Chapter # 4

UNIT # 4

ALGEBRAIC EXPRESSIONS & ALGEBRAIC FORMULAS

Ex # 4.1

Algebraic Expressions

When variables and constants are connected by algebraic operations like addition, subtraction, multiplication, division, root extraction & rising integral or fractional powers is called algebraic expressions.

Variable:

A quantity that value may change within the context of problem. It is unknown value.

Normally, we use English letters for variables

Example:

a, d, e, x, y, z

Constant:

A quantity that value doesn't change. It is a fixed value.

Example:

4, 6, 267, 983384

Constant

جس کی value تبدیل نہیں ہوتی یعنی 1,2,3,9,22

Variable

جس کی value تبدیل ہوتی یعنی a,b,c,x,y,z

For Addition and Subtraction and other important terminologies

Visit this video:

https://youtu.be/4jFH9OMmjXI

Polynomial

The algebraic expression in which powers of variables are whole numbers is called polynomial.

Rational Expression:

An expression of form of $\frac{p(x)}{q(x)}$ where $p(x)$ & $q(x)$ are polynomials and $q(x) \neq 0$.

Example:

$$\frac{x^2 - 6x + 1}{x + 9}$$

$$\frac{4x^2 + 10x + 11}{5}$$

Note:

Every polynomial $p(x)$ is a rational expression but every rational expression need not to be a polynomial.

Irrational Expression:

An expression which cannot be written in the form of $\frac{p(x)}{q(x)}$

Term

Different parts of an algebraic expression joined by the operations of addition and subtraction are called term.

Example

$3x^3 + 5\sqrt{x} - 7$. The terms are $\mathbf{3x^3}$, $\mathbf{5\sqrt{x}}$, $\mathbf{-7}$

Rules to express a rational expression in its lowest term

$$Let \ \frac{p(x)}{q(x)}$$

Step 1: Factorize both the polynomial in the numerator and denominator.

Step 2: cancel the common factors between them.

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Q1: Which of the following expressions are polynomials?

(i) $1-5y+8y^2+6y^3$

Ans: Polynomial and also Rational

(ii) $\frac{5}{x^2}+\frac{3}{4x+1}$

Ans: Non-Polynomial but Rational

(iii) $\frac{\sqrt{x}}{6x-1}$

Ans: Non-Polynomial but Irrational

Q2: Which of the following rational expressions are in their lowest terms?

(i) $\frac{5y^2-5}{y-1}$

Solution:

$\frac{5y^2--5}{y-1}$

$\frac{5y^2-5}{y-1}=\frac{5(y^2-1)}{y-1}$

$\frac{5y^2-5}{y-1}=\frac{5(y+1)(y-1)}{y-1}$

$\frac{5y^2-5}{y-1}=5(y+1)$

So it is **Not** in Lowest Term:

(ii) $\frac{x^2-9}{x-2}$

Solution:

$\frac{x^2-9}{x-2}$

$\frac{x^2-9}{x-2}=\frac{(x+3)(x-3)}{x-2}$

We can't solve it more

So it is in Lowest Term

(iii) $\frac{x+y}{x^2-y^2}$

Solution:

$\frac{x+y}{x^2-y^2}$

$\frac{x+y}{x^2-y^2}=\frac{x+y}{(x+y)(x-y)}$

$\frac{x+y}{x^2-y^2}=\frac{1}{x-y}$

So it is **Not** in Lowest Term:

Q3: Reduce the following rational expression to their lowest term:

(i) $\frac{x-5}{x^2-5x}$

Solution:

$\frac{x-5}{x^2-5x}$

$\frac{x-5}{x^2-5x}=\frac{x-5}{x(x-5)}$

$\frac{x-5}{x^2-5x}=\frac{1}{x}$

(ii) $\frac{t^3(t-3)}{(t-3)(t+5)}$

Solution:

$\frac{t^3(t-3)}{(t-3)(t+5)}$

$\frac{t^3(t-3)}{(t-3)(t+5)}=\frac{t^3}{(t+5)}$

(iii) $\frac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$

Solution:

$\frac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$

Ans: It cannot be reduced further

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(iv) $\mathbf{\dfrac{2a+6}{a^2-9}}$

Solution:

$\dfrac{2a+6}{a^2-9}$

$\dfrac{2a+6}{a^2-9} = \dfrac{2(a+3)}{(a+3)(a-3)}$

$\dfrac{2a+6}{a^2-9} = \dfrac{2}{(a-3)}$

Q4: Add the following rational expressions:

(i) $\mathbf{4x^2-5x-10,\quad 2x^2+5x+10}$

Solution:

$4x^2-5x-10,\quad 2x^2+5x+10$

Now

$(4x^2-5x-10)+(2x^2+5x+10)$

$= 4x^2-5x-10+2x^2+5x+10$

Write the like term

$= 4x^2+2x^2-5x+5x-10+10$

$= 6x^2$

(ii) $\mathbf{\dfrac{y+9}{y^2+3}, \quad \dfrac{-7y+7}{y^2+3}}$

Solution:

$\dfrac{y+9}{y^2+3}, \quad \dfrac{-7y+7}{y^2+3}$

$= \dfrac{y+9}{y^2+3} + \dfrac{-7y+7}{y^2+3}$

$= \dfrac{(y+9)+(-7y+7)}{y^2+3}$

$= \dfrac{y+9-7y+7}{y^2+3}$

$= \dfrac{y-7y+9+7}{y^2+3}$

$= \dfrac{-6y+16}{y^2+3}$

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(iii) $\mathbf{\dfrac{y}{y+4}, \quad \dfrac{2y}{y-4}}$

Solution:

$\dfrac{y}{y+4}, \quad \dfrac{2y}{y-4}$

$= \dfrac{y}{y+4} + \dfrac{2y}{y-4}$

$= \dfrac{y(y-4)+2y(y+4)}{(y+4)(y-4)}$

$= \dfrac{y^2-4y+2y^2+8y}{(y+4)(y-4)}$

$= \dfrac{y^2+2y^2-4y++8y}{x^2-4^2}$

$= \dfrac{3y^2+4y}{x^2-16}$

(iv) $\mathbf{\dfrac{t}{t^2-25}, \quad \dfrac{3t}{t+5}}$

Solution:

$\dfrac{t}{t^2-25}, \quad \dfrac{3t}{t+5}$

$\dfrac{t}{t^2-25} + \dfrac{3t}{t+5}$

$\dfrac{t}{(t+5)(t-5)} + \dfrac{3t}{t+5}$

$\dfrac{t+3t(t-5)}{(t+5)(t-5)}$

$\dfrac{t+3t^2-15t}{t^2-5^2}$

$\dfrac{3t^2+t-15t}{t^2-25}$

$\dfrac{3t^2-14t}{t^2-25}$

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Ex # 4.1

Q5: Subtract the first expression from the second in the following.

(i) $y^2 + 4y - 15\,,\quad 8y^2 + 2$

Solution:

$y^2 + 4y - 15\,,\quad 8y^2 + 2$

$= (8y^2 + 2) - (y^2 + 4y - 15)$

$= 8y^2 + 2 - y^2 - 4y + 15$

$= 8y^2 - y^2 - 4y + 2 + 15$

$= 7y^2 - 4y + 17$

(ii) $\dfrac{8x^2 - 7}{x^2 + 1}\,,\quad \dfrac{8x^2 + 7}{x^2 + 1}$

Solution:

$\dfrac{8x^2 - 7}{x^2 + 1}\,,\quad \dfrac{8x^2 + 7}{x^2 + 1}$

$= \dfrac{8x^2 + 7}{x^2 + 1} - \dfrac{8x^2 - 7}{x^2 + 1}$

$= \dfrac{(8x^2 + 7) - (8x^2 - 7)}{x^2 + 1}$

$= \dfrac{8x^2 + 7 - 8x^2 + 7}{x^2 + 1}$

$= \dfrac{8x^2 - 8x^2 + 7 + 7}{x^2 + 1}$

$= \dfrac{14}{x^2 + 1}$

(iii) $\dfrac{1}{a - 3}\,,\quad \dfrac{2a}{a^2 - 9}$

Solution:

$\dfrac{1}{a - 3}\,,\quad \dfrac{2a}{a^2 - 9}$

$= \dfrac{2a}{a^2 - 9} - \dfrac{1}{a - 3}$

$= \dfrac{2a}{(a + 3)(a - 3)} - \dfrac{1}{a - 3}$

$= \dfrac{2a - 1(a + 3)}{(a + 3)(a - 3)}$

$= \dfrac{2a - a - 3}{(a + 3)(a - 3)}$

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$= \dfrac{a - 3}{(a + 3)(a - 3)}$

$= \dfrac{1}{(a + 3)}$

(iv) $\dfrac{x}{3x - 6}\,,\quad \dfrac{x + 2}{x - 2}$

Solution:

$\dfrac{x}{3x - 6}\,,\quad \dfrac{x + 2}{x - 2}$

$= \dfrac{x + 2}{x - 2} - \dfrac{x}{3x - 6}$

$= \dfrac{x + 2}{x - 2} - \dfrac{x}{3(x - 2)}$

$= \dfrac{3(x + 2) - x}{3(x - 2)}$

$= \dfrac{3x + 6 - x}{3(x - 2)}$

$= \dfrac{3x - x + 6}{3(x - 2)}$

$= \dfrac{2x + 6}{3(x - 2)}$

$= \dfrac{2(x + 3)}{3(x - 2)}$

Q6: Simplify the following.

