = \{1, 2, 3, \dots\}$

Set of Whole Numbers = $W = \{0, 1, 2, 3, \dots\}$

Set of Integers = $Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$

Set of Even Numbers = $E = \{\dots -4, -2, 0, 2, 4, \dots\}$

Set of Odd Numbers = $O = \{\dots -3, -1, 1, 3, 5, \dots\}$

Set of Prime Numbers = $P = \{2, 3, 5, 7, 11, 13, 17, \dots\}$

Q.2- If $A = \{2, 3, 5, 7, 11\}$

$B = \{1, 3, 5, 7, 9\}$

Find $A \cup B$ and $A \cap B$

Solution:-

$$\begin{aligned} A \cup B &= \{2, 3, 5, 7, 11\} \cup \{1, 3, 5, 7, 9\} \\ &= \{1, 2, 3, 5, 7, 9, 11\} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{2, 3, 5, 7, 11\} \cap \{1, 3, 5, 7, 9\} \\ &= \{3, 5, 7\} \end{aligned}$$

Q.3- If $A = \{2,3,4,5\}$, $B = \{2,4,6,8\}$. Then find $A - B$ and $B - A$.

Solution:-

$$\begin{aligned} A - B &= \{2,3,4,5\} - \{2,4,6,8\} \\ &= \{3,5\} \end{aligned}$$

$$\begin{aligned} B - A &= \{2,4,6,8\} - \{2,3,4,5\} \\ &= \{6,8\} \end{aligned}$$

Q.4- If $U = \{1,2,3,4,5,6,7\}$, $A = \{3,4,5\}$, $B = \{1,3,5,7\}$. Find $(A \cup B)'$ and $(A \cap B)'$.

Solution:-

$$\begin{aligned} A \cup B &= \{3,4,5\} \cup \{1,3,5,7\} \\ &= \{1,3,4,5,7\} \end{aligned}$$

$$\begin{aligned} (A \cup B)' &= U - (A \cup B) \\ &= \{1,2,3,4,5,6,7\} - \{1,3,4,5,7\} \\ &= \{2,6\} \end{aligned}$$

$$\begin{aligned} A \cap B &= \{3,4,5\} \cap \{1,3,5,7\} \\ &= \{3,5\} \end{aligned}$$

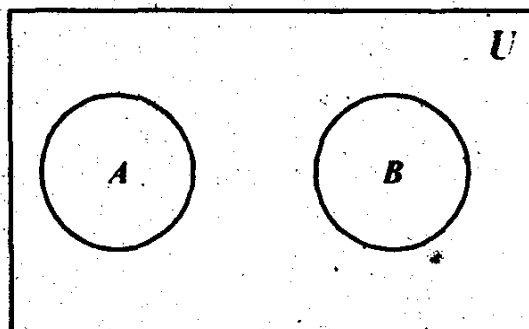
$$\begin{aligned} (A \cap B)' &= U - (A \cap B) \\ &= \{1,2,3,4,5,6,7\} - \{3,5\} \\ &= \{1,2,4,6,7\} \end{aligned}$$

Q.5- Show two sets A and B by Venn Diagram When.

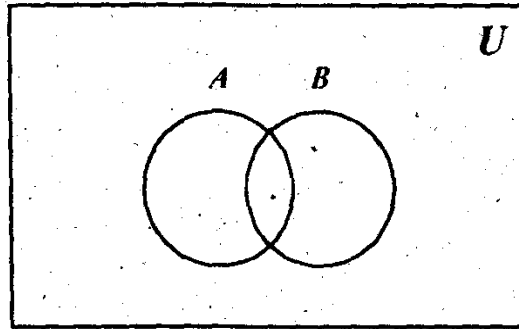
- (i) They are disjoint
- (ii) They are overlapping

Solution:-

- (i) The figure shows that A and B are disjoint.



- (ii) The figure given below shows that A and B are overlapping.



Q.6- State De-Morgan's Laws.

Ans. These laws state that

$$(i) (A \cup B)^c = A^c \cap B^c$$

$$(ii) (A \cap B)^c = A^c \cup B^c$$

Q.7- If $A = \{3,5,6\}$, $B = \{1,3\}$ then find $A \times B$ and $B \times A$.

Ans. $A \times B = \{3,5,6\} \times \{1,3\}$.

$$= \{(3,1), (3,3), (5,1), (5,3), (6,1), (6,3)\}$$

$$B \times A = \{1,3\} \times \{3,5,6\}$$

$$= \{(1,3), (1,5), (1,6), (3,3), (3,5), (3,6)\}$$

Q.8- Define a binary relation from a set A to set B .

Ans. If A and B are two non empty sets then any subset of $A \times B$ is called a binary relation from A to B .

Q.9- If $A = \{1,2,3\}$, $B = \{3,4\}$. Find any two binary relations from A to B .

Ans. $A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

$$R_1 = \{(1,3), (2,4), (3,3)\}$$

$$R_2 = \{(1,4), (3,4)\}$$

Q.10- Define Domain and Range of a binary relation.

Ans. If R is a binary relation. Then Domain of R is the set of all first elements of ordered pairs in R . The set of all second elements of ordered pairs in R is called Range of R .

