UNIT # 3
LOGARITHM

Exercise # 3.1

SCIENTIFIC NOTATION:

Scientific notation is a way of writing numbers that are too big or too small to be easily written in decimal form.

Representation

The positive number "x" is represented in scientific notation as the product of two numbers where the first number "a" is a real number greater than 1 and less than 10 and the second is the integral power of "n" of 10.

$$x = a \times 10^n$$

Rules for Standard Notation to Scientific Notation

(i) In a given number, place the decimal after first non-zero digit.

(ii) If the decimal point is moved towards left, then the power of "10" will be positive.

(iii) If the decimal is moved towards right, then the power of "10" will be negative.

The numbers of digits through which the decimal point has been moved will be the exponent.

Rules for Standard Notation to Scientific Notation

(i) If the exponent of 10 is Positive, move the decimal towards Right.

(ii) If the exponent of 10 is Negative, move the decimal toward Left.

(iii) Move the decimal point to the same number of digits as the exponent of 10.

Example # 7 Page # 80

How many miles does light travel in 1 day? The speed of the light is 186,000 mi/ sec. write the answer in scientific notation.

Solution:

$Time = t = 1\ day = 24\ hr$

$t = 24 \times 60 \times 60\ sec = 86400$

$t = 8.64 \times 10^4\ sec$

$Speed = v = 186000\ mi/sec$

$v = 1.86 \times 10^5\ \ mi/sec$

As we know that

$$s = vt$$

Put the values

$s = 1.86 \times 10^5 \times 8.64 \times 10^4$

$s = 1.86 \times 8.64 \times 10^5 \times 10^4$

$s = 16.0704 \times 10^{5+4}$

$s = 16.0704 \times 10^9$

$s = 1.60704 \times 10^1 \times 10^9$

$s = 1.60704 \times 10^{10}$

Thus light travels $1.60704 \times 10^1 \times 10^9$ miles in a day

Exercise # 3.1

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Q1: Write each number in scientific notation.

(i) 405,000

Solution:

405,000

In Scientific Form:

$$4.05 \times 10^4$$

(ii) 1,670,000

Solution:

1,670,000

In Scientific Form:

$$1.67 \times 10^6$$

(iii) 0.00000039

Solution:

0.00000039

In Scientific Form:

$$3.9 \times 10^{-7}$$

(iv) 0.00092

Solution:

0.00092

In Scientific Form:

$$9.2 \times 10^{-4}$$

Ex # 3.1

(v) **234,600,000,000**

Solution:

234,600,000,000

In Scientific Form:

$$2.346 \times 10^{11}$$

(vi) **8,904,000,000**

Solution:

8,904,000,000

In Scientific Form:

$$8.904 \times 10^{9}$$

(vii) **0.00104**

Solution:

0.00104

In Scientific Form:

$$1.04 \times 10^{-3}$$

(viii) **0.00000000514**

Solution:

0.00000000514

In Scientific Form:

$$5.14 \times 10^{-9}$$

(ix) $\mathbf{0.05 \times 10^{-3}}$

Solution:

$$0.05 \times 10^{-3}$$

In Scientific Form:

$$5.0 \times 10^{-2} \times 10^{-3}$$

$$5.0 \times 10^{-2-3}$$

$$5.0 \times 10^{-5}$$

Q2: **Write each number in standard notation.**

(i) $\mathbf{8.3 \times 10^{-5}}$

Solution:

8.3×10^{-5}

In Standard Form:

0.000083

(ii) $\mathbf{4.1 \times 10^{6}}$

Solution:

4.1×10^{6}

In Standard Form:

410000

Ex # 3.1

(iii) $\mathbf{2.07 \times 10^{7}}$

Solution:

2.07×10^{7}

In Standard Form:

20700000

(iv) $\mathbf{3.15 \times 10^{-6}}$

Solution:

3.15×10^{-6}

In Standard Form:

0.00000315

(v) $\mathbf{6.27 \times 10^{-10}}$

Solution:

6.27×10^{-10}

In Standard Form:

0.000000000627

(vi) $\mathbf{5.41 \times 10^{-8}}$

Solution:

5.41×10^{-8}

In Standard Form:

0.0000000541

(vii) $\mathbf{7.632 \times 10^{-4}}$

Solution:

7.632×10^{-4}

In Standard Form:

0.0007632

(viii) $\mathbf{9.4 \times 10^{5}}$

Solution:

9.4×10^{5}

In Standard Form:

940000

(ix) $\mathbf{-2.6 \times 10^{9}}$

Solution:

-2.6×10^{9}

In Standard Form:

-2600000000

Ex # 3.1

Q3: How long does it take light to travel to Earth from the sun? The sun is 9.3×10^7 miles from Earth, and light travels 1.86×10^5 mi/s.