(i) $\dfrac{2x}{6x - 9} \cdot \dfrac{4x - 6}{x^2 + x}$

Solution:

$\dfrac{2x}{6x - 9} \cdot \dfrac{4x - 6}{x^2 + x}$

$= \dfrac{2x}{3(2x - 3)} \cdot \dfrac{2(2x - 3)}{x(x + 1)}$

$= \dfrac{2}{3} \cdot \dfrac{2}{(x + 1)}$

$= \dfrac{4}{3(x + 1)}$

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(ii) $\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$

Solution:

$$\frac{x+4}{3-x} \cdot \frac{x^2-9}{x^2-16}$$
$$= \frac{x+4}{-x+3} \cdot \frac{x^2-3^2}{x^2-4^2}$$
$$= \frac{x+4}{-(x-3)} \cdot \frac{(x+3)(x-3)}{(x+4)(x-4)}$$
$$= \frac{1}{-1} \cdot \frac{(x+3)}{(x-4)}$$
$$= \frac{1(x+3)}{-1(x-4)}$$
$$= \frac{x+3}{-x+4}$$
$$= \frac{x+3}{4-x}$$

(iii) $\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$

Solution:

$$\frac{3x-15}{2x+6} \cdot \frac{x^2-9}{x^2-25}$$
$$= \frac{3(x-5)}{2(x+3)} \cdot \frac{(x+3)(x-3)}{(x+5)(x-5)}$$
$$= \frac{3}{2} \cdot \frac{(x-3)}{(x-5)}$$
$$= \frac{3(x-3)}{2(x-5)}$$

Q7: Simplify the following.

(i) $\frac{2y-10}{3y} \div (y-5)$

Solution:

$$\frac{2y-10}{3y} \div (y-5)$$
$$= \frac{2(y-5)}{3y} \times \frac{1}{y-5}$$
$$= \frac{2}{3y}$$

Ex # 4.1

(ii) $\frac{p}{q} \div \frac{r}{q} \cdot \frac{p}{q}$

Solution:

$$\frac{p}{q} \div \frac{r}{q} \cdot \frac{p}{q}$$
$$= \frac{p}{q} \cdot \frac{q}{r} \cdot \frac{p}{q}$$
$$= \frac{p}{q} \cdot \frac{1}{r} \cdot \frac{p}{1}$$
$$= \frac{p^2}{qr}$$

(iii) $\frac{a^2-9}{(a-6)(a+4)} \div \frac{a-3}{a-6}$

Solution:

$$\frac{a^2-9}{(a-6)(a+4)} \div \frac{a-3}{a-6}$$
$$= \frac{(a+3)(a-3)}{(a-6)(a+4)} \times \frac{a-6}{a-3}$$
$$= \frac{(a+3)}{(a+4)}$$
$$= \frac{a+3}{a+4}$$

Ex # 4.2

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Q1: Evaluate the following when $a = 3$, $b = -1$, $c = 2$.

(i) $5a - 10$

Solution:

$5a - 10$

$5a - 10 = 5(3) - 10$

$5a - 10 = 15 - 10$

$5a - 10 = 5$

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Ex # 4.2

(ii) $\mathbf{3b + 5c}$

Solution:

$3b + 5c$

$3b + 5c = 3(-1) + 5(2)$

$3b + 5c = -3 + 10$

$3b + 5c = 7$

(iii) $\mathbf{2a - 3b + 2c}$

Solution:

$2a - 3b + 2c$

$2a - 3b + 2c = 2(3) - 3(-1) + 2(2)$

$2a - 3b + 2c = 6 + 3 + 4$

$2a - 3b + 2c = 13$

Q2: Evaluate the following for $x = -5$ and $y = 2$.

(i) $\mathbf{7 - 3xy}$

Solution:

$7 - 3xy$

$7 - 3xy = 7 - 3(-5)(2)$

$7 - 3xy = 7 - 3(-10)$

$7 - 3xy = 7 + 30$

$7 - 3xy = 37$

(ii) $\mathbf{x^2 + xy + y^2}$

Solution:

$x^2 + xy + y^2$

$x^2 + xy + y^2 = (-5)^2 + (-5)(2) + (2)^2$

$x^2 + xy + y^2 = 25 + (-10) + 4$

$x^2 + xy + y^2 = 25 - 10 + 4$

$x^2 + xy + y^2 = 15 + 4$

$x^2 + xy + y^2 = 19$

(iii) $\mathbf{(3x)^2 - (4y)^2}$

Solution:

$(3x)^2 - (4y)^2$

$(3x)^2 - (4y)^2 = [3(-5)]^2 - [4(2)]^2$

$(3x)^2 - (4y)^2 = [-15]^2 - [8]^2$

$(3x)^2 - (4y)^2 = 225 - 64$

$(3x)^2 - (4y)^2 = 161$

Ex # 4.2

Q3: Evaluate the following when $k = -2$, $l = 3$, $m = 4$.

(i) $\mathbf{k^2(2l - 3m)}$

Solution:

$k^2(2l - 3m)$

$k^2(2l - 3m) = (-2)^2[2(3) - 3(4)]$

$k^2(2l - 3m) = 4(6 - 12)$

$k^2(2l - 3m) = 4(-6)$

$k^2(2l - 3m) = -24$

(ii) $\mathbf{5m\sqrt{k^2 + l^2}}$

Solution:

$5m\sqrt{k^2 + l^2}$

$5m\sqrt{k^2 + l^2} = 5(4)\sqrt{(-2)^2 + (3)^2}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{4 + 9}$

$5m\sqrt{k^2 + l^2} = 20\sqrt{13}$

(iii) $\mathbf{\dfrac{k + l + m}{k^2 + l^2 + m^2}}$

Solution:

$\dfrac{k + l + m}{k^2 + l^2 + m^2}$

Put the values

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{(-2) + (3) + (4)}{(-2)^2 + (3)^2 + (4)^2}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{-2 + 3 + 4}{4 + 9 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{1 + 4}{13 + 16}$

$\dfrac{k + l + m}{k^2 + l^2 + m^2} = \dfrac{5}{29}$

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Ex # 4.2

Q4: ***Evaluate*** $\boldsymbol{\frac{a+1}{4a^2+1}}$ ***when*** $\boldsymbol{a=\frac{1}{2}}$ **and** $\boldsymbol{a=-\frac{1}{2}}$.

Solution:

For $\boldsymbol{a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}+1}{4\left(\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{1+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{3}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{3}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{3}{4}$$

For $\boldsymbol{a=-\frac{1}{2}}$

$$\frac{a+1}{4a^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{-\frac{1}{2}+1}{4\left(-\frac{1}{2}\right)^2+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{-1+2}{2}}{4\left(\frac{1}{4}\right)+1}$$

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{1+1}$$

Ex # 4.2

$$\frac{a+1}{4a^2+1}=\frac{\frac{1}{2}}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\div 2$$

$$\frac{a+1}{4a^2+1}=\frac{1}{2}\times\frac{1}{2}$$

$$\frac{a+1}{4a^2+1}=\frac{1}{4}$$

Q5: ***If*** $\boldsymbol{a=9}$, $\boldsymbol{b=12, c=15}$ ***and*** $\boldsymbol{s=\frac{a+b+c}{2}}$.

Find the value of $\boldsymbol{\sqrt{s(s-a)(s-b)(s-c)}}$

Solution:

Given:

$$a=9, b=12, c=15 \text{ and } s=\frac{a+b+c}{2}$$

To Find:

$$\sqrt{s(s-a)(s-b)(s-c)}=?$$

First we find:

$$s=\frac{a+b+c}{2}$$

Put the values:

$$s=\frac{a+b+c}{2}$$

$$s=\frac{9+12+15}{2}$$

$$s=\frac{36}{2}$$

$$s=18$$

Now

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(18-9)(18-12)(18-15)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{18(9)(6)(3)}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{2\times 9\times 9\times 2\times 3\times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9\times 9\times 2\times 2\times 3\times 3}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{9^2\times 2^2\times 3^2}$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times 2\times 3$$

$$\sqrt{s(s-a)(s-b)(s-c)}=9\times 6$$

$$\sqrt{s(s-a)(s-b)(s-c)}=54$$

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Ex # 4.3

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$ **Q2, Q3(i)**
5. $(a+b)^2 - (a-b)^2 = 4ab$ **Q2, Q3(ii)**
6. $(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$ **Q1, Q5**
7. $(x+y)^2 - (x-y)^2 = 4xy$ **Q1, Q4, Q5**
8. $(u+v)^2 - (u-v)^2 = 4uv$ **Q6**

Ex # 4.3

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Q1: Find the value of $x^2 + y^2$ and xy, when:

(i) $\boldsymbol{x+y=8,\quad x-y=3}$

Solution:

$x+y=8,\quad x-y=3$

To Find:

$x^2+y^2 =?$ and $xy =?$

$\underline{x^2+y^2}$

As we have

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

Put the values

$(8)^2 + (3)^2 = 2(x^2+y^2)$

$64 + 9 = 2(x^2+y^2)$

$73 = 2(x^2+y^2)$

Divide B.S by 2

$\frac{73}{2} = \frac{2(x^2+y^2)}{2}$

$\frac{73}{2} = x^2 + y^2$

$x^2 + y^2 = \frac{73}{2}$

$\underline{xy}$

Also we have

$$(x+y)^2 - (x-y)^2 = 4xy$$

Put the values

$(8)^2 - (3)^2 = 4xy$

$64 - 9 = 4xy$

$55 = 4xy$

Divide B.S by 4

$\frac{55}{4} = \frac{4xy}{4}$

$\frac{55}{4} = xy$

$xy = \frac{55}{4}$

Ex # 4.3

(ii) $\boldsymbol{x+y=10,\quad x-y=7}$

Solution:

$x+y=10,\quad x-y=7$

To Find:

$x^2+y^2 =?$ And $xy =?$

$\underline{x^2+y^2}$

As we have

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

Put the values

$(10)^2 + (7)^2 = 2(x^2+y^2)$

$100 + 49 = 2(x^2+y^2)$

$149 = 2(x^2+y^2)$

Divide B.S by 2

$\frac{149}{2} = \frac{2(x^2+y^2)}{2}$

$\frac{149}{2} = x^2 + y^2$

$x^2 + y^2 = \frac{149}{2}$

$\underline{xy}$

Also we have

$$(x+y)^2 - (x-y)^2 = 4xy$$

Put the values

$(10)^2 - (7)^2 = 4xy$

$100 - 49 = 4xy$

$51 = 4xy$

Divide B.S by 4

$\frac{51}{4} = \frac{4xy}{4}$

$\frac{51}{4} = xy$

$xy = \frac{51}{4}$

(iii) $\boldsymbol{x+y=11,\quad x-y=5}$

Solution:

$x+y=11,\quad x-y=5$

To Find:

$x^2+y^2 =?$ and $xy =?$

$\underline{x^2+y^2}$

As we have

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

Put the values

$(11)^2 + (5)^2 = 2(x^2+y^2)$

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Ex # 4.3

$121 + 25 = 2(x^2 + y^2)$
$146 = 2(x^2 + y^2)$
Divide B.S by 2
$\frac{146}{2} = \frac{2(x^2 + y^2)}{2}$
$73 = x^2 + y^2$
$x^2 + y^2 = 73$

xy

Also we have
$$(x + y)^2 - (x - y)^2 = 4xy$$
Put the values
$(11)^2 - (5)^2 = 4xy$
$121 - 25 = 4xy$
$96 = 4xy$
Divide B.S by 4
$\frac{96}{4} = \frac{4xy}{4}$
$24 = xy$
$xy = 24$

(iv) $\boldsymbol{x + y = 7, \quad x - y = 4}$

Solution:
$x + y = 7, \quad x - y = 4$
To Find:
$x^2 + y^2 =$? and $xy =$?

x² + y²

As we have
$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$
Put the values
$(7)^2 + (4)^2 = 2(x^2 + y^2)$
$49 + 16 = 2(x^2 + y^2)$
$65 = 2(x^2 + y^2)$
Divide B.S by 2
$\frac{65}{2} = \frac{2(x^2 + y^2)}{2}$
$\frac{65}{2} = x^2 + y^2$
$x^2 + y^2 = \frac{65}{2}$

xy

Also we have
$$(x + y)^2 - (x - y)^2 = 4xy$$
Put the values
$(7)^2 - (4)^2 = 4xy$

Ex # 4.3

$49 - 16 = 4xy$
$33 = 4xy$
Divide B.S by 4
$\frac{33}{4} = \frac{4xy}{4}$
$\frac{33}{4} = xy$
$xy = \frac{33}{4}$

Q2: Find the value of $a^2 + b^2$ and ab, when

(i) $\boldsymbol{a + b = 7, \qquad a - b = 3}$

Solution:
$a + b = 7$ and $a - b = 3$
To Find:
$a^2 + b^2 =$? and $ab =$?

a² + b²

As we have
$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$
Put the values
$(7)^2 + (3)^2 = 2(a^2 + b^2)$
$49 + 9 = 2(a^2 + b^2)$
$58 = 2(a^2 + b^2)$
Divide B.S by 2
$\frac{58}{2} = \frac{2(a^2 + b^2)}{2}$
$29 = a^2 + b^2$
$a^2 + b^2 = 29$

ab

Also we have
$$(a + b)^2 - (a - b)^2 = 4ab$$
Put the values
$(7)^2 - (3)^2 = 4ab$
$49 - 9 = 4ab$
$40 = 4ab$
Divide B.S by 4
$\frac{40}{4} = \frac{4ab}{4}$
$10 = ab$
$ab = 10$

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Ex # 4.3

Q2: (ii) **Find the value of a^2+b^2 and ab, when $a+b=9,\ a-b=1$.**

Solution:

$a+b=9$ and $a-b=1$

To Find:

$a^2+b^2=?$ and $ab=?$

a^2+b^2

As we have

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put the values

$(9)^2+(1)^2=2(a^2+b^2)$

$81+1=2(a^2+b^2)$

$82=2(a^2+b^2)$

Divide B.S by 2

$\frac{82}{2}=\frac{2(a^2+b^2)}{2}$

$41=a^2+b^2$

$a^2+b^2=41$

ab

Also we have

$$(a+b)^2-(a-b)^2=4ab$$

Put the values

$(9)^2-(1)^2=4ab$

$81-1=4ab$

$80=4ab$

Divide B.S by 4

$\frac{80}{4}=\frac{4ab}{4}$

$20=ab$

$ab=20$

Q3: (i) **If $a+b=10,\ a-b=6$, then find the value of a^2+b^2.**

Solution:

$a+b=10$ and $a-b=6$

To Find:

$a^2+b^2=?$

As we have

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Put the values

$(10)^2+(6)^2=2(a^2+b^2)$

$100+36=2(a^2+b^2)$

$136=2(a^2+b^2)$

Ex # 4.3

Divide B.S by 2

$\frac{136}{2}=\frac{2(a^2+b^2)}{2}$

$68=a^2+b^2$

$a^2+b^2=68$

Q3: (ii) **If $a+b=5,\ a-b=\sqrt{17}$, then find the value of ab.**

Solution:

$a+b=5$ and $a-b=\sqrt{17}$

To Find:

$$ab=?$$

Also we have

$$(a+b)^2-(a-b)^2=4ab$$

Put the values

$(5)^2-\left(\sqrt{17}\right)^2=4ab$

$25-17=4ab$

$8=4ab$

Divide B.S by 4

$\frac{8}{4}=\frac{4ab}{4}$

$2=ab$

$ab=2$

Q4: **Find the value of $4xy$ when $x+y=17$, $x-y=5$.**

Solution:

$$x+y=17,\quad x-y=5$$

To find:

$$4xy=?$$

Also we have

$$(x+y)^2-(x-y)^2=4xy$$

Put the values

$(17)^2-(5)^2=4xy$

$289-25=4xy$

$264=4xy$

OR

$4xy=264$

Chapter # 4

Ex # 4.3

Q5: **If $+y = 11$ and $x - y = 3$, find $8xy(x^2 + y^2)$.**

Solution:

$x + y = 11, \quad x - y = 3$

To Find:

$8xy(x^2 + y^2) = ?$

As we have

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

Put the values

$(11)^2 + (3)^2 = 2(x^2 + y^2)$

$121 + 9 = 2(x^2 + y^2)$

$130 = 2(x^2 + y^2)$

$2(x^2 + y^2) = 130 \quad --equ(i)$

Also we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(11)^2 - (3)^2 = 4xy$

$121 - 9 = 4xy$

$112 = 4xy$

$4xy = 112 \quad --equ(ii)$

Multiply equ (i) and (ii)

$2(x^2 + y^2) \times 4xy = 130 \times 112$

$8xy(x^2 + y^2) = 14560$

Q6: **If $u + v = 7$ and $uv = 12$, find $u - v$.**

Solution:

$u + v = 7, \; uv = 12$

To Find:

$u - v = ?$

As we know that

$$(u + v)^2 - (u - v)^2 = 4uv$$

Put the values

$(7)^2 - (u - v)^2 = 4(12)$

$49 - (u - v)^2 = 48$

$-(u - v)^2 = 48 - 49$

$-(u - v)^2 = -1$

$(u - v)^2 = 1$

Taking square root on B.S

$\sqrt{(u - v)^2} = \sqrt{1}$

$u - v = \pm 1$

Ex # 4.4

1. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ **Q1, Q2, Q3**
2. $2(x^2 + y^2 + z^2 - xy - yz - zx) = (x - y)^2 + (y - z)^2 + (z - x)^2$ **Q4, Q5**
3. $2(a^2 + b^2 + c^2 - ab - bc - ca) = (a - b)^2 + (b - c)^2 + (c - a)^2$ **Q6**

Ex # 4.4

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Q1: **Find the values of $a^2 + b^2 + c^2$, when**

(i) **$a + b + c = 5$ and $ab + bc + ca = -4$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -4$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-4)$

$25 = a^2 + b^2 + c^2 - 8$

$25 + 8 = a^2 + b^2 + c^2$

$33 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 33$

(ii) **$a + b + c = 5$ and $ab + bc + ca = -2$**

Solution:

$a + b + c = 5$ and $ab + bc + ca = -2$

To Find:

$a^2 + b^2 + c^2 = ?$

As we know that

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Put the values

$(5)^2 = a^2 + b^2 + c^2 + 2(-2)$

$25 = a^2 + b^2 + c^2 - 4$

$25 + 4 = a^2 + b^2 + c^2$

$29 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 29$

Chapter # 4

Ex # 4.4

Q2: Find the values of $a+b+c$, when

(i) $a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

Solution:

$a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

To Find:

$a+b+c=?$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(a+b+c)^2=38+2(-1)$

$(a+b+c)^2=38-2$

$(a+b+c)^2=36$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{36}$

$a+b+c=6$

(ii) $a^2+b^2+c^2=10$ and $ab+bc+ca=11$

Solution:

$a^2+b^2+c^2=10$ and $ab+bc+ca=11$

To Find:

$a+b+c=?$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(a+b+c)^2=10+2(11)$

$(a+b+c)^2=10+22$

$(a+b+c)^2=32$

Taking square root on B.S

$\sqrt{(a+b+c)^2}=\sqrt{32}$

$a+b+c=\sqrt{16\times 2}$

$a+b+c=\sqrt{16}\times\sqrt{2}$

$a+b+c=4\sqrt{2}$

Q3: Find the values of $ab+bc+ca$, when

(i) $a^2+b^2+c^2=56$ and $a+b+c=12$

Solution:

$a^2+b^2+c^2=56$ and $a+b+c=12$

To Find:

$ab+bc+ca=?$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Ex # 4.4

Put the values

$(12)^2=56+2(ab+bc+ca)$

$144=56+2(ab+bc+ca)$

Subtract 56 from B.S

$144-56=56-56+2(ab+bc+ca)$

$88=2(ab+bc+ca)$

Divide B.S by 2

$\frac{88}{2}=\frac{2(ab+bc+ca)}{2}$

$44=ab+bc+ca$

$ab+bc+ca=44$

(ii) $a^2+b^2+c^2=12$ and $a+b+c=5$

Solution:

$a^2+b^2+c^2=12$ and $a+b+c=5$

To Find:

$ab+bc+ca=?$

As we know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Put the values

$(5)^2=12+2(ab+bc+ca)$

$25=12+2(ab+bc+ca)$

Subtract 12 from B.S

$25-12=12-12+2(ab+bc+ca)$

$13=2(ab+bc+ca)$

Divide B.S by 2

$\frac{13}{2}=\frac{2(ab+bc+ca)}{2}$

$\frac{13}{2}=ab+bc+ca$

$ab+bc+ca=\frac{13}{2}$

Chapter # 4

Ex # 4.4

Q #4 **Prove that $x^2+y^2+y^2-xy-yz-zx = \left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$**

Solution:

$x^2+y^2+y^2-xy-yz-zx =$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

R.H.S

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

$$=\frac{(x-y)^2}{(\sqrt{2})^2}+\frac{(y-z)^2}{(\sqrt{2})^2}+\frac{(z-x)^2}{(\sqrt{2})^2}$$

$$=\frac{x^2+y^2-2xy}{2}+\frac{y^2+z^2-2yz}{2}+\frac{z^2+x^2-2zx}{2}$$

$$=\frac{x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx}{2}$$

$$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{2}$$

$$=x^2+y^2+z^2-xy-yz-zx$$

$=$ L.H.S

Q #5 **Write $2[x^2+y^2+y^2-xy-yz-zx]$ as the sum of three squares.**

Solution:

$2[x^2+y^2+y^2-xy-yz-zx]$

$2x^2+2y^2+2z^2-2xy-2yz-2zx$

$x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$

Re-arranging the terms

$x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$

As we have

$$\boldsymbol{a^2+b^2-2ab=(a-b)^2}$$

$(x-y)^2+(y-z)^2+(z-x)^2$

Ex # 4.4

Q #6 **Find the value of $a^2+b^2+c^2-ab-bc-ca$ when $a-b=2,\ b-c=3, c-a=4.$**

Solution:

Given that:

$$a-b=2,\qquad b-c=3, c-a=4$$

To find

$$a^2+b^2+c^2-ab-bc-ca=?$$

As we have

$\boldsymbol{2(a^2+b^2+c^2-ab-bc-ca)=(a-b)^2+(b-c)^2+(c-a)^2}$

Put the values

$2(a^2+b^2+c^2-ab-bc-ca)=(2)^2+(3)^2+(4)^2$

$2(a^2+b^2+c^2-ab-bc-ca)=4+9+16$

$2(a^2+b^2+c^2-ab-bc-ca)=29$

Divide B.S by 2

$$\frac{2(a^2+b^2+c^2-ab-bc-ca)}{2}=\frac{29}{2}$$

$$a^2+b^2+c^2-ab-bc-ca=\frac{29}{2}$$

Ex # 4.5

1. $(a+b)^3=a^3+b^3+3ab(a+b)$ **Q#1, 7**
2. $(a-b)^3=a^3-b^3-3ab(a-b)$ **Q#2**
3. $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$ **Q#3**
4. $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$ **Q#4**
5. $\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\left(\frac{1}{a}\right)\left(3a+\frac{1}{a}\right)$ **Q#5**
6. $\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$ **Q#6**
7. $(u-v)^3=u^3-v^3-3uv(u-v)$ **Q#8**
8. $\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$ **Q#9**
9. $\left(a^2+\frac{1}{a^2}\right)^2=a^4+\frac{1}{a^4}+2(a^2)\left(\frac{1}{a^2}\right)$ **Q#9**
10. $\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)\left(a+\frac{1}{a}\right)$

Chapter # 4

Ex # 4.5

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Q1: Find the value of $a^3 + b^3$, when

(i) $a + b = 4$ and $ab = 5$.

Solution:

$$a + b = 4, \qquad ab = 5$$

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(5)(4)$

$64 = a^3 + b^3 + 60$

Subtract 60 from B.S

$64 - 60 = a^3 + b^3 + 60 - 60$

$4 = a^3 + b^3$

$a^3 + b^3 = 4$

(ii) $a + b = 3$ and $ab = 20$.

Solution:

$a + b = 3$ and $ab = 20$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(3)^3 = a^3 + b^3 + 3(3)(20)$

$27 = a^3 + b^3 + 180$

Subtract 180 from B.S

$27 - 180 = a^3 + b^3 + 180 - 180$

$-153 = a^3 + b^3$

$a^3 + b^3 = -153$

(iii) $a + b = 4$ and $ab = 2$.

Solution:

$a + b = 4$ and $ab = 2$.

To Find:

$$a^3 + b^3 = ?$$

As we have

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Put the values

$(4)^3 = a^3 + b^3 + 3(2)(4)$

$64 = a^3 + b^3 + 24$

Ex # 4.5

Subtract 24 from B.S

$64 - 24 = a^3 + b^3 + 24 - 24$

$40 = a^3 + b^3$

$a^3 + b^3 = 40$

Q2: Find the value of $a^3 - b^3$, when

(i) $a - b = 5$ and $ab = 7$.

Solution:

$a - b = 5, \; ab = 7$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Put the values

$(5)^3 = a^3 - b^3 - 3(7)(5)$

$125 = a^3 - b^3 - 105$

Add 105 on B.S

$125 + 105 = a^3 - b^3 - 105 + 105$

$230 = a^3 - b^3$

$a^3 - b^3 = 230$

(ii) $a - b = 2$ and $ab = 15$.

Solution:

$a - b = 2, \quad ab = 15$

To Find:

$$a^3 - b^3 = ?$$

As we have

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Put the values

$(2)^3 = a^3 - b^3 - 3(15)(2)$

$8 = a^3 - b^3 - 90$

Add 90 on B.S

$8 + 90 = a^3 - b^3 - 90 + 90$

$98 = a^3 - b^3$

$a^3 - b^3 = 98$

Chapter # 4

Ex # 4.5

(iii) $\boldsymbol{a-b=7}$ **and** $\boldsymbol{ab=6.}$

Solution:

$a-b=7, \quad ab=6$

To Find:

$$a^3-b^3=?$$

As we have

$$\boldsymbol{(a-b)^3=a^3-b^3-3ab(a-b)}$$

Put the values

$(7)^3=a^3-b^3-3(6)(7)$

$343=a^3-b^3-126$

Add 126 on B.S

$343+126=a^3-b^3-126+126$

$469=a^3-b^3$

$a^3+b^3=469$

Q3: Find the value of $\boldsymbol{x^3+\frac{1}{x^3}}$, when

(i) $\boldsymbol{x+\frac{1}{x}=\frac{5}{2}}$

Solution:

$x+\frac{1}{x}=\frac{5}{2}$

To Find:

$x^3+\frac{1}{x^3}=?$

As we have

$$\boldsymbol{\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)}$$

Put the values

$\left(\frac{5}{2}\right)^3=x^3+\frac{1}{x^3}+3\left(\frac{5}{2}\right)$

$\frac{125}{8}=x^3+\frac{1}{x^3}+\frac{15}{2}$

Subtract $\frac{15}{2}$ from B.S

$\frac{125}{8}-\frac{15}{2}=x^3+\frac{1}{x^3}+\frac{15}{2}-\frac{15}{2}$

$\frac{125-60}{8}=x^3+\frac{1}{x^3}$

$\frac{65}{8}=x^3+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=\frac{65}{8}$

Ex # 4.5

(ii) $\boldsymbol{x+\frac{1}{x}=2}$

Solution:

$$x+\frac{1}{x}=2$$

To Find:

$$x^3+\frac{1}{x^3}=?$$

As we have

$$\boldsymbol{\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3(x)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)}$$

Put the values

$(2)^3=x^3+\frac{1}{x^3}+3(2)$

$8=x^3+\frac{1}{x^3}+6$

Subtract 6 from B.S

$8-6=x^3+\frac{1}{x^3}+6-6$

$2=x^3+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=2$

Q3: Find the value of $\boldsymbol{x^3-\frac{1}{x^3}}$, when

(i) $\boldsymbol{x-\frac{1}{x}=\frac{3}{2}}$

Solution:

$$x-\frac{1}{x}=\frac{3}{2}$$

To Find:

$$x^3-\frac{1}{x^3}=?$$

As we have

$$\boldsymbol{\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right)}$$

Put the values

$\left(\frac{3}{2}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{3}{2}\right)$

