Chapter # 5

Gravitation

COMPREHENSIVE QUESTIONS:

1. State and explain the law of universal Gravitation. Also show that the law obeys Newton's third law of motion.

Ans. Law of universal Gravitation:

Statement:

Everybody in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Derivation:

Consider two spherical bodies of masses 'm_1' and 'm_2' separated by distance 'r' as shown in fig.

According to the Newton's law of universal gravitation, the force of gravity 'F_g' between them is:

$$F_g \propto m_1 m_2 \quad \text{---------(i)}$$

And $$F_g \propto \frac{1}{r^2} \quad \text{----------(ii)}$$

Combining eq(i) and (ii), we get

$$F_g \propto \frac{m_1 m_2}{r2}$$

$$\boldsymbol{F_g = \frac{Gm_1 m_2}{r2}}$$

Where 'G' is the constant of proportionality and is known as universal gravitational constant. The value of 'G' is 6.67×10^{-11} $Nm^2 kg^{-2}$. It does not depend on the medium between the two bodies.

Newton's third law of motion and universal gravitation:

The law of universal gravitation also obeys the Newton's third law of motion. We can see that the force acting on mass 'm_2' due to mass m_1 is F_{12}. Also the force acting on mass m_1 due to mass "m_2" is same force 'F_{21}', Both these forces are equal but opposite in direction. Therefore, we can say that the forces acting on two bodies due to gravitation force is the example of action and reaction i-e

$$F_{12} = - F_{21}$$

2. Determine the mass of earth by applying the law of gravitation.

Ans. Determination of Mass of Earth:

The mass of the earth can be determined with the help of law of universal gravitation. Let an object of mass 'm_0' be placed on the surface of the earth. The distance between the centre of the body and the earth is equal to the radius of earth 'r_E'. If the mass of earth is 'm_E' then the force 'F_g'

with which the earth attracts the body towards its centre is given by law of gravitation.

$$\mathbf{F_g = G\, \frac{m_o m_E}{r_E^2}}$$

Also, we know that force of gravity (F_g) is equal to the weight of the body (W). i-e

$$F_g = W$$

So,

$$W = G\,\frac{m_o m_E}{r_E 2}$$

We know that $W = m_o g$

$$m_o g = G\,\frac{m_0 m_E}{r_E 2}$$

$$g = G\,\frac{m_E}{r_E 2}$$

By cross Multiplication, we get

$$Gm_E = gr_E^2$$

Divide 'G' on both sides,

$$\frac{Gm_E}{G} = \frac{gr_E^2}{G}$$

$$\boldsymbol{m_E = \frac{gr_E^2}{G}} \text{ ------ (i)}$$

By putting the values of g, r_E^2 and G in eq (i) we will obtain value of m_E.

Gravitational constant = G= 6.67×10^{-11} Nm^2 kg^{-2}

Acceleration due to gravity = g = 9.8 ms^{-2}

And, radius of earth $r_E = 6.4 \times 10^6$m

Now,

$$m_E = \frac{9.8 \times (6.4 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$m_E = \frac{9.8 \times 40.96 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$m_E = \frac{401.40 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$m_E = \frac{401.40 \times 10^{12} \times 10^{11}}{6.67}$$

$$m_E = 60.18 \times 10^{12+11}$$

$$m_E = 60.18 \times 10^{23}$$

$$m_E = 6.018 \times 10^{23+1}$$

$$\boldsymbol{m_E = 6 \times 10^{24}\ Kg}$$

Thus, the mass of earth is approximately 6×10^{24}kg.

3. What is gravitational field and gravitational field strength Show that weight of an object changes with location.

Ans. <u>Gravitational Field (Gravity as a field force):</u>

Gravitational field is region surrounding the earth in which another object feels force of attraction toward its centre According to the field theory; every mass creates a gravitational field composed of field lines that permeates outward into space. The earth creates a gravitational field that pulls objects towards its centre by force of gravity. At any point, earth's gravitational Field can be described by the gravitational field strength (g).

The gravitational field or gravitational field strength is defined as a measure of gravitational force (F_g) exerted on a per unit mass (m) of an object.