Example:

$$R = \{(1,3), (2,4), (3,5), (4,6)\}$$

Dom $R = \{1, 2, 3, 4, \}$

Rng $R = \{3, 4, 5, 6, \}$

Q.11- Define a function from a set A to the set B.

Ans. Let A and B are two non empty sets and f is a binary relation from A to B such that

(i) Domain, $f = A$

(ii) There is no repetition in the first elements of ordered pairs in f . Then f is said to be a function from A to B . It is expressed as $f: A \rightarrow B$

Q.12- Let $A = \{l, m, n\}$, $B = \{3, 5, 7\}$

Show that $f = \{(l, 3), (m, 3), (n, 3)\}$ is a function from A to B .

Solution:-

(i) Domain, $f = \{l, m, n\} = A$

First condition is satisfied.

(ii) All the three ordered pairs in f have different first elements and there is no repetition of first elements.

So 2nd condition is also satisfied.

Thus f is a function from A to B

Q.13- Define an into function?

Solution:-

Let f be a function from A to B then f is called a function from A into B if

Range of $f \neq B$

Example:

If $A = \{a, b, c\}$, $B = \{x, y\}$

Then $f = \{(a, x), (b, x), (c, x)\}$ is an into function (from A into B)

Q.14- Define an Onto function.

Ans. Let f be a function from A to B such that

Range : $f = B$.

Then f is called a function from A onto B .

Example:

Let $A = \{p, q, r\}$, $B = \{x, y, z\}$

Then $f = \{(p, x), (q, y), (r, z)\}$ is a function from A onto B

Because, Range $f = \{x, y, z\} = B$

Q.15- Define a one-one function.

Ans. Let $f : A \rightarrow B$ is a function such that second element of each ordered pairs in f is also not repeated.

Example:

$f = \{(a, x), (b, y), (c, z)\}$

It is a one-one function.

**Q.16- Let $X = \{7, 8, 9\}$, $Y = \{d, e, f\}$
and $h = \{(7, e), (8, d), (9, f)\}$**

Show that h is a one-one function from A onto B .

Solution:-

(i) Domain $h = \{7, 8, 9\} = X$

(ii) No first element is repeated in h . So h is a function from x to y .

(iii) Range : $h = \{d, e, f\} = Y$

So h is an onto function.

Now again non of the second elements is repeated.

So this function is one-one function.

SOLVED EXERCISES**EXERCISE 8.1**

Q.1- If $A = \{1, 4, 7, 8\}$, $B = \{4, 6, 8, 9\}$

and $C = \{3, 4, 5, 7\}$ Find:

(i) $A \cup B$ (ii) $B \cup C$ (iii) $A \cap C$ (iv) $A \cap (B \cap C)$

(v) $A \cup (B \cup C)$ (vi) $A \cap (B \cap C)$

Solution:-

(i) $A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\} = \{1, 4, 6, 7, 8, 9\}$ Ans.

(ii) $B \cup C = \{4, 6, 8, 9\} \cup \{3, 4, 5, 7\} = \{3, 4, 5, 6, 7, 8, 9\}$ Ans.

(iii) $A \cap C = \{1, 4, 7, 8\} \cap \{3, 4, 5, 7\} = \{4, 7\}$ Ans.

(iv) $A \cap (B \cap C) = ?$

$(B \cap C) = \{4, 6, 8, 9\} \cap \{3, 4, 5, 7\} = \{4\}$

Now $A \cap (B \cap C)$

$= \{1, 4, 7, 8\} \cap \{4\} \quad \because (B \cap C) = \{4\} = \{4\}$ Ans.

(v) $(A \cup B) \cup C = ?$

$(A \cup B) = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\} = \{1, 4, 6, 7, 8, 9\}$

Now

$(A \cup B) \cup C = \{1, 4, 6, 7, 8, 9\} \cup \{3, 4, 5, 7\}$
 $= \{1, 3, 4, 5, 6, 7, 8, 9\}$ Ans.

(vi) $(A \cap B) \cap C = ?$

$A \cap B = \{1, 4, 7, 8\} \cap \{4, 6, 8, 9\} = \{4, 8\}$

Now $(A \cap B) \cap C = \{4, 8\} \cap \{3, 4, 5, 7\} = \{4\}$ Ans.

Q.2- If $A = \{1, 7, 11, 15, 17, 21\}$, $B = \{11, 17, 19, 23\}$
 and $C = \{2, 3, 5\}$.

Verify that: $(A \cap B) \cap C = A \cap (B \cap C)$

Solution:-

$A \cap B = \{1, 7, 11, 15, 17, 21\} \cap \{11, 17, 19, 23\}$

$A \cap B = \{11, 17\}$

Now $(A \cap B) \cap C = \{11, 17\} \cap \{2, 3, 5\}$

$(A \cap B) \cap C = \{\} = \phi \dots (1)$

Now $B \cap C = \{11, 17, 19, 23\} \cap \{2, 3, 5\} = \{\} = \phi$

$A \cap (B \cap C) = \{1, 7, 11, 15, 17, 21\} \cap \phi$

$A \cap (B \cap C) = \phi \dots (2)$

Results (1) and (2) show that

$(A \cap B) \cap C = A \cap (B \cap C)$

Q.3- If $A = \{2, 4, 6\}$, $B = \{3, 6, 9, 12\}$ and $C = \{4, 6, 8, 10\}$
 verify that: $A \cup (B \cap C) = (A \cup B) \cap C$