$\frac{27}{8}=x^3-\frac{1}{x^3}-\frac{9}{2}$

Add $\frac{9}{2}$ on B.S

$\frac{27}{8}+\frac{9}{2}=x^3-\frac{1}{x^3}-\frac{9}{2}+\frac{9}{2}$

Chapter # 4

$\frac{27+36}{8} = x^3 - \frac{1}{x^3}$

$\frac{63}{8} = x^3 - \frac{1}{x^3}$

$x^3 - \frac{1}{x^3} = \frac{63}{8}$

(ii) $\boldsymbol{x - \frac{1}{x} = \frac{7}{3}}$

Solution:

$x - \frac{1}{x} = \frac{7}{3}$

To Find:

$x^3 - \frac{1}{x^3} = ?$

As we have

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Put the values

$\left(\frac{7}{3}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{7}{3}\right)$

$\frac{343}{27} = x^3 - \frac{1}{x^3} - \frac{21}{3}$

Add $\frac{21}{3}$ on B.S

$\frac{343}{27} + \frac{21}{3} = x^3 - \frac{1}{x^3} - \frac{21}{3} + \frac{21}{3}$

$\frac{343+189}{27} = x^3 - \frac{1}{x^3}$

$\frac{532}{27} = x^3 - \frac{1}{x^3}$

$x^3 - \frac{1}{x^3} = \frac{532}{27}$

(iii) $$\boldsymbol{x - \frac{1}{x} = \frac{15}{4}}$$

Solution:

$$x - \frac{1}{x} = \frac{15}{4}$$

To Find:

$$x^3 - \frac{1}{x^3} = ?$$

As we have

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

Ex # 4.5

Put the values

$\left(\frac{15}{4}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(\frac{15}{4}\right)$

$\frac{3375}{64} = x^3 - \frac{1}{x^3} - \frac{45}{4}$

Add $\frac{45}{4}$ on B.S

$\frac{3375}{64} + \frac{45}{4} = x^3 - \frac{1}{x^3} - \frac{45}{4} + \frac{45}{4}$

$\frac{3375+720}{64} = x^3 - \frac{1}{x^3}$

$\frac{4095}{64} = x^3 - \frac{1}{x^3}$

$x^3 - \frac{1}{x^3} = \frac{4095}{64}$

Q5: **If $\boldsymbol{3a + \frac{1}{a} = 4}$, find $\boldsymbol{27a^3 + \frac{1}{a^3}}$**

Solution:

$$3a + \frac{1}{a} = 4$$

To Find:

$$27a^3 + \frac{1}{a^3} = ?$$

As we have

$$\left(3a + \frac{1}{a}\right)^3 = 27a^3 + \frac{1}{a^3} + 3(3a)\left(\frac{1}{a}\right)\left(3a + \frac{1}{a}\right)$$

Put the values

$(4)^3 = 27a^3 + \frac{1}{a^3} + 9(4)$

$64 = 27a^3 + \frac{1}{a^3} + 36$

Subtract 36 from B.S

$64 - 36 = 27a^3 + \frac{1}{a^3} + 36 - 36$

$28 = 27a^3 + \frac{1}{a^3}$

$27a^3 + \frac{1}{a^3} = 28$

Chapter # 4

Ex # 4.5

Q6: **If $x-\frac{1}{2x}=6$, find $x^3-\frac{1}{8x^3}$**

Solution:

$x-\frac{1}{2x}=6$

To Find:

$x^3-\frac{1}{8x^3}=?$

As we have

$\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3(x)\left(\frac{1}{2x}\right)\left(x-\frac{1}{2x}\right)$

Put the values

$(6)^3=x^3-\frac{1}{8x^3}-\frac{3}{2}(6)$

$216=x^3-\frac{1}{8x^3}-3(3)$

$216=x^3-\frac{1}{8x^3}-9$

Add 9 on B.S

$216+9=x^3-\frac{1}{8x^3}-9+9$

$225=x^3-\frac{1}{8x^3}$

$x^3-\frac{1}{8x^3}=225$

Q7: **If $a+b=6$, show that $a^3+b^3+18ab=216$.**

Solution:

$$a+b=6$$

To Prove:

$$a^3+b^3+18ab=216$$

As we have

$$(a+b)^3=a^3+b^3+3ab(a+b)$$

Put the values

$$(6)^3=a^3+b^3+3ab(6)$$
$$216=a^3+b^3+18ab$$
$$a^3+b^3+18ab=216$$

Q8: **If $u-\text{v}=3$ then prove that $u^3-\text{v}^3-9uv=27$.**

Solution:

$$u-\text{v}=3$$

To Prove:

$$u^3-\text{v}^3-9uv=27$$

As we have

$$(u-v)^3=u^3-v^3-3uv(u-v)$$

Ex # 4.5

Put the values

$(2)^3=a^3-b^3-3(15)(2)$

$8=a^3-b^3-90$

Add 90 on B.S

$8+90=a^3-b^3-90+90$

$98=a^3-b^3$

$a^3-b^3=98$

Q9: **If $a+\frac{1}{a}=2$, find the values of $a^2+\frac{1}{a^2}$, $a^4+\frac{1}{a^4}$, $a^3+\frac{1}{a^3}$**

Solution:

Given

$$a+\frac{1}{a}=2$$

To prove

$a^2+\frac{1}{a^2}=?$

$a^4+\frac{1}{a^4}=?$

$a^3+\frac{1}{a^3}=?$

As we have

$\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2(a)\left(\frac{1}{a}\right)$

Put the values

$(2)^2=a^2+\frac{1}{a^2}+2$

$4=a^2+\frac{1}{a^2}+2$

Subtract 2 from B.S

$4-2=a^2+\frac{1}{a^2}+2-2$

$2=a^2+\frac{1}{a^2}$

$a^2+\frac{1}{a^2}=2$

Now take square on B.S

$\left(a^2+\frac{1}{a^2}\right)^2=(2)^2$

$(a^2)^2+\left(\frac{1}{a^2}\right)^2+2(a^2)\left(\frac{1}{a^2}\right)=4$

$a^4+\frac{1}{a^4}+2=4$

Chapter # 4

Ex # 4.5

Subtract 2 from B.S

$$a^4 + \frac{1}{a^4} + 2 - 2 = 4 - 2$$

$$a^4 + \frac{1}{a^4} = 2$$

$$\boldsymbol{Now\ a^3 + \frac{1}{a^3}}$$

Also we have

$$\left(\boldsymbol{a + \frac{1}{a}}\right)^3 = \boldsymbol{a^3 + \frac{1}{a^3} + 3(a)\left(\frac{1}{a}\right)\left(a + \frac{1}{a}\right)}$$

Put the values

$$(2)^3 = a^3 + \frac{1}{a^3} + 3(2)$$

$$8 = a^3 + \frac{1}{a^3} + 6$$

Subtract 6 from B.S

$$8 - 6 = a^3 + \frac{1}{a^3} + 6 - 6$$

$$2 = a^3 + \frac{1}{a^3}$$

$$a^3 + \frac{1}{a^3} = 2$$

Hence

$$a^2 + \frac{1}{a^2} = a^4 + \frac{1}{a^4} = a^3 + \frac{1}{a^3} = 2$$

Ex # 4.6

1. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
2. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
3. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - (x)\left(\frac{1}{x}\right)\right)$
4. $x^3 - \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + (x)\left(\frac{1}{x}\right)\right)$

OR

1. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$
2. $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} - 1\right)$
3. $(x + y)(x^2 - xy + y^2) = x^3 + y^3$
4. $(x - y)(x^2 + xy + y^2) = x^3 - y^3$
5. $(x + y)(x - y) = x^2 - y^2$

Ex # 4.6

Page # 118

Q1: Find the following product.

(i) $\boldsymbol{(a - 1)(a^2 + a + 1)}$

Solution:

$$(a - 1)(a^2 + a + 1)$$
$$= (a - 1)[(a)^2 + (a)(1) + (1)^2]$$

As we know that

$$\boldsymbol{(a - b)(a^2 + ab + b^2) = a^3 - b^3}$$

Here $a = a$ and $b = 1$

So

$= (a)^3 - (1)^3$

$= a^3 - 1$

(ii) $\boldsymbol{(3 - b)(9 + 3b + b^2)}$

Solution:

$$(3 - b)(9 + 3b + b^2)$$
$$= (3 - b)[(3)^2 + (3)(b) + (b)^2]$$

As we know that

$$\boldsymbol{(a - b)(a^2 + ab + b^2) = a^3 - b^3}$$

Here $a = 3$ and $b = b$

So

$= (3)^3 - (b)^3$

$= 27 - b^3$

(iii) $\boldsymbol{(8 + b)(64 - 8b + b^2)}$

Solution:

$$(8 + b)(64 - 8b + b^2)$$
$$= (8 + b)[(8)^2 - (8)(b) + (b)^2]$$

As we know that

$$\boldsymbol{(a + b)(a^2 - ab + b^2) = a^3 + b^3}$$

Here $a = 8$ and $b = b$

So

$= (8)^3 + (b)^3$

$= 512 + b^3$

(iv) $(a + 2)(a^2 - 2a + 4)$

Solution:

$$(a + 2)(a^2 - 2a + 4)$$
$$= (a + 2)[(a)^2 - (a)(2) + (2)^2]$$

As we know that

$$\boldsymbol{(a + b)(a^2 - ab + b^2) = a^3 + b^3}$$

Here $a = a$ and $b = 2$

So

$= (a)^3 + (2)^3$

$= a^3 + 8$

Chapter # 4

Ex # 4.6

Q2: Find the following product.