<u>Mathematical Form:</u>

$$\text{Gravitational Field} = \frac{Gravitational\ force}{unit\ mass}$$

Or

$$g = \frac{\boldsymbol{F_g}}{\boldsymbol{m}}$$

<u>Quantity and Unit:</u>

The gravitational field (g) is a vector quantity and its unit is Newton per kilogram i-e Nkg^{-1}

<u>Gravitational Field strength:</u>

The gravitational field strength tells us how strong a gravitational field is. The gravitational field is represented by field lines as shown in figure, which shows the strength of gravitational field decreases, as the distance from the earth increase. The gravitational field strength of earth near its surface is equal to the acceleration of free fall (g) at its surface i-e g = 9.8 ms^{-2}

According to Newton's second law, a = F/m. As gravitational field strength (g) is defined as $\frac{F_g}{m}$, so the value of 'g'at any given point is equal to the acceleration due to gravity. For this reason, the gravitational field strength is 9.8 Nkg^{-1} means 9.8N force is exerted on every 1 kg mass of an object. Therefore, the acceleration is same for any object, regardless of mass.

<u>Weight changes with location:</u>

As we know that weight is the magnitude of force due to gravity i-e W = m_og. Now we can refine our definition of weight as mass time gravitational field strength this new definition helps to explain why our weight changes with our location in the universe by calculating the value of g i-e.

$$\mathbf{g} = \frac{\boldsymbol{Gm_E}}{\boldsymbol{r_E^2}}$$

This equation shows that gravitational field strength depends only on mass of earth 'm_E' and radius of earth 'r_E'. Greater will be the value of 'g' if lesser the distance from earth's centre. Therefore, on the surface of any planet, the value of earth's centre varies as we change location. 'g' as well as our weight will depends on the planet's mass and radius.

4.How is the Value of 'g' changing by going to higher altitude? Write the relevant formula.

Ans. <u>Variation of 'g' with altitude:</u>

The value of 'g' doesn't depend upon the mass of the body. It means that light and heavy bodies should fall towards the centre of earth with constant acceleration. However, the value of 'g' depends upon the distance of the body from the centre of the earth. Greater the distance from the centre of the earth, smaller will be the value of 'g'. That is why the value of 'g' at the poles is greater than at equator because earth is not a perfect square; its equatorial radius is greater than the radius at the poles.

<u>Derivation:</u>

Consider a body of mass 'm_o' placed on earth's surface, as shown in the figure, now we know that law of universal gravitation is given by.

$$\boldsymbol{F_g = G\frac{m_o m_E}{r_E^2}}$$

As,

$F_g = W = m_0 g$

So, $$m_0 g = G\frac{m_o m_E}{r_E^2}$$

$$g = G\frac{m_E}{r_E^2} \quad \dots\dots\dots\dots (i)$$

If we lift the body to a height "h", then the value of "g" at the height is given by

$$g = G\frac{M_e}{(R_e+h)^2}$$

From equ(i), $Gm_E = gr^2_E$

So,

$$g = G\frac{\boldsymbol{gr_E^2}}{(R_e+h)^2} \quad \dots\dots\dots\dots \text{equ(ii)}$$

Both eq (i) and (ii) shows that 'g' is inversely proportional to the square of distance of the body from the earth centre. Thus, the value of 'g' decreases with altitude (height).

5.Derive the formula for the orbital speed of an artificial satellite.

Ans: <u>Satellites:</u>

Satellites are the objects revolving around the planet in fixed orbits. Artificial satellites are man-made objects that revolve around the earth or other planets in different orbit with uniform speed due to gravitational force.

A satellite requires a centripetal force (F_c) to revolve around the earth and this necessary centripetal force is provided by the gravitational force of attraction between the earth and satellite.

Derivation:

Consider a satellite of mass 'm_s' revolving in a circular orbit with uniform velocity 'v' from earth of mass 'm_E' Let 'r' be the distance between the centre of earth and centre of satellite as shown in figure.

Now, the gravitational force by Newton's law of universal gravitation is:

$$\mathbf{F_g}=\frac{Gm_Em_S}{r^2} \quad \text{.........eq (i)}$$

The necessary centripetal force 'F_c' required for uniform circular motion is given by:

$$F_c=\frac{m_sv^2}{r} \quad \text{.........eq (ii)}$$

As Centripetal force is provided by gravitational force, therefore

$$F_c = F_g \quad \text{.........eq (iii)}$$

Now, putting the value of F_g and F_c in eq (iii),
We get

$$\frac{m_sv^2}{r}=\frac{Gm_sm_E}{r^2}$$

$$v^2=\frac{Gm_E}{r}$$

Taking square root on both sides

$$\sqrt{v^2}=\sqrt{\frac{Gm_E}{r}}$$

$$v=\sqrt{\frac{Gm_E}{r}}$$

Where $r = r_E + h$, so

$$\boldsymbol{v}=\sqrt{\frac{\boldsymbol{Gm_E}}{\boldsymbol{r_E+h}}}$$

This equation represents the orbital speed of satellite where h is the height of satellite from surface of earth and 'r_E'is the radius of earth. So, the orbital of satellite speed depends upon the mass of earth and the distance from the centre of the earth to the centre of mass of the satellite and doesn't depend upon mass of satellite.