Solution:-

$A = \{2, 4, 6\}$, $B = \{3, 6, 9, 12\}$

$$C = \{4, 6, 8, 10\}$$

We have to show that $A \cup (B \cup C) = (A \cup B) \cup C$

To solve the L.H.S.

$$B \cup C = \{3, 6, 9, 12\} \cup \{4, 6, 8, 10\}$$

$$= \{3, 4, 6, 8, 9, 10, 12\}$$

$$A \cup (B \cup C) = \{2, 4, 6\} \cup \{3, 4, 6, 8, 9, 10, 12\}$$

$$A \cup (B \cup C) = \{2, 3, 4, 6, 8, 9, 10, 12\} \dots (1)$$

Now to solve the R.H.S. Consider

$$A \cup B = \{2, 4, 6\} \cup \{3, 6, 9, 12\}$$

$$A \cup B = \{2, 3, 4, 6, 9, 12\}$$

$$(A \cup B) \cup C = \{2, 3, 4, 6, 9, 12\} \cup \{4, 6, 8, 10\}$$

$$(A \cup B) \cup C = \{2, 3, 4, 6, 8, 9, 10, 12\} \dots (2)$$

Results (1) and (2) show that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Q.4- If $A = \{2, 3, 5, 7, 9\}$, $B = \{1, 3, 5, 7\}$

and $C = \{2, 3, 4, 5, 6\}$

verify that: $(A \cap B) \cap C = A \cap (B \cap C)$

Solution:- We are given that

$$A = \{2, 3, 5, 7, 9\}, B = \{1, 3, 5, 7\}$$

$$C = \{2, 3, 4, 5, 6\}$$

We have to prove that

$$(A \cap B) \cap C = A \cap (B \cap C)$$

First we will solve L.H.S. Consider

$$A \cap B = \{2, 3, 5, 7, 9\} \cap \{1, 3, 5, 7\} = \{3, 5, 7\}$$

$$(A \cap B) \cap C = \{3, 5\} \dots (1)$$

Now we will solve R.H.S. Consider

$$B \cap C = \{1, 3, 5, 7\} \cap \{2, 3, 4, 5, 6\}$$

$$B \cap C = \{3, 5\}$$

$$\text{Now } A \cap (B \cap C) = \{2, 3, 5, 7, 9\} \cap \{3, 5\}$$

$$A \cap (B \cap C) = \{3, 5\} \dots (2)$$

Results (1) and (2) show that

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Q.5- If $U = \{7, 8, 9, 10, 11, 12, 13, 14\}$

$$A = \{7, 10, 13, 14\}$$

and $B = \{7, 8, 11, 12\}$ then

verify $(A \cap B)^c = A^c \cup B^c$

Solution:- We are given that

$$U = \{7, 8, 9, 10, 11, 13, 14\}$$

$$A = \{7, 10, 13, 14\}$$

$$B = \{7, 8, 11, 12\}$$

We are to verify $(A \cap B)^c = (A^c \cup B^c)$.

To solve L.H.S.

$$A \cap B = \{7, 10, 13, 14\} \cap \{7, 8, 11, 12\} = \{7\}$$

$$(A \cap B)^c = U - (A \cap B)$$

$$= \{7, 8, 9, 10, 11, 12, 13, 14\} - \{7\}$$

$$(A \cap B)^c = \{8, 9, 10, 11, 12, 13, 14\} \dots (1)$$

Now to solve R.H.S.

$$A^c = U - A = \{7, 8, 9, \dots, 14\} - \{7, 10, 13, 14\}$$

$$= \{8, 9, 11, 12\}$$

$$B^c = U - B = \{7, 8, 9, \dots, 14\} - \{7, 8, 11, 12\}$$

$$= \{9, 10, 13, 14\}$$

$$A^c \cup B^c = \{8, 9, 11, 12\} \cup \{9, 10, 13, 14\}$$

$$A^c \cup B^c = \{8, 9, 10, 11, 12, 13, 14\} \dots (2)$$

Results (1) and (2) show that

$$(A \cap B)^c = A^c \cup B^c$$

Q.6- If $U = \{4, 6, 8, 9, 10\}$ $A = \{4, 6\}$ $B = \{6, 8, 9\}$

We are to verify De Morgans Laws

$$(A \cup B)^c = A^c \cap B^c \text{ and } (A \cap B)^c = A^c \cup B^c$$

Solution:-

First Consider $(A \cup B)^c = A^c \cap B^c$

To solve L.H.S.

$$A \cup B = \{4, 6\} \cup \{6, 8, 9\}$$

$$A \cup B = \{4, 6, 8, 9\}$$

$$\begin{aligned}(A \cup B)^c &= U - (A \cup B) \\ &= \{4, 6, 8, 9, 10\} - \{4, 6, 8, 9\}\end{aligned}$$

$$(A \cup B)^c = \{10\} \dots (1)$$

Now to solve R.H.S.