(i) $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

Solution:

$\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

$\left(2p+\frac{1}{2p}\right)\left[(2p)^2+\frac{1}{(2p)^2}-(2p)\left(\frac{1}{2p}\right)\right]$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$=(2p)^3+\left(\frac{1}{2p}\right)^3$

$=8p^3+\frac{1}{8p^3}$

(ii) $\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

Solution:

$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left(\frac{9}{4}p^2+\frac{4}{9p^2}+1\right)$

$\left(\frac{3}{2}p-\frac{2}{3p}\right)\left[\left(\frac{3}{2}p\right)^2+\left(\frac{2}{3p}\right)^2+\left(\frac{3}{2}p\right)\left(\frac{2}{3p}\right)\right]$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

So

$=\left(\frac{3}{2}p\right)^3-\left(\frac{2}{3p}\right)^3$

$=\frac{27}{8}p^3-\frac{8}{27p^3}$

(iii) $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

Solution:

$\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

$\left(3p-\frac{1}{3p}\right)\left[(3p)^2+\frac{1}{(3p)^2}+(3p)\left(\frac{1}{3p}\right)\right]$

As we know that

$$\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+(x)\left(\frac{1}{x}\right)\right)=x^3-\frac{1}{x^3}$$

Ex # 4.6

So

$=(3p)^3-\left(\frac{1}{3p}\right)^3$

$=27p^3+\frac{1}{27p^3}$

(iv) $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

Solution:

$\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

$\left(5p+\frac{1}{5p}\right)\left[(5p)^2+\frac{1}{(5p)^2}-(5p)\left(\frac{1}{5p}\right)\right]$

As we know that

$$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-(x)\left(\frac{1}{x}\right)\right)=x^3+\frac{1}{x^3}$$

So

$=(5p)^3+\left(\frac{1}{5p}\right)^3$

$=125p^3+\frac{1}{125p^3}$

Q3: Find the following continued product.

(i) $(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

Solution:

$(x^2-y^2)(x^2-xy+y^2)(x^2+xy+y^2)$

Using $a^2-b^2=(a+b)(a-b)$

$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

Arrange it

$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

By Using Formulas

$=(x^3+y^3)(x^3-y^3)$

Again by Formula

$=(x^3)^2-(y^3)^2$

$=x^6-y^6$

Chapter # 4

Ex # 4.6

(ii) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Solution:

$(x+y)(x-y)(x^2+y^2)(x^4+y^4)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^2-y^2)(x^2+y^2)(x^4+y^4)$

Again by Formula

$[(x^2)^2-(y^2)^2](x^4+y^4)$

$(x^4-y^4)(x^4+y^4)$

Now again by Formula

$(x^4)^2-(y^4)^2$

x^8-y^8

iii. $(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Solution:

$(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$

Arrange it

$(2x-y)(4x^2+2xy+y^2)(2x+y)(4x^2-2xy+y^2)$

$(2x-y)[(2x)^2+(2x)(y)+(y)^2](2x+y)[(2x)^2-(2x)(y)+(y)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(2x)^3-(y)^3][(2x)^3+(y)^3]$

$(8x^3-y^3)(8x^3+y^3)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(8x^3)^2-(y^3)^2$

$64x^6-y^6$

iv. $(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Solution:

$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Arrange it

$(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

$(x-2)[(x)^2+(x)(2)+(2)^2](x+2)[(x)^2-(x)(2)+(2)^2]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

and $(x+y)(x^2-xy+y^2)=x^3+y^3$

$[(x)^3-(2)^3][(x)^3+(2)^3]$

$(x^3-8)(x^3+8)$

Using Formula $(a+b)(a-b)=a^2-b^2$

$(x^3)^2-(8)^2$

x^6-64

Ex # 4.6

Q4: Find the product with the help of formula. $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

Solution:

$(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

$=(\sqrt{x}-\sqrt{y})\left[(\sqrt{x})^2+(\sqrt{x})(\sqrt{y})+(\sqrt{y})^2\right]$

As $(x-y)(x^2+xy+y^2)=x^3-y^3$

$=(\sqrt{x})^3-(\sqrt{y})^3$

$=\left(x^{\frac{1}{2}}\right)^3-\left(y^{\frac{1}{2}}\right)^3$

$=x^{\frac{3}{2}}-y^{\frac{3}{2}}$

Q5: Simplify with the help of formula. $(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

Solution:

$(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

$=(x^p+y^q)[(x^p)^2-(x^p)(y^q)+(y^q)^2]$

As $(x+y)(x^2-xy+y^2)=x^3+y^3$

$=(x^p)^3+(y^p)^3$

$=x^{3p}+y^{3p}$

Examples Page # 116 and 117

Ex # 4.7

<u>SURDS</u>

A number of the form of $\sqrt[n]{a}$ is called Surd, where a is a positive rational number.

A number will be a surd, if

i. It is irrational
ii. It is a root
iii. A root of a rational number.

Examples:

$\sqrt{3}$ and $\sqrt{5+\sqrt{3}}$

In the above examples, both are irrational numbers. First number is a root of rational number 3, whereas the second number is a root of irrational number $5+\sqrt{3}$.

Thus $\sqrt{3}$ is a surd and $\sqrt{5+\sqrt{3}}$ is not a surd.

$\sqrt[3]{8}$ is not a surd because its value is 2 which is rational.

$\sqrt{-2}$, $\sqrt{-3}$ are not surds because -2 *and* -3 are negative.

Conjugate of Surds

The conjugate of $a\sqrt{x}+b\sqrt{y}$ is $a\sqrt{x}-b\sqrt{y}$.

Similarly the conjugate of $5+\sqrt{3}$ is $5-\sqrt{3}$

Ex # 4.7

Page # 122

Q1: State which of the following are surd quantities

(i) $\sqrt[3]{81}$

As 81 is a rational number and the result is irrational.

So it is surd.

(ii) $\sqrt{1+\sqrt{5}}$

As $1+\sqrt{5}$ is irrational.

So it is not surd.

(iii) $\sqrt{\sqrt{5}}$

As $\sqrt{5}$ is irrational.

So it is not surd.

(iv) $\sqrt[4]{32}$

As 32 is a rational number and the result is irrational.

So it is surd.

Ex # 4.7

(v) π

As π is irrational.

So it is not surd.

(vi) $\sqrt{1+\pi^2}$

As $1+\pi^2$ is irrational.

So it is not surd.

Q2: Express the following as the simplest possible surds.

(i) $\sqrt{12}$

Solution:

$\sqrt{12}$

$\sqrt{2\times2\times3}$

$\sqrt{2^2\times3}$

$\sqrt{2^2}\sqrt{3}$

$2\sqrt{3}$

2	12
2	6
3	3
	1

(ii) $\sqrt{48}$

Solution:

$\sqrt{48}$

$\sqrt{2\times2\times2\times2\times3}$

$\sqrt{2^2\times2^2\times3}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3}$

$2\times2\sqrt{3}$

$4\sqrt{3}$

2	48
2	24
2	12
2	6
3	3
	1

(iii) $\sqrt{240}$

Solution:

$\sqrt{240}$

$\sqrt{2\times2\times2\times2\times3\times5}$

$\sqrt{2^2\times2^2\times3\times5}$

$\sqrt{2^2}\sqrt{2^2}\sqrt{3\times5}$

$2\times2\sqrt{15}$

$4\sqrt{15}$

2	240
2	120
2	60
2	30
3	15
5	5
	1

Chapter # 4

Ex # 4.7

Q3: Simplify the following surds.

(i) $\left(2-\sqrt{3}\right)\left(3+\sqrt{5}\right)$

Solution:

$\left(2-\sqrt{3}\right)\left(3+\sqrt{5}\right)$

$2\left(3+\sqrt{5}\right)-\sqrt{3}\left(3+\sqrt{5}\right)$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{3\times 5}$

$6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$

(ii) $\left(\sqrt{3}-4\right)\left(\sqrt{2}+1\right)$

Solution:

$\left(\sqrt{3}-4\right)\left(\sqrt{2}+1\right)$

$\sqrt{3}\left(\sqrt{2}+1\right)-4\left(\sqrt{2}+1\right)$

$\sqrt{3\times 2}+1\sqrt{3}-4\sqrt{2}-4$

$\sqrt{6}+\sqrt{3}-4\sqrt{2}-4$

(iii) $\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Solution:

$\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{2}\right)$

$\sqrt{2}\left(\sqrt{5}+\sqrt{2}\right)+\sqrt{3}\left(\sqrt{5}+\sqrt{2}\right)$

$\sqrt{2\times 5}+\sqrt{2\times 2}+\sqrt{3\times 5}+\sqrt{3\times 2}$

$\sqrt{10}+2+\sqrt{15}+\sqrt{6}$

(iv) $\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

Solution:

$\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

Using Formula: $(a+b)(a+b)=a^2-b^2$

So

$(3)^2-\left(2\sqrt{3}\right)^2$

$9-(2)^2\left(\sqrt{3}\right)^2$

$9-4(3)$

$9-12$

-3

Q4: Rationalize the denominator and simplify.