Topic Wise Question

7. Derive a formula to calculate the value of g.

Value of g:

The Newton's law of Universal gravitation shows that value of g depends on mass of all reacting bodies and distance to it. So the value of 'g' can be determined by using law of gravitation.

Consider an object of mass 'm_o' placed on the surface of earth and r_E is the distance between their centers as shown in figure The gravitational force between the object and earth is as follow.

$F_g = G\frac{m_o m_E}{r_E^2}$ ---------- (i)

As, we know that

$$F_g = W = m_o g \text{ ----------- (ii)}$$

Comparing eq (i) and (ii)

$m_o g = G\frac{m_o m_E}{r_E^2}$

$$\mathbf{g = G\frac{m_E}{r_E^2}}$$

By putting the values of G, m_E & r_E, we will obtain the value of g

As,

$m_E = 6 \times 10^{24}$ kg

$r_E = 6.4 \times 10^{6}$ m

$G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$$

$$g = \frac{40.02 \times 10^{-11+24}}{40.96 \times 10^{12}}$$

$$g = \frac{40.02 \times 10^{13}}{40.96 \times 10^{12}}$$

$$g = \frac{40.02 \times 10^{13} \times 10^{-12}}{40.96}$$

$g = 0.977 \times 10^{1}$

$g = 9.77 \times 10^{1-1}$

$g = 9.77 \times 10^{0}$ (..10^0=1)

$g = 9.77$

or

$\mathbf{g = 9.8ms^{-2}}$

This is value of 'g' at the surface of earth.

CONCEPTUAL QUESTIONS:

Q1. If there is an attractive force between all objects, why don't we feel ourselves gravitating toward nearby massive buildings?

Ans. Gravitational force pulls us to massive buildings but the forces are relatively small due to the small masses of us and buildings when compared to the mass of the earth. Therefore, the attractive force would be almost unnoticeable.

Q2. Does the sun exert a larger force on the Earth than that exerted on the sun by the earth? Explain.

Ans. By Newton's third law, the force exerted on the Sun by the Earth is exactly equal to the force the Sun exerts on the Earth but in opposite direction.

Q3. What is the importance of gravitational constant 'G'? Why is it difficult to calculate?

Ans. Constant always play an important in bringing the equality in the dimensions on both sides of an equation and its value depends upon the factors relating the interaction between the two bodies similar is the case with gravitational constant 'G' here it maintains the equality on both sides of the universal gravitational law's equation. It's difficult to calculate it because there's no theoretical derivation to it is just an experimentally measured value.

Q4. If Earth somehow expanded to a larger radius, with no charge in mass, how would your weight be affected? How would it be affected if earth instead shrunk?

Ans: According to law of universal Gravitation

$$F = \frac{Gmm_E}{r_E^2} \quad \text{------------ (i)}$$

As we know that

F=W=mg

So eq (i) becomes

$$mg = \frac{Gmm_E}{r_E^2}$$

$$g = \frac{Gm_E}{r_E^2} \quad \text{------------ (ii)}$$

From eq (ii), we say that force of gravity is inversely proportional to the radius of earth. So if radius of Earth gets larger weight would get smaller. If the earth shrunk, the radius of earth decreases and as a result weight gets increases.

Q5. What would happen to your weight on earth if the mass of the earth doubled but its radius stayed the same?

Ans: As we know that

$$g = \frac{Gm_E}{r_E^2} \quad \text{------------ (i)}$$

Above eq (i) shows that value of 'g' depends on mass of earth 'm_E' and radius of earth 'r_E'. If we only double the mass of earth the value of 'g' becomes double and weight depends on the value of 'g'. i.e.

W= mg ------------ (ii)

Eq (ii) shows that if value of ‘g’ is doubled, the weight will also double.

Q6. Why lighter and heavier objects fall at the same rate toward the earth?