$$\begin{aligned}A^c &= U - A = \{4, 6, 8, 9, 10\} - \{4, 6\} \\ &= \{8, 9, 10\}\end{aligned}$$

$$\begin{aligned}B^c &= U - B = \{4, 6, 8, 9, 10\} - \{6, 8, 9\} \\ &= \{4, 10\}\end{aligned}$$

$$\begin{aligned}\text{Now } A^c \cap B^c &= \{8, 9, 10\} \cap \{4, 10\} \\ &= \{10\} \dots (2)\end{aligned}$$

Results (1) and (2) show that

$$(A \cup B)^c = A^c \cap B^c$$

Now take De. Morgans 2nd law

$$(A \cap B)^c = A^c \cup B^c$$

To solve the L.H.S.

$$A \cap B = \{4, 6\} \cap \{6, 8, 9\} = \{6\}$$

$$\begin{aligned}(A \cap B)^c &= U - (A \cap B) \\ &= \{4, 6, 8, 9, 10\} - \{6\}\end{aligned}$$

$$(A \cap B)^c = \{4, 8, 9, 10\} \dots (1)$$

Now

$$\begin{aligned}A^c &= U - A = \{4, 6, 8, 9, 10\} - \{4, 6\} \\ &= \{8, 9, 10\}\end{aligned}$$

$$B^c = U - B = \{4, 6, 8, 9, 10\} - \{6, 8, 9\}$$

$$B^c = \{4, 10\}$$

$$\begin{aligned}A^c \cup B^c &= \{8, 9, 10\} \cup \{4, 10\} \\ &= \{4, 8, 9, 10\} \dots (2)\end{aligned}$$

Results (1) and (2) show that

$$(A \cap B)^c = A^c \cup B^c$$

Q.7- If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 $A = \{2, 3, 6, 9\}$
 and $B = \{1, 3, 6, 7, 8\}$ then
 verify $(A \cup B)^c = A^c \cap B^c$

Solution:- We are to prove that

$$(A \cup B)^c = A^c \cap B^c$$

To solve L.H.S.

$$\begin{aligned} A \cup B &= \{2, 3, 6, 9\} \cup \{1, 3, 6, 7, 8\} \\ &= \{1, 2, 3, 6, 7, 8, 9\} \end{aligned}$$

$$\begin{aligned} (A \cup B)^c &= U - (A \cup B) \\ &= \{1, 2, 3, \dots, 9\} - \{1, 2, 3, 6, 7, 8, 9\} \end{aligned}$$

$$(A \cup B)^c = \{4, 5\} \dots (1)$$

Now to solve R.H.S.

$$\begin{aligned} A^c &= U - A = \{1, 2, 3, \dots, 9\} - \{2, 3, 6, 9\} \\ &= \{1, 4, 5, 7, 8\} \end{aligned}$$

$$\begin{aligned} B^c &= U - B = \{1, 2, 3, \dots, 9\} - \{1, 3, 6, 7, 8\} \\ &= \{2, 4, 5, 9\} \end{aligned}$$

$$A^c \cap B^c = \{1, 4, 5, 7, 8\} \cap \{2, 4, 5, 9\}$$

$$A^c \cap B^c = \{4, 5\} \dots (2)$$

From (1) and (2). We get.

$$(A \cup B)^c = A^c \cap B^c$$

Q.8- Fill in the blanks:

- | | |
|--|---|
| (i) $A \cup A = \underline{\hspace{2cm}}$ | (ii) $A \cap A = \underline{\hspace{2cm}}$ |
| (iii) $A \cup \Phi = \underline{\hspace{2cm}}$ | (iv) $A \cap \Phi = \underline{\hspace{2cm}}$ |
| (v) $\Phi \cup \Phi = \underline{\hspace{2cm}}$ | (vi) $(A \cap B)' = \underline{\hspace{2cm}}$ |
| (vii) $(A \cap B)' = \underline{\hspace{2cm}}$ | (viii) $(A')' = \underline{\hspace{2cm}}$ |
| (ix) $\Phi \cap \Phi = \underline{\hspace{2cm}}$ | (x) $A \cap A' = \underline{\hspace{2cm}}$ |

Solution:-

- | | |
|---|---|
| (i) $A \cup A = \underline{A}$ | (ii) $A \cap A = \underline{A}$ |
| (iii) $A \cup \Phi = \underline{A}$ | (iv) $A \cap \Phi = \underline{\Phi}$ |
| (v) $\Phi \cap \Phi = \underline{\Phi}$ | (vi) $(A \cap B)' = \underline{A' \cup B'}$ |

$$(vii) \quad (A \cup B)' = \underline{A' \cap B'} \quad (viii) \quad (A')' = \underline{A}$$

$$(ix) \quad \Phi \cap \Phi' = \underline{\Phi} \quad (x) \quad A \cap A' = \underline{\Phi}$$

EXERCISE 8.2

Q.1- If $A = \{3, 5, 6\}$, $B = \{1, 3\}$, Find $A \times B$ and $B \times A$ also the domains and ranges of the two binary relations established at your own for each case.