(i) $\dfrac{\mathbf{1}}{\sqrt{\mathbf{7}}}$

Solution:

$\dfrac{1}{\sqrt{7}}$

Ex # 4.7

Multiply and divide by $\sqrt{7}$

$\dfrac{1}{\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}}$

$\dfrac{1\sqrt{7}}{\left(\sqrt{7}\right)^2}$

$\dfrac{\sqrt{7}}{7}$

(ii) $\dfrac{\mathbf{3}}{\sqrt{\mathbf{45}}}$

Solution:

$\dfrac{3}{\sqrt{45}}$

$\dfrac{3}{\sqrt{3\times 3\times 5}}$

$\dfrac{3}{3\sqrt{5}}$

$\dfrac{1}{\sqrt{5}}$

Multiply and divide by $\sqrt{5}$

$\dfrac{1}{\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}$

$\dfrac{1\sqrt{5}}{\left(\sqrt{5}\right)^2}$

$\dfrac{\sqrt{5}}{5}$

(iii) $\dfrac{\mathbf{1}}{\sqrt{\mathbf{2}}-\mathbf{1}}$

Solution:

$\dfrac{1}{\sqrt{2}-1}$

Multiply and divide by $\sqrt{2}+1$

$\dfrac{1}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

$\dfrac{1\left(\sqrt{2}+1\right)}{\left(\sqrt{2}\right)^2-(1)^2}$

$\dfrac{\sqrt{2}+1}{2-1}$

$\sqrt{2}+1$

Chapter # 4

Ex # 4.7

(iv) $\mathbf{\dfrac{5}{2+\sqrt{5}}}$

Solution:

$\dfrac{5}{2+\sqrt{5}}$

Multiply and divide by $2-\sqrt{5}$

$\dfrac{5}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$

$\dfrac{5(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2}$

$\dfrac{5(2-\sqrt{5})}{4-5}$

$\dfrac{5(2-\sqrt{5})}{-1}$

$-5(2-\sqrt{5})$

(v) $\mathbf{\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}}$

Solution:

$\dfrac{1}{\sqrt{5}-2}+\dfrac{1}{\sqrt{5}+2}$

$\dfrac{1(\sqrt{5}+2)+1(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$\dfrac{\sqrt{5}+2+\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{\sqrt{5}+\sqrt{5}}{5-4}$

$\dfrac{2\sqrt{5}}{1}$

$2\sqrt{5}$

Q5: **If $x=\sqrt{5}+2$, find the value of $x+\dfrac{1}{x}$ and $x^2+\dfrac{1}{x^2}$**

Solution:

$x=\sqrt{5}+2$

To find:

$$x+\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}$

Ex # 4.7

Multiply and divide by $\sqrt{5}-2$

$\dfrac{1}{x}=\dfrac{1}{\sqrt{5}+2}\times\dfrac{\sqrt{5}-2}{\sqrt{5}-2}$

$\dfrac{1}{x}=\dfrac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{5-4}$

$\dfrac{1}{x}=\dfrac{\sqrt{5}-2}{1}$

$\dfrac{1}{x}=\sqrt{5}-2$

Now

$x+\dfrac{1}{x}=(\sqrt{5}+2)+(\sqrt{5}-2)$

$x+\dfrac{1}{x}=\sqrt{5}+2+\sqrt{5}-2$

$x+\dfrac{1}{x}=2\sqrt{5}$

Taking Square on B.S

$\left(x+\dfrac{1}{x}\right)^2=(2\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2(x)\left(\dfrac{1}{x}\right)=(2)^2(\sqrt{5})^2$

$x^2+\dfrac{1}{x^2}+2=4(5)$

$x^2+\dfrac{1}{x^2}+2=20$

Subtract 2 from B.S

$x^2+\dfrac{1}{x^2}+2-2=20-2$

$x^2+\dfrac{1}{x^2}=18$

Answers:

$$x+\frac{1}{x}=2\sqrt{5}$$

$$x^2+\frac{1}{x^2}=18$$

Chapter # 4

Ex # 4.7

Q6: **If $x=\sqrt{2}+\sqrt{3}$, find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x=\sqrt{2}+\sqrt{3}$

To find:

$$x-\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}$

Multiply and divide by $\sqrt{2}-\sqrt{3}$

$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$\frac{1}{x}=\frac{1(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2})^2-(\sqrt{3})^2}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{2-3}$

$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{-1}$

$\frac{1}{x}=-(\sqrt{2}-\sqrt{3})$

$\frac{1}{x}=-\sqrt{2}+\sqrt{3}$

Now

$x-\frac{1}{x}=(\sqrt{2}+\sqrt{3})-(-\sqrt{2}+\sqrt{3})$

$x-\frac{1}{x}=\sqrt{2}+\sqrt{3}+\sqrt{2}-\sqrt{3}$

$x-\frac{1}{x}=2\sqrt{2}$

Taking Square on B.S

$\left(x-\frac{1}{x}\right)^2=(2\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2(\sqrt{2})^2$

$x^2+\frac{1}{x^2}-2=4(2)$

$x^2+\frac{1}{x^2}-2=8$

Ex # 4.7

Add 2 on B.S

$x^2+\frac{1}{x^2}-2+2=8+2$

$x^2+\frac{1}{x^2}=10$

Answers:

$$x-\frac{1}{x}=2\sqrt{2}$$

$$x^2+\frac{1}{x^2}=10$$

Q7: **If $x=5-2\sqrt{6}$, find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$**

Solution:

$x=5-2\sqrt{6}$

To find:

$$x+\frac{1}{x}=?\text{ and } x^2+\frac{1}{x^2}=?$$

$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}$

Multiply and divide by $5+2\sqrt{6}$

$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}\times\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$

$\frac{1}{x}=\frac{1(5+2\sqrt{6})}{(5-2\sqrt{6})(5+2\sqrt{6})}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-(2)^2(\sqrt{6})^2}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-(4)(6)}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-24}$

$\frac{1}{x}=\frac{5+2\sqrt{6}}{1}$

$\frac{1}{x}=5+2\sqrt{6}$

Chapter # 4

Ex # 4.7

Now

$$x+\frac{1}{x}=(5-2\sqrt{6})+(5+2\sqrt{6})$$
$$x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}$$
$$x+\frac{1}{x}=10$$

Taking Square on B.S

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$
$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right)=100$$
$$x^2+\frac{1}{x^2}+2=100$$

Subtract 2 from B.S

$$x^2+\frac{1}{x^2}+2-2=100-2$$
$$x^2+\frac{1}{x^2}=98$$

Answers:

$$x+\frac{1}{x}=10$$
$$x^2+\frac{1}{x^2}=98$$

Q8: If $x=\frac{1}{\sqrt{2}-1}$ find the value of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$

Solution:

$$x=\frac{1}{\sqrt{2}-1}$$

To find

$$x-\frac{1}{x}=?\text{ and }x^2+\frac{1}{x^2}=?$$

Now

$$\frac{1}{x}=\sqrt{2}-1$$
$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}$$

Multiply and divide by $\sqrt{5}-2$

$$\frac{1}{x}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

Ex # 4.7

$$\frac{1}{x}=\frac{1(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)}$$
$$\frac{1}{x}=\frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$
$$\frac{1}{x}=\frac{\sqrt{5}-2}{5-4}$$
$$\frac{1}{x}=\frac{\sqrt{5}-2}{1}$$
$$\frac{1}{x}=\sqrt{5}-2$$

Now

$$x-\frac{1}{x}=(\sqrt{2}+1)-(\sqrt{2}-1)$$
$$x-\frac{1}{x}=\sqrt{2}+1-\sqrt{2}+1$$
$$x-\frac{1}{x}=2$$

Taking Square on B.S

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$
$$x^2+\frac{1}{x^2}-2(x)\left(\frac{1}{x}\right)=(2)^2$$
$$x^2+\frac{1}{x^2}-2=4$$

Add 2 on B.S

$$x^2+\frac{1}{x^2}-2+2=4+2$$
$$x^2+\frac{1}{x^2}=6$$

Answers:

$$x-\frac{1}{x}=2$$
$$x^2+\frac{1}{x^2}=6$$

Chapter # 4

Ex # 4.7

Q9: **If $x = \sqrt{10} + 3$, find the value of $x - \frac{1}{x}$ and $x^2 + \frac{1}{x^2}$**

Solution:

$x = \sqrt{10} + 3$

To find

$$x - \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$$\frac{1}{x} = \frac{1}{\sqrt{10} + 3}$$

Multiply and divide by $\sqrt{10} - 3$

$$\frac{1}{x} = \frac{1}{\sqrt{10} + 3} \times \frac{\sqrt{10} - 3}{\sqrt{10} - 3}$$

$$\frac{1}{x} = \frac{1(\sqrt{10} - 3)}{(\sqrt{10} + 3)(\sqrt{10} - 3)}$$

$$\frac{1}{x} = \frac{\sqrt{10} - 3}{(\sqrt{10})^2 - (3)^2}$$

$$\frac{1}{x} = \frac{\sqrt{10} - 3}{10 - 9}$$

$$\frac{1}{x} = \frac{\sqrt{10} - 3}{1}$$

$$\frac{1}{x} = \sqrt{10} - 3$$

Now

$$x - \frac{1}{x} = (\sqrt{10} + 3) - (\sqrt{10} - 3)$$

$$x - \frac{1}{x} = \sqrt{10} + 3 - \sqrt{10} + 3$$

$$x - \frac{1}{x} = \sqrt{10} - \sqrt{10} + 3 + 3$$

$$x - \frac{1}{x} = 6$$

Taking Square on B.S

$$\left(x - \frac{1}{x}\right)^2 = (6)^2$$

$$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 36$$

Ex # 4.7

$$x^2 + \frac{1}{x^2} - 2 = 36$$

Add 2 on B.S

$$x^2 + \frac{1}{x^2} - 2 + 2 = 36 + 2$$

$$x^2 + \frac{1}{x^2} = 38$$

Answers:

$$x - \frac{1}{x} = 6$$

$$x^2 + \frac{1}{x^2} = 38$$

Q10: **If $x = 2 - \sqrt{3}$, find the value of $x^4 + \frac{1}{x^4}$**

Solution:

$x = 2 - \sqrt{3}$

To find

$$x + \frac{1}{x} = ? \text{ and } x^2 + \frac{1}{x^2} = ?$$

Now

$$\frac{1}{x} = \frac{1}{2 - \sqrt{3}}$$

Multiply and divide by $2 + \sqrt{3}$

$$\frac{1}{x} = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$\frac{1}{x} = \frac{1(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$$

$$\frac{1}{x} = \frac{2 + \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2 + \sqrt{3}}{4 - 3}$$

$$\frac{1}{x} = \frac{2 + \sqrt{3}}{1}$$

$$\frac{1}{x} = 2 + \sqrt{3}$$

Now

$$x + \frac{1}{x} = (2 - \sqrt{3}) + (2 + \sqrt{3})$$

$$x + \frac{1}{x} = 2 - \sqrt{3} + 2 + \sqrt{3}$$

Chapter # 4

Ex # 4.7

$x + \frac{1}{x} = 2 + 2 - \sqrt{3} + \sqrt{3}$

$x + \frac{1}{x} = 4$

Taking Square on B.S

$\left(x + \frac{1}{x}\right)^2 = (4)^2$

$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 16$

$x^2 + \frac{1}{x^2} + 2 = 16$

Subtract 2 from B.S

$x^2 + \frac{1}{x^2} + 2 - 2 = 16 - 2$

$x^2 + \frac{1}{x^2} = 14$

Again take the square on B.S

$\left(x^2 + \frac{1}{x^2}\right)^2 = (14)^2$

$x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right) = 196$

$x^4 + \frac{1}{x^4} + 2 = 196$

Subtract 2 from B.S

$x^4 + \frac{1}{x^4} + 2 - 2 = 196 - 2$

$x^4 + \frac{1}{x^4} = 194$

Answer:

$$x^4 + \frac{1}{x^4} = 194$$

Review Exercise # 4

Page # 124

Q2: **Simplify** $\frac{12x^4y^5}{25a^3b^4} \cdot \frac{15a^5b^4}{16x^7y^2}$

Solution:

$\frac{12x^4y^5}{25a^3b^4} \cdot \frac{15a^5b^4}{16x^7y^2}$

$\frac{3y^3}{5} \cdot \frac{3a^2}{4x^3}$

$\frac{9y^3a^2}{20x^3}$

$\frac{9a^2y^3}{20x^3}$

Q3: **Evaluate** $\frac{2x-3}{x^2-x+1}$ **for** $x = 2$

Solution:

$\frac{2x-3}{x^2-x+1}$

Put the value

$\frac{2x-3}{x^2-x+1} = \frac{2(2)-3}{(2)^2-(2)+1}$

$\frac{2x-3}{x^2-x+1} = \frac{4-3}{4-2+1}$

$\frac{2x-3}{x^2-x+1} = \frac{1}{2+1}$

$\frac{2x-3}{x^2-x+1} = \frac{1}{3}$

Q4: **Find the value of $x^2 + y^2$ and xy when $x + y = 7$, $x - y = 3$.**

Solution:

$x + y = 7 \ , \ x - y = 3$

To Find:

$x^2 + y^2 =?$ and $xy =?$

$x^2 + y^2$

As we have

$$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$$

Put the values

$(7)^2 + (3)^2 = 2(x^2 + y^2)$

$49 + 9 = 2(x^2 + y^2)$

$58 = 2(x^2 + y^2)$

Chapter # 4

Review Ex # 4

Divide B.S by 2

$\frac{58}{2} = \frac{2(x^2 + y^2)}{2}$

$29 = x^2 + y^2$

$29 = x^2 + y^2$

<u>xy</u>

As we have

$$(x + y)^2 - (x - y)^2 = 4xy$$

Put the values

$(7)^2 - (3)^2 = 4xy$

$49 - 9 = 4xy$

$40 = 4xy$

Divide B.S by 4

$\frac{40}{4} = \frac{4xy}{4}$

$10 = xy$

$xy = 10$

Q5: Find the value of $a + b + c$ when $a^2 + b^2 + c^2 = 43$ and $ab + bc + ca = 3$.

<u>Solution:</u>

$a^2 + b^2 + c^2 = 43$ and $ab + bc + ca = 3$

To Find:

$$a + b + c = ?$$

As we know that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

Put the values

$(a + b + c)^2 = 43 + 2(3)$

$(a + b + c)^2 = 43 + 6$

$(a + b + c)^2 = 49$

Taking square root on B.S

$\sqrt{(a + b + c)^2} = \sqrt{49}$

$a + b + c = 7$

Q6: If $a + b + c = 6$ and $a^2 + b^2 + c^2 = 24$, then find the value of $ab + bc + ca$

<u>Solution:</u>

$a + b + c = 6 \; and \; a^2 + b^2 + c^2 = 24$

To Find:

$$ab + bc + ca = ?$$

As we know that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

Review Ex # 4

Put the values

$(6)^2 = 24 + 2(ab + bc + ca)$

$36 = 25 + 2(ab + bc + ca)$

Subtract 24 from B.S

$36 - 24 = 24 - 24 + 2(ab + bc + ca)$

$12 = 2(ab + bc + ca)$

Divide B.S by 2

$\frac{12}{2} = \frac{2(ab + bc + ca)}{2}$

$6 = ab + bc + ca$

$ab + bc + ca = 6$

Q7: If $2x - 3y = 8$ and $xy = 2$, then find the values of $8x^3 - 27y^3$.

<u>Solution:</u>

$2x - 3y = 8$ and $xy = 2$

To Find:

$$8x^3 - 27y^3 = ?$$

As we have

$$(2x - 3y)^3 = (2x)^3 - (3y)^3 - 3(2x)(3y)(2x - 3y)$$

Put the values

$(8)^3 = 8x^3 - 27y^3 - 18xy(8)$

$512 = 8x^3 - 27y^3 - 18(2)(8)$

$512 = 8x^3 - 27y^3 - 288$

Add 288 on B.S

$512 + 288 = 8x^3 - 27y^3 - 288 + 288$

$800 = 8x^3 - 27y^3$

$8x^3 - 27y^3 = 800$

Chapter # 4

Review Ex # 4

Q8: **Find the product** $\left(\frac{4}{5}x - \frac{5}{4x}\right)\left(\frac{16}{25}x^2 - \frac{25}{16x^2} + 1\right)$

Solution:

$$\left(\frac{4}{5}x - \frac{5}{4x}\right)\left(\frac{16}{25}x^2 - \frac{25}{16x^2} + 1\right)$$

$$\left(\frac{4}{5}x - \frac{5}{4x}\right)\left[\left(\frac{4}{5}x\right)^2 + \left(\frac{5}{4x}\right)^2 + \left(\frac{4}{5}x\right)\left(\frac{5}{4x}\right)\right]$$

As we know that

$$\left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + (x)\left(\frac{1}{x}\right)\right) = x^3 - \frac{1}{x^3}$$

$$= \left(\frac{4}{5}x\right)^3 - \left(\frac{5}{4x}\right)^3$$

$$= \frac{64}{125}x^3 - \frac{125}{64x^3}$$

Q9: **Find the value of** $x^3 + \frac{1}{x^3}$ **, when** $x + \frac{1}{x} = 8$

Solution:

$$x + \frac{1}{x} = 8$$

To Find:

$$x^3 + \frac{1}{x^3} = ?$$

As we have

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

Put the values

$$(8)^3 = x^3 + \frac{1}{x^3} + 3(8)$$

$$512 = x^3 + \frac{1}{x^3} + 24$$

Subtract 24 from B.S

$$512 - 24 = x^3 + \frac{1}{x^3} + 24 - 24$$

$$488 = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = 488$$

Review Ex # 4

Think **Trick**

Q10: **Simplify** $\frac{2x^2}{x^4 - 16} - \frac{x}{x^2 - 4} + \frac{1}{x + 2}$

Solution:

$$\frac{2x^2}{x^4 - 16} - \frac{x}{x^2 - 4} + \frac{1}{x + 2}$$

$$\frac{2x^2}{x^4 - 16} + \frac{1}{x + 2} - \frac{x}{x^2 - 4}$$

$$\frac{2x^2}{(x^2)^2 - (4)^2} + \frac{1}{x + 2} - \frac{x}{(x + 2)(x - 2)}$$

$$\frac{2x^2}{(x^2 + 4)(x^2 - 4)} + \frac{1(x - 2) - x}{(x + 2)(x - 2)}$$

$$\frac{2x^2}{(x^2 + 4)(x^2 - 4)} + \frac{x - 2 - x}{(x + 2)(x - 2)}$$

$$\frac{2x^2}{(x^2 + 4)(x^2 - 4)} + \frac{x - x - 2}{x^2 - 4}$$

$$\frac{2x^2}{(x^2 + 4)(x^2 - 4)} + \frac{-2}{x^2 - 4}$$

$$\frac{2x^2}{(x^2 + 4)(x^2 - 4)} - \frac{2}{x^2 - 4}$$

$$\frac{2x^2 - 2(x^2 + 4)}{(x^2 + 4)(x^2 - 4)}$$

$$\frac{2x^2 - 2x^2 - 8}{(x^2)^2 - (4)^2}$$

$$\frac{-8}{x^4 - 16}$$