Ans: Because value of ‘g’ does not depend upon the mass of the body, it depends only on mass of earth and radius of earth. Therefore, lighter and heavier bodies fall towards earth with same acceleration.

Q7. The value of ‘g’ changes with location on earth, however we take the same value of ‘g’ as 9.8ms^{-2} for ordinary calculations why?

Ans: The value of ‘g’ depends upon the distance from the centre of earth. Greater the distance from the centre of earth, smaller will be the value of ‘g’ and vice versa. The change in the value of ‘g’ is significant only at very large distance. Therefore, we take same value of ‘g’ for ordinary calculation.

Q8. Moon is attracted by the earth, why it does not fall on earth?

Ans: The moon is natural satellite of the earth. It revolves around the earth in a specific orbit. The earth attracts the moon towards itself with gravitational force. The gravitational force of earth provides necessarily centripetal force which compels the moon to move in the circular path. Because of this orbital motion moon does not fall on earth.

Q9. Why for some height larger and smaller satellites must have same orbital speeds?

The orbital speed depends upon the mass of earth and distance from the centre of earth to the centre of mass of satellite and does not depend upon the mass of satellite. Therefore, for some particular distance from the centre of earth, all the satellites have the same orbital speed irrespective of the size of satellite.

Chapter No 05

Assignments

5.1 The mass of earth is 6×10^{24} kg and that of the moon is 7.4 $\times 10^{22}$ kg. If the distance between the earth and the moon is 3.84$\times 10^{5}$ km, calculate the force exerted by the earth on the moon,

Data:

Mass of earth = m_1= 6$\times$10^{24} kg

Mass of moon = m_2 = 7.4 $\times$10^{22} kg

Distance = r= 3.84$\times$10^{5}km

= 3.84 $\times$10^{5} $\times$ 10^{3}m

= 3.84 $\times$ 10^{8}cm

Gravitational constant = G = 6.67 $\times$ 10^{-11}Nm^2kg^{-2}

Gravitational force = F_g = ?

Solution:

By using formula

$$\mathbf{F_g = G\frac{m_1 m_2}{r^2}}$$

Putting Values

$$= \frac{6.67\times10^{-11}\times6\times10^{24}\times7.4\times10^{22}}{(3.84\times10^{8})^2}$$

$$= \frac{6.67\times6\times7.4\times10^{-11+24+22}}{14.7456\times10^{16}}$$

$$= \frac{296.148\times10^{35}}{14.7456\times10^{16}}$$

$$= 20.083 \times 10^{35-16}$$

$$= 20.083 \times 10^{19}\ \text{N}$$

$F_g \quad = 2.008 \times 10^{20}$N

$\mathbf{F_g \quad = 2 \times 10^{20}N}$

5.2 If the radius of the moon is 1.74 $\times10^6$ m and have acceleration due to gravity on its surface as 1.6ms^{-2}. Calculate the mass of moon.

Data:

Radius of Moon = r = 1.74 × 10^6 m

Acceleration due to gravity on moon = g = 1.6ms^{-2}

Gravitational constant = G = 6.67 × 10^{-11} N$m^2\ kg^{-2}$

Find: Mass of moon = m = ?

Solution:

By using formula

$$\mathbf{m = \frac{gr^2}{G}}$$

Putting Values

$$= \frac{1.6\times(1.74\times10^{6})^2}{6.67\times10^{-11}}$$

$$= \frac{1.6\times3.027\times10^{12}}{6.67\times10^{-11}}$$

$$= \frac{4.843\times10^{12}}{6.67\times10^{-11}}$$

$$= 0.726\times10^{12+11}$$

$$= 0.73 \times 10^{23}\ \text{kg}$$

$\mathbf{m = 7.3 \times 10^{22}\ kg}$

5.3 An astronaut of mass 65.0 kg (weighting 637N on earth) is walking on the surface of the moon, which has a mean radius of 1.74 $\times$ 10^{6}m and a mass of 7.35 $\times$10^{22}kg. What is the weight of the astronaut on moon? What is the free – fall acceleration at the surface of the moon?

Data:

mass of astronaut = m = 65kg

Radius of moon = r_M = 1.74×10^6m

Mass of moon = $m_M = 7.35 \times 10^{22}$ kg

Gravitational constant = G = 6.67×10^{-11} Nm^2kg^{-2}

Find:

Weight of astronaut = W= ?