Solution:-

$$A = \{3, 5, 6\}, \quad B = \{1, 3\}$$

$$A \times B = \{(3, 1), (3, 3), (5, 1), (5, 3), (6, 1), (6, 3)\}$$

$$B \times A = \{(1, 3), (1, 5), (1, 6), (3, 3), (3, 5), (3, 6)\}$$

Two binary relations in $A \times B$ are

$$R_1 = \{(3, 1), (5, 3), (5, 1)\}$$

$$R_2 = \{(3, 1), (3, 3), (5, 3), (6, 3)\}$$

$$\text{Dom } R_1 = \{3, 5\}, \quad \text{Range } R_1 = \{1, 3\}$$

$$\text{Dom } R_2 = \{3, 5, 6\}, \quad \text{Range } R_2 = \{1, 3\}$$

Two binary relations in $B \times A$ are

$$R_3 = \{(1, 3), (1, 6), (3, 3)\}$$

$$R_4 = \{(1, 5), (3, 5)\}$$

$$\text{Dom } R_3 = \{1, 3\}, \quad \text{Range } R_3 = \{3, 6\}$$

$$\text{Dom } R_4 = \{1, 3\}, \quad \text{Range } R_4 = \{5\}$$

Q.2- If $A = \{-2, 1, 4\}$, then write two binary relations in A also write their domains and ranges.

Solution:-

$$A = \{-2, 1, 4\}$$

$$A \times A = \{-2, 1, 4\} \times \{-2, 1, 4\}$$

$$= \{(-2, -2), (-2, 1), (-2, 4), (1, -2), (1, 1)$$

$$(1, 4), (4, -2), (4, 1), (4, 4)\}$$

Now any subset of $A \times A$ is a binary relation in A .

Thus two binary relations are

$$R_1 = \{(-2, -2), (1, -2), (4, 1)\}$$

$$R_1 = \{(-2, 1), (1, 1), (4, 1)\}$$

$$\text{Dom } R_1 = \text{set of first elements of ordered pairs in } R_1 \\ = \{-2, 1, 4\}$$

$$\text{Rang } R_1 = \text{set of 2nd elements of ordered pairs in } R_1 \\ = \{1\}$$

Similarly,

$$\text{Dom } R_2 = \{-2, 1, 4\}, \text{ Rang } R_2 = \{1\}$$

Q.3- Write the number of binary relations possible in each of following cases.

- (i) In $C \times C$ when the number of elements in C is 3.
 (ii) In $A \times B$ if the number of elements in set A is 3 and in set B is 4.

Solution:-

- (i) Numbers of elements in $C = 3$
 Numbers of elements in $C \times C = 3 \times 3 = 9$
 So, number of binary relations in $C \times C$
 $=$ Number of all subsets of $C \times C$
 $= 2^9$ Ans.

- (ii) Numbers of elements in $A = 3$
 Numbers of elements in $B = 4$
 Thus, Numbers of elements in $A \times B = 3 \times 4 = 12$
 So, Number of all subsets of $A \times B = 2^{12}$
 and number of all possible binary relations in
 $A \times B = 2^{12}$ Ans.

Q.4- If $L = \{1, 2, 3\}$, and $M = \{2, 3, 4\}$, then write a binary relation R such that

$$R = \{(x, y) \mid x \in L, y \in M \wedge y \leq x\}$$

Also write $\text{Dom}(R)$ and $\text{Range}(R)$.

Solution:-

$$L = \{1, 2, 3\}, M = \{2, 3, 4\}$$

$$L \times M = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3)\}$$

$$\{(2, 4), (3, 2), (3, 3), (3, 4)\}$$

$$\text{Now } R = \{(x, y) / x \in L, y \in M \wedge y \leq x\}$$

$$R = \{(2, 2), (3, 2), (3, 3)\}$$

$$\text{Dom}(R) = \{2, 3\}, \quad \text{Rng}(R) = \{2, 3\}$$

Q.5- If $X = \{0, 3, 5\}$ and $Y = \{2, 4, 8\}$, then establish any four binary relations in $X \times Y$.

Solution:-

$$X \times Y = \{(0, 2), (0, 4), (0, 8), (3, 2), (3, 4), (3, 8), (5, 2), (5, 4), (5, 8)\}$$

Binary relation in $X \times Y$ is any subset of $X \times Y$. So four binary relations in $X \times Y$ are.

$$R_1 = \{(0, 2), (3, 2), (5, 2)\}$$

$$R_2 = \{(0, 4), (0, 8), (3, 2), (5, 8)\}$$

$$R_3 = \{(0, 8), (3, 4), (5, 2)\}$$

$$R_4 = \{(5, 2), (5, 4), (5, 8)\}$$

Q.6- If $A = \{a, b, c\}$ and $B = \{2, 4, 6\}$ and $f = \{(a, 4), (b, 4), (c, 4)\}$ is a binary relation from $A \times B$ then show that "f" is a function from A into B

Solution:-

$$f = \{(a, 4), (b, 4), (c, 4)\}$$

$$\text{Dom } f = \{a, b, c\} = A$$

Now we see that non of the 1st elements of ordered pairs in f is repeated. So f is a function from A to B .

$$\text{Now Range}(f) = \{4\} \neq B$$

It means f is a function from A into B .

Q.7- If $A = \{l, m, n\}$ and $B = \{2, 4, 6\}$ and $g = \{(l, 3), (m, 1), (n, 1)\}$ is a binary relation in $A \times B$, then show that "g" is A into B function.

Solution:-

$$g = \{(l, 3), (m, 1), (n, 1)\}$$

$$\text{Dom } (g) = \{l, m, n\} = A$$

We see that non of the first elements ordered pairs in g is repeated.