Free – fall acceleration on moon = g_M= ?

Solution:

By Using Formula

$$g = \frac{Gm_M}{r_M^2}$$

Putting values

$$= \frac{6.67 \times10^{-11} \times7.35 \times10^{22}}{(1.74\times10^6)^2}$$

$$= \frac{6.67 \times7.35 \times10^{-11+22}}{3.027\times10^{12}}$$

$$= \frac{49.02 \times10^{11}}{3.027\times10^{12}}$$

$$= 16.19 \times 10^{11-12}$$

= 16.19×10^{-1} m/s^2

= 16.2×10^{-1} m/s^2

g = 1.62 m/s^2

Now we find weight of Astronaut

As we know that

W = mg

Putting values

= 65×1.62 N

= 105.3 N

W = 105 N

5.4 Calculate the value of 'g' at 1000 km and 35900 km above the earth surface

i. Calculate value of 'g' at 1000 km.

Data:

Radius of earth $= r_E = 6.4 \times 10^6$m

Acceleration due to gravity = g= $9.8 ms^{-2}$

Height above earth surface = h= 1000km

$= 1000 \times 10^3 m$

$= 10^3 \times 10^3$m

$= 10^{3+3}$m

$= 1\times10^6$m

FIND:

Value of 'g' at height h = g_h =?

SOLUTION:

By using formula

$$g_h = \frac{g r_E^2}{(r_E + h)^2}$$

Putting Values

$$= \frac{9.8\times(6.4\times10^6)^2}{(6.4\times10^6 + 1\times10^6)^2}$$

$$= \frac{9.8\times40.96\times10^{12}}{(7.4\times10^6)^2}$$

$$= \frac{401.408\times10^{12}}{54.76\times10^{12}}$$

$= 7.33 \times10^{12-12}$ m/s^2

g_h = 7.33 m/s^2

ii. Calculate value of 'g' at 35900 km

Data:

Radius of earth = r_E= 6.4×10^6m

Acceleration due to gravity = g = 9.8 ms^{-2}

Height above earth surface = h = 35900 km

$= 35900 \times 10^3$m

$=35.9\times10^3 \times10^3$m

$= 35.9 \times10^6$m

FIND:

Value of 'g' at height h = g_h= ?

SOLUTION:

By using formula

$$g_h = \frac{g r_E^2}{(r_E + h)^2}$$

Putting Values

=

$\boldsymbol{g_h}$**= 0.22 m/$\boldsymbol{s^2}$**

5.5 If a satellite orbits the earth at 2,000 km above sea level, how fast must the orbiting satellite travel to maintain a circular orbit?

Data:

Height of satellite above earth surface = h = 2000 km

$= 2000\times10^3$m

$= 2.0 \times 10^3 \times10^3$m

= 2 $\times 10^6$m

Mass of Earth = m_E = 6 $\times10^{24}$kg

Radius of Earth = r_E= 6.4 $\times10^6$m

Gravitational constant = G = 6.67×10^{-11}Nm^2kg^2

Find: Orbital Speed = v = ?

Solution:

By using formula

$$\mathbf{v} = \sqrt{\frac{Gm_E}{r_E+h}}$$

$$= \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6.4\times10^6+2\times10^6}}$$

$$= \sqrt{\frac{40.02\times10^{-11+24}}{8.4\times10^6}}$$

$$= \sqrt{\frac{40.02\times10^{13}}{8.4\times10^6}}$$

$$= \sqrt{4.76} \times 10^{13-6}$$

$$= \sqrt{4.76} \times 10^7$$

$$= \sqrt{47.6} \times 10^6$$

$= 6.899 \times 10^3$m/s

v = 6.899 x 10³ m/s

<u>Numericals</u>

1.Pluto's moon Charon is unusually large considering Pluto's size, giving them the character of a double planet. Their masses are 1.25 $\times$ 10^{22}kg and 1.9 $\times10^{21}$kg, and their average distance from one another is 1.96 $\times$ 10^4km. What is the gravitational force between them?

Data:

Mass of Pluto's Moon= m_1 = 1.25×10^{22}kg

Mass of Pluto's = m_2 = 1.9×10^{21}kg

Distance between Pluto's and Pluto's Moon = r = 1.96$\times$ 10^4km

$= 1.96 \times 10^4 \times 10^3$m

$= 1.96 \times10^{4+3}$m

$= 1.96 \times 10^{7}$m

Gravitational constant = G = 6.67×10^{-11}Nm^2kg^{-2}

FIND:

Gravitational force = F_g = ?

Solution:

$$\mathbf{F_g = G\frac{m_1m_2}{r^2}}$$

$$= \frac{6.67\times10^{-11}(1.25\times10^{22})(1.9\times10^{21})}{(1.96\times10^{7})^2}$$

$$= \frac{6.67\times1.25\times1.9\times10^{-11+22+21}}{3.84\times10^{14}}$$

$$= \frac{15.84\times10^{32}}{3.84\times10^{14}}$$

$$= 4.12 \times 10^{32-14}\text{N}$$

$$\mathbf{F_g = 4.12 \times 10^{18}N}$$

2.The mass of Mars is 6.4 $\times10^{23}$kg and having radius of 3.4 $\times$ 10^{6}m. Calculate the gravitational field strength (g) on Mars surface.

Data:

Mass of Mars =m_M = 6.4×10^{23}kg

Radius of Mars = r_M= 3.4×10^{6}m

Gravitational constant = G = 6.67×10^{-11} Nm^2 kg^{-2}

Gravitational strength on Mars = g_M= ?

Solution:

By using formula

$$\mathbf{g_M = \frac{Gm_M}{r_M^2}}$$

$$= \frac{6.67\times10^{-11}\times6.4\times10^{23}}{(3.4\times10^{6})^2}$$

$$= \frac{6.67\times6.4\times10^{-11+23}}{11.56\times10^{12}}$$

$$= \frac{42.68\times10^{12}}{11.56\times10^{12}}$$

$$= 3.69\times10^{12-12}\ \text{m}/s^2$$

$$\mathbf{g_m = 3.69\ m/s^2}$$

3.Titan is the largest moon of Saturn and the only moon in the solar system known to have a substantial atmosphere. Find the acceleration due to gravity on Titan's surface, given that its mass is 1.35$\times$ 10^{18} and its radius is 2570km.

DATA:

Mass of Titan = m_T = 1.35×10^{18}kg

Radius of Titan = r_T = 2570km

$= 2570 \times 10^{3}$m

$= 2.57 \times 10^{3} \times 10^{3}$ m

$= 2.57 \times 10^{6}$ m

Gravitational constant = G = 6.67×10^{-11} Nm^2kg^{-2}

FIND:

Gravity on Titan's surface = g_T = ?

SOLUTION:

By using Formula

$$g_T = \frac{Gm_T}{r_T^2}$$

$$= \frac{6.67\times10^{-11}\times1.35\times10^{18}}{(2.57\times10^6)^2}$$

$$= \frac{6.67\times1.35\times10^{-11+18}}{6.6049\times10^{12}}$$

$$= \frac{9.0045\times10^7}{6.6049\times10^{12}}$$

$$g_T = 1.36\times10^{7-12}\ m/s^2$$

$$\mathbf{g_T = 1.36\times10^{-5}\ m/s^2}$$

4.At which altitude above Earth's surface would the gravitational acceleration be 4.9m/s^2

Data:

Gravitational acceleration at height = g_h= 4.9m/s^2

Acceleration due to gravity = g = 9.8m/s^2

Radius of Earth = r_E= 6.4×10^6m

Find:

Altitude above earth's Surface = h = ?

Solution:

By Using Formula

$$g_h = \frac{g\,r_E^2}{(r_E+h)^2}$$

Rearrange the formula

$$(r_E+h)^2 = \frac{gr_E^2}{g_h}$$

$$r_E + h = \sqrt{\frac{gr_E^2}{g_h}}$$

$$\mathbf{h = \sqrt{\frac{gr_E^2}{g_h}} - r_E}$$

Putting Value

$$= \sqrt{\frac{9.8\times(6.4\times10^6)^2}{4.9}} - 6.4\times10^6$$

$$= \sqrt{\frac{9.8\times40.96\times10^{12}}{4.9}} - 6.4\times10^6$$

$$= \sqrt{\frac{401.408\times10^{12}}{4.9}} - 6.4\times10^6$$

$= \sqrt{81.92 \times 10^{12}}$ - 6.4×10^6

$= 9.05 \times 10^6 - 6.4 \times 10^6$

$= (9.05-6.4) \times 10^6$

h $= 2.65 \times 10^6$ m

or

h $= 2.6 \times 10^6$ m

5.Assume that a satellite orbits Earth 225km above its surface. Given that the mass of Earth is 6×10^{24} kg and the radius of Earth is $6.4 \times 10^6 m$, What is the satellite's orbital speed?