So g is a function from A to B .

Now $\text{Rng}(g) = \{1, 3\} \neq B$

It shows that g is a funtion from A into B .

Q.8- If $A = \{1, 3, 5\}$ and $B = \{x, y, z\}$
and $g = \{(1, x), (3, y), (5, z)\}$ is a binary relation from
 $A \times B$, then show that " g " is A onto B function.

Solution:-

$g = \{(1, x), (3, y), (5, z)\}$

$\text{Dom}(g) = \{1, 3, 5\}$

Also non of 1st elemnets of ordered pairs in g is repeated. So g is a function from A to B .

Now $\text{Rng}(g) = \{x, y, z\} = B$.

It shows that g is a function from A onto B .

Review Exercise 8

Q.1- Encircle the correct answer.

- (i) If A and B are two non-empty sets, then $A \cup B = ?$
 (a) Φ (b) $B \cup A$ (c) $A \cap B$ (d) $B \cap A$
- (ii) If A and B are two non-empty overlapping sets, then
 $A \cap B = ?$
 (a) Φ (b) $B \cap A$ (c) $A \cup B$ (d) $B \cup A$
- (iii) For any two sets A and B , $A \cup B = B \cup A$ is called:
 (a) Commutative law (b) Associative law
 (c) De-morgan's law (d) Intersection of two sets
- (iv) $A \cup (B \cap C) = (A \cup B) \cap C$ is called
 (a) Commutative law (b) Associative law
 (c) De-morgan's law (d) Intersection of sets
- (v) If $U = \{1, 2, 3, 4\}$, $A = \{4\}$, then $A' = ?$
 (a) $\{1, 2, 3\}$ (b) Φ (c) $\{1\}$ (d) $\{1, 2, 3, 4\}$

- (vi) If $U = \{1, 2, 3\}$, $A = \{1\}$, then $U - A = ?$
 (a) $\{2, 3\}$ (b) $\{1, 2\}$
 (c) $\{1, 3\}$ (d) Φ
- (vii) $(A \cup B)' = ?$
 (a) $A' \cup B'$ (b) $A' \cap B'$
 (c) $(A \cap B)'$ (d) Φ
- (viii) $(A \cap B)' = ?$
 (a) $A' \cap B'$ (b) $A' \cup B'$
 (c) $A \cap B$ (d) $A \cup B$
- (ix) If $R = \{(4, 5), (5, 4), (5, 6), (6, 4)\}$ then domain of R .
 (a) $\{4, 6\}$ (b) $\{4, 5\}$ (c) $\{4, 5, 6\}$ (d) $\{5, 6\}$
- (x) If $R = \{(4, 5), (5, 4), (5, 6), (6, 4)\}$ then range of R is:
 (a) $\{4\}$ (b) $\{5\}$ (c) $\{6\}$ (d) $\{4, 5, 6\}$

Ans.

| | | | | |
|--------|---------|----------|--------|-------|
| (i) b | (ii) b | (iii) a | (iv) b | (v) a |
| (vi) a | (vii) b | (viii) b | (ix) c | (x) d |

Q.2- Fill in the blanks.

- (i) $(A \cup B)' =$ _____
- (ii) $(A \cap B)' = ?$ _____
- (iii) $A \cup (B \cap C) =$ _____
- (iv) $A \cap (B \cap C) =$ _____
- (v) If A and B be the two non-empty sets, then
 $A \cup B = B \cup A$ is called the _____
- (vi) If A and B be the two non-empty sets, then
 $A \cap B = B \cap A$ is called _____
- (vii) Any sub-set of a cartesian product is called a _____
- (viii) If $R_1 = \{(1, 2), (3, 4), (5, 6)\}$, then domain of R_1 is _____
- (xi) If $R_1 = \{(1, 2), (3, 4), (5, 6)\}$, then range of R_1 is _____
- (x) If $f : A \rightarrow B$ then every element of a set A has its image in _____

| | |
|---------------------------|--------------------------|
| (i) $(A' \cap B')$ | (ii) $A' \cup B'$ |
| (iii) $(A \cup B) \cup C$ | (iv) $(A \cap B) \cap C$ |
| (v) Commutative Law | (vi) Commutative Law |
| (vii) Binary relation | (viii) $\{1, 3, 5\}$ |
| (ix) $\{2, 4, 6\}$ | (x) Set B |

Q.3- If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 3, 4, 6\}$ and $C = \{2, 3, 4, 7, 8, 9\}$.
 Verify that : $(A \cap B) \cap C = A \cap (B \cap C)$

Solution:-

$$A = \{1, 2, 3, 4, 5, 6\}, B = \{2, 3, 4, 6\}$$

$$C = \{2, 3, 4, 7, 8, 9\}$$

We have to prove that

$$(A \cap B) \cap C = A \cap (B \cap C)$$

To solve L.H.S

$$A \cap B = \{1, 2, 3, 4, 5, 6\} \cap \{2, 3, 4, 6\} = \{2, 3, 4, 6\}$$

$$(A \cap B) \cap C = \{2, 3, 4, 6\} \cap \{2, 3, 4, 7, 8, 9\} = \{2, 3, 4\} \dots (1)$$