Data:

Height above earth surface=h=225km

$= 225 \times 10^3$m

$= 0.225 \times 10^3 \times 10^3$m

$= 0.255 \times 10^6$m

Mass of Earth = $m_E = 6 \times 10^{24}$kg

Radius of Earth = $r_E = 6.4 \times 10^6$m

Gravitational Constant = G = 6.67×10^{-11} Nm^2 kg^{-2}

Find:

Satellite's orbital speed =v=?

Solution:

By using formula

$$v = \sqrt{\frac{Gm_E}{r_E + h}}$$

Putting values

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 0.225 \times 10^6}}$$

$$v = \sqrt{\frac{6.67 \times 6 \times 10^{-11+24}}{(6.4 + 0.225) \times 10^6}}$$

$$v = \sqrt{\frac{40.02 \times 10^{13}}{6.625 \times 10^6}}$$

$$v = \sqrt{6.040 \times 10^{13-6}}$$

$$v = \sqrt{6.040 \times 10^7}$$

$$v = \sqrt{60.4 \times 10^6} \text{ m/s}$$

v= 7.77×10^3m/s

6.The distance from centre of earth to centre of moon is 3.8×10^8m. Mass of earth is 6×10^{24}kg. What is the orbital speed of moon?

Data:

Mass of earth = m_E = 6 × 10^{24}kg

Distance between Earth and moon = r = 3.8× 10^8m

Gravitational constant = G = 6.67 × 10^{-11}Nm^{2}kg^{-2}

FIND:

Orbital speed of moon = v= ?

Solution:

By using formula

$$\mathbf{v} = \sqrt{\frac{Gm_E}{r}}$$

Putting values

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{3.8 \times 10^{8}}}$$

$$v = \sqrt{\frac{40.02 \times 10^{-11+24}}{3.8 \times 10^{8}}}$$

$$v = \sqrt{\frac{40.02 \times 10^{13}}{3.8 \times 10^{8}}}$$

$$v = \sqrt{10.53 \times 10^{13-8}}$$

$$v = \sqrt{10.53 \times 10^{5}}$$

$$v = \sqrt{105.3 \times 10^{4}}\ m/s$$

v= 10.26 ×10^2 m/s

v = 1.026×10^3 m/s

v = 1.02 ×10^3 m/s

7.The Hubble space telescope orbits Earth (m_E=6× $10^{24}kg$) with an orbital speed of 7.6× 10^3m/s. Calculate its altitude above Earth's Surface.

Data:

Mass of Earth = m_E = 6× $10^{24}kg$

Radius of Earth = r_E = 6.4× $10^6 m$

Gravitational constant = G = 6.67 × 10^{-11}Nm^{2}kg^{-2}

Orbital speed = v = 7.6 ×10^3m/s

Find:

Altitude = h = ?

Solution:

By using Formula

$\mathbf{v} = \sqrt{\frac{Gm_E}{r_E + h}}$

Taking square on both sides

$$v^2 = \left(\sqrt{\frac{Gm_E}{r_E + h}}\right)^2$$

$$v^2 = \frac{Gm_E}{r_E + h}$$

Rearranging equation

$$r_E + h = \frac{Gm_E}{v^2}$$

$$h = \frac{Gm_E}{v^2} - r_E$$

Putting Values

$$h = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(7.6 \times 10^3)^2} - 6.4 \times 10^6$$

$$h = \frac{6.67 \times 6 \times 10^{-11+24}}{57.76 \times 10^6} - 6.4 \times 10^6$$

$$h = \frac{40.02 \times 10^{13}}{57.76 \times 10^6} - 6.4 \times 10^6$$

$$h = 0.6928 \times 10^{13-6} - 6.4 \times 10^6$$

$$h = 00.6928 \times 10^7 - 6.4 \times 10^6$$

$$h = 6.928 \times 10^6 - 6.4 \times 10^6$$

$$h = (6.928 - 6.4) \times 10^6$$

$$h = 0.5286 \times 10^6$$

$$h = 528.6 \times 10^{6-3}$$

$$h = 528 \times 10^3$$

$$h = 528 \times 10^3\, m$$

$$h = 528\, km \qquad \therefore 10^3 = \text{kilo}$$