Now to solve R.H.S

$$B \cap C = \{2, 3, 4, 6\} \cap \{2, 3, 4, 7, 8, 9\} = \{2, 3, 4\}$$

$$A \cap (B \cap C) = \{1, 2, 3, 4, 5, 6\} \cap \{2, 3, 4\} = \{2, 3, 4\} \dots (2)$$

Results (1) and (2) show that $(A \cap B) \cap C = A \cap (B \cap C)$

Q.4- If $A = \{2, 3, 4\}$, $B = \{3, 6, 9, 12\}$ and $C = \{4, 6, 8, 10\}$.
 Verify that : $A \cup (B \cap C) = (A \cup B) \cap C$

Solution:-

$$A = \{2, 3, 4\}, B = \{3, 6, 9, 12\}$$

$$C = \{4, 6, 8, 10\}$$

We have to prove that

$$A \cup (B \cap C) = (A \cup B) \cap C$$

To solve L.H.S

$$B \cap C = \{3, 6, 9, 12\} \cap \{4, 6, 8, 10\} = \{6\}$$

$$\begin{aligned} A \cup (B \cap C) &= \{2, 3, 4\} \cup \{6\} \\ &= \{2, 3, 4, 6\} \dots (1) \end{aligned}$$

Now to solve R.H.S

$$\begin{aligned}(A \cup B) &= \{2, 3, 4\} \cup \{3, 6, 9, 12\} \\ &= \{2, 3, 4, 6, 9, 12\}\end{aligned}$$

$$\begin{aligned}(A \cup B) \cup C &= \{2, 3, 4, 6, 9, 12\} \cup \{4, 6, 8, 10\} \\ &= \{2, 3, 4, 6, 8, 9, 10, 12\} \dots (2)\end{aligned}$$

Results (1) and (2) show that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Q.5- If $A = \{2, 3, 4\}$ and $B = \{1, 3\}$. Find $A \times B$ and $B \times A$. Also establish two binary relations each from these cartesian products.

Solution:-

$$\begin{aligned}A &= \{2, 3, 4\}, B = \{1, 3\} \\ A \times B &= \{2, 3, 4\} \times \{1, 3\} \\ &= \{(2, 1), (2, 3), (3, 1), (3, 3), (4, 1), (4, 3)\}\end{aligned}$$

Two binary relations in $A \times B$ are

$$\begin{aligned}R_1 &= \{(2, 1), (3, 1), (4, 1)\} \\ R_2 &= \{(2, 3), (3, 1), (3, 3), (4, 1)\}\end{aligned}$$

Now $B \times A = \{1, 3\} \times \{2, 3, 4\}$
 $= \{(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4)\}$

Two binary relations in $B \times A$ are

$$\begin{aligned}R_3 &= \{(1, 2), (1, 4), (3, 3)\} \\ R_4 &= \{(1, 3), (1, 4), (3, 4), (3, 2)\}\end{aligned}$$

Q.6- Write the number of binary relations possible in each of the following cases.

(i) In $C \times C$, when the number of elements in C are 4.

(ii) In $A \times B$, if number of elements in A are 2 and in B are 3.

Solution:-

- (i) Number of elements in $C = 4$
 Number of elements in $C \times C = 4 \times 4 = 16$

Thus Number of all subsets of $C \times C = 2^{16}$

So Number of all Binary relations $= 2^{16}$

(ii) Number of elements of $A = 2$

Number of elements of $B = 3$

Number of elements of $A \times B = 2 \times 3 = 6$

Thus Number of all subsets of $A \times B = 2^6 = 64$

So Number of all binary relation in $A \times B = 64$

Q.7- If $R = \{(a,b) | a, b \in W, 3a + 2b = 16\}$. Find its domain and range R.

Solution:-

$$R = \{(a,b) | a, b \in W, 3a + 2b = 16\}$$

Consider the equation

$$3a + 2b = 16$$

Put $a = 0, 2$ and 4

$$\text{For } a = 0 \Rightarrow b = 8 \Rightarrow (0, 8) \in R$$

$$\text{For } a = 2 \Rightarrow b = 5 \Rightarrow (2, 5) \in R$$

$$\text{For } a = 4 \Rightarrow b = 2 \Rightarrow (4, 2) \in R$$

Now $R = \{(0, 8), (2, 5), (4, 2)\}$

Thus

$$\text{Dom } (R) = \{0, 2, 4\}$$

$$\text{Rang } (R) = \{2, 5, 8\}$$

Multiple Choice Question

Q.1- The set $\left[\frac{p}{q} : p, q \in Z \wedge q \neq 0 \right]$ is the set of

- | | |
|------------------------|----------------------|
| (a) Real Numbers | (b) Rational Numbers |
| (c) Irrational Numbers | (d) Prime Numbers |

Q.2- Zero = 0, is

- | | |
|-----------------------|------------------------|
| (a) An even number | (b) Odd numbers |
| (c) Imaginary numbers | (d) Irrational numbers |

Q.3- $A \cup B =$

- (a) $\{x / x \in A \vee x \in B\}$ (b) $\{x / x \in A \wedge x \in B\}$
 (c) $\{x / x \in A \wedge x \notin B\}$ (d) $\{x / x \notin A \wedge x \in B\}$

Q.4- The set $\{x / x \in U \wedge x \notin A\}$ is equal to

- (a) A (b) A^c (c) A' (d) $A - B$

Q.5- The set $\{x / x \in A \wedge x \notin B\}$ is equal to

- (a) A^c (b) B^c (c) $A - B$ (d) $B - A$

Q.6- $A \cup (B \cap C) = (A \cup B) \cap C$ is the law

- (a) De Morgan (b) Commutative
 (c) Associative (d) Distributive

Q.7- In the venn diagram two sets A and B are such that

- (a) $A \subseteq B$ (b) $B \subseteq A$ (c) Overlapping (d) Disjoint

Q.8- The statement $(A \cup B)^c = A^c \cap B^c$ is of

- (a) Distributive law (b) Associative law
 (c) De-Morgans law (d) Commutative law

Q.9- If $A = \{1, 2, 3, 4, 5, 6\}$ and $U = \{1, 2, 3, \dots, 10\}$

Then A^c is equal to

- (a) $\{2, 4, 6, 8, 10\}$ (b) $\{1, 3, 5, 7, 9\}$
 (c) $\{7, 8, 9, 10\}$ (d) $\{1, 2, 3, 4\}$

Q.10- If $A = \{1, 2, 3\}$, $B = \{y, z\}$, then all the binary relations in $A \times B$ are

- (a) 6 (b) 9 (c) 32 (d) 64

Q.11- $R = \{(1, 2), (1, 3), (2, 5), (3, 10)\}$ is a binary relations.

Its Domain is

- (a) $\{1, 1, 2, 3\}$ (b) $\{1, 2, 3\}$
 (c) $\{2, 3, 5, 10\}$ (d) $\{1, 2, 3, 5, 10\}$

Q.12- If $A = \{a, b\}$, $B = \{x, y\}$, Then the function from A onto B is

- (a) $\{(a, x), (b, x)\}$ (b) $\{(b, x), (a, y)\}$
 (c) $\{(a, x), (a, y)\}$ (d) $\{(b, x), (b, y)\}$

Q.13- If f is a function from A to B such that $\text{Rang } F = B$
Then it is a function

- (a) Into (b) Onto
(c) One-One (d) Corresponding

Q.14- A one-one and onto function is called

- (a) Injective (b) Surjective
(c) Bijective (d) Objective

Q.15- If A and B are disjoint sets then

- (a) $A \cap B = \Phi$ (b) $A \cup B = \Phi$
(c) $A^c = B$ (d) $B^c = A$

MODEL CLASS TEST

Time : 40 mins

Max Marks : 25

Q.1- Tick the best choice.

- (i) The law $A \cup B = B \cup A$ is called
(a) De-Morgan (b) Associative
(c) Commutative (d) Distributive
- (ii) If $R = \{(1,3), (1,4), (2,3)\}$ Then $\text{Dom}(R) =$
(a) $\{1,1,2\}$ (b) $\{1,2\}$
(c) $\{3,3,4\}$ (d) $\{3,4\}$
- (iii) If "f" is a function, such that non of 2nd element of ordered pairs in f is repeated. Then f is called
(a) Onto (b) into (c) One-One (d) Bijective.
- (iv) Complement of universal set is equal to
(a) Universal set (b) Empty set
(c) Sub set (d) Super set
- (v) $(A \cup B)^c$ is equal to
(a) $A^c \cup B^c$ (b) $(A \cap B)^c$
(c) $A^c \cap B^c$ (d) Φ
- (vi) $A \cup \Phi$ is equal to
(a) A (b) Φ (c) $A \cap \Phi$ (d) A^c

(vii) $\{2,4\} \cap \{1,3,5\}$ is equal to

(a) $\{3\}$ (b) $\{1,2,4\}$ (c) Φ (d) $\{1,2,3,4,5\}$

Q.2- Attempt any five of the following short questions.

(i) If $A = \{a,b,c\}$ and $B = \{a,e,i,o,u\}$

Then find $A \cup B$ and $A \cap B$

(ii) If $U = \{1,2,3,\dots,10\}$ and $A = \{1,2,3,4\}$

Then find A^c

(iii) If $U = \{1,2,3,\dots,10\}$ and $B = \{1,2,3,4\}$

Then find $B \cup B^c$

(iv) If $R = \{(1,5), (2,6), (2,7), (3,7)\}$

Then find $\text{Dom}(R)$ and $\text{Rng}(R)$

(v) If $A = \{5,6,7\}$, $B = \{1,2\}$ Then find the function from A onto B

(vi) If $A = \{a,b,c,d\}$ and $B = \{1,3\}$

Write a binary relation from A to B which is not a function.

Attempt any two of the following questions.

Q.3- If $U = \{1,2,3,\dots,9\}$, $A = \{2,3,6,9\}$, $B = \{1,3,6,7,8\}$

Then verify $(A \cup B)^c = A^c \cap B^c$

Q.4- If $A = \{2,3,5,7,9\}$, $B = \{1,3,5,7\}$, $C = \{2,3,4,5,6\}$

Then verify $(A \cap B) \cap C = A \cap (B \cap C)$

Q.5- If $A = \{1,3,5\}$, $B = \{2,4,6\}$ Then find $A \times B$ and a bijective function from A to B.