Solution:

Given:

Distance between earth and sun $= 9.3 \times 10^7$ miles

Speed of light $= 1.86 \times 10^5$ mi/s

As we have:

$s = vt$

$\frac{s}{v} = t$

Or

$t = \frac{s}{v}$

Put the values:

$t = \frac{9.3 \times 10^7}{1.86 \times 10^5}$

$t = 5 \times 10^7 \times 10^{-5}$

$t = 5 \times 10^{7-5}$

$t = 5 \times 10^2$

$t = 500 \ sec$

$t = 480 \ \text{sec} + 20 \ sec$

$t = 8 \ \text{min} \ 20 \ sec$

Exercise # 3.2

Logarithm

If $a^x = y$ then the index x is called the logarithm of y to the base a and written as $\log_a y = x$.

We called $\log_a y = x$ like log of y to the base a equal to x.

Logarithm Form	Exponential Form
$\log_a y = x$	$a^x = y$
$\log_8 64 = 2$	$8^2 = 64$

Ex # 3.2

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Q1: Write the following in logarithm form.

(i) $\mathbf{4^4 = 256}$

Solution:

$4^4 = 256$

In logarithm form

$\log_4 256 = 4$

(ii) $\mathbf{2^{-6} = \frac{1}{64}}$

Solution:

$2^{-6} = \frac{1}{64}$

In logarithm form

$\log_2 \frac{1}{64} = -6$

(iii) $\mathbf{10^0 = 1}$

Solution:

$10^0 = 1$

In logarithm form

$\log_{10} 1 = 0$

(iv) $x^{\frac{3}{4}} = y$

Solution:

$x^{\frac{3}{4}} = y$

In logarithm form

$\log_x y = \frac{3}{4}$

(v) $\mathbf{3^{-4} = \frac{1}{81}}$

Solution:

$3^{-4} = \frac{1}{81}$

In logarithm form

$\log_3 \frac{1}{81} = -4$

(vi) $\mathbf{64^{\frac{2}{3}} = 16}$

Solution:

$64^{\frac{2}{3}} = 16$

In logarithm form

$\log_{64} 16 = \frac{2}{3}$

Ex # 3.2

Q2: Write the following in exponential form.

(i) $\log_a\left(\frac{1}{a^2}\right) = -1$

Solution:

$\log_a\left(\frac{1}{a^2}\right) = -1$

In exponential form

$a^{-1} = \frac{1}{a^2}$

(ii) $\log_2 \frac{1}{128} = -7$

Solution:

$\log_2 \frac{1}{128} = -7$

In exponential form

$2^{-7} = \frac{1}{128}$

(iii) $\log_b 3 = 64$

Solution:

$\log_b 3 = 64$

In exponential form

$b^{64} = 3$

(iv) $\log_a a = 1$

Solution:

$\log_a a = 1$

In exponential form

$a^1 = 1$

(v) $\log_a 1 = 0$

Solution:

$\log_a 1 = 0$

In exponential form

$a^0 = 1$

(vi) $\log_4 \frac{1}{8} = \frac{-3}{2}$

Solution:

$\log_4 \frac{1}{8} = \frac{-3}{2}$

In exponential form

$4^{\frac{-3}{2}} = \frac{1}{8}$

Ex # 3.2

Q3: Solve:

(i) $\log_{\sqrt{5}} 125 = x$

Solution:

$\log_{\sqrt{5}} 125 = x$

In exponential form

$(\sqrt{5})^x = 125$

$\left(5^{\frac{1}{2}}\right)^x = 5 \times 5 \times 5$

$5^{\frac{x}{2}} = 5^3$

Now

$\frac{x}{2} = 3$

Multiply B.S by 2

$2 \times \frac{x}{2} = 2 \times 3$

$x = 6$

(ii) $\log_4 x = -3$

Solution:

$\log_4 x = -3$

In exponential form

$4^{-3} = x$

Now

$\frac{1}{4^3} = x$

$\frac{1}{4 \times 4 \times 4} = x$

$\frac{1}{64} = x$

Or

$x = \frac{1}{64}$

(iii) $\log_{81} 9 = x$

Solution:

$\log_{81} 9 = x$

In exponential form

$81^x = 9$

$(9^2)^x = 9^1$

$9^{2x} = 9^1$

Now $2x = 1$

Divide B.S by 2

$\frac{2x}{2} = \frac{1}{2}$

$2x = \frac{1}{2}$

Ex # 3.2

(iv) $\log_3(5x+1)=2$

Solution:

$\log_3(5x+1)=2$

In exponential form

$3^2=5x+1$

$9=5x+1$

Subtract 1 form B.S

$9-1=5x+1-1$

$8=5x$

Divide B.S by 5

$\frac{8}{5}=\frac{5x}{5}$

$\frac{8}{5}=x$

$x=\frac{8}{5}$

(v) $\log_2 x=7$

Solution:

$\log_2 x=7$

In exponential form

$2^7=x$

Now

$2\times2\times2\times2\times2\times2\times2=x$

$128=x$

$x=128$

(vi) $\log_x 0.25=2$

Solution:

$\log_x 0.25=2$

In exponential form

$x^2=0.25$

$x^2=\frac{25}{100}$

Taking square root on B.S

$\sqrt{x^2}=\sqrt{\frac{25}{100}}$

$x=\frac{5}{10}$

$x=\frac{1}{2}$

Ex # 3.2

(vii) $\log_x(0.001)=-3$

Solution:

$\log_x(0.001)=-3$

In exponential form

$x^{-3}=0.001$

$x^{-3}=\frac{1}{1000}$

$x^{-3}=\frac{1}{10^3}$

$x^{-3}=10^{-3}$

So

$x=10$

(viii) $\log_x\frac{1}{64}=-2$

Solution:

$\log_x\frac{1}{64}=-2$

In exponential form

$x^{-2}=\frac{1}{64}$

$x^{-2}=\frac{1}{8\times8}$

$x^{-2}=\frac{1}{8^2}$

$x^{-2}=8^{-2}$

So

$x=8$

(ix) $\log_{\sqrt{3}}x=16$

Solution:

$\log_{\sqrt{3}}x=16$

In exponential form

$(\sqrt{3})^{16}=x$

$\left(3^{\frac{1}{2}}\right)^{16}=x$

$3^{\frac{16}{2}}=x$

$3^8=x$

$3\times3\times3\times3\times3\times3\times3\times3=x$

$6561=x$

$x=6561$

Exercise # 3.3

COMMON LOGARITHM

Introduction

The common logarithm was invented by a British Mathematician Prof. Henry Briggs (1560-1631).

Definition

Logarithms having base 10 are called common logarithms or Briggs logarithms.

Note:

The base of logarithm is not written because it is considered to be 10.

Logarithm of the number consists of two parts.

Characteristics and Mantissa

Example: 1.5377

Characteristics

The digit before the decimal point or Integral part is called characteristics

Mantissa

The decimal fraction part is mantissa.

In above example

1 is Characteristics and .5377 is Mantissa.

USE OF LOG TABLE TO FIND MANTISSA:

A logarithm table is divided into three parts.

(i) The first part of the table is the extreme left column contains number from10 to 99.

(ii) The second part of the table consists of 10 columns headed by 0, 1, 2, …. 9. The number under these columns are taken to find mantissa.

(iii) The third part consists of small columns known as mean difference headed by 1, 2, 3, … 9. These columns are added to the Mantissa found in second column.

To Find Mantissa

Let we have an example: 763.5

Solution:

(i) First ignore the decimal point.

(ii) Take first two digits e.g. 76 and proceed along this row until we come to column headed by third digit 3 of the number which is 8825.

(iii) Now take fourth digit i.e. 5 and proceed along this row in mean difference column which is 5.

(iv) Now add $8825 + 3 = 8828$

Ex # 3.3

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Q1: Find the characteristics of the common logarithm of each of the following numbers.

(i) 57

In Scientific form:

5.7×10^1

Thus Characteristics $= 1$

(ii) 7.4

In Scientific form:

7.4×10^0

Thus Characteristics $= 0$

(iii) 5.63

In Scientific form:

5.63×10^0

Thus Characteristics $= 0$

(iv) 56.3

In Scientific form:

5.63×10^1

Thus Characteristics $= 1$

(v) 982.5

In Scientific form:

9.825×10^2

Thus Characteristics $= 2$

(vi) 7824

In Scientific form:

7.824×10^3

Thus Characteristics $= 3$

(vii) 186000

In Scientific form:

1.86×10^5

Thus Characteristics $= 5$

viii. 0.71

In Scientific form:

7.1×10^{-1}

Thus Characteristics $= -1$

Ex # 3.3

Q2: Find the following.

(i) $\log 87.2$

Solution:

$\log 87.2$

In Scientific form:

8.72×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9405$

Hence $\log 87.2 = 1.9405$

(ii) $\log 37300$

Solution:

$\log 37300$

In Scientific form:

3.73×10^4

Thus Characteristics $= 4$

To find Mantissa, using Log Table:

So Mantissa $= .5717$

Hence $\log 37300 = 4.5717$

(iii) $\log 753$

Solution:

$\log 753$

In Scientific form:

7.53×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8768$

Hence $\log 753 = 2.8768$

(iv) $\log 9.21$

Solution:

$\log 9.21$

In Scientific form:

9.21×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .9643$

Hence $\log 9.21 = 0.9643$

Ex # 3.3

(v) $\log 0.00159$

Solution:

$\log 0.00159$

In Scientific form:

1.59×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .2014$

Hence $\log 0.00159 = \bar{3}.2014$

(vi) $\log 0.0256$

Solution:

$\log 0.0256$

In Scientific form:

2.56×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .4082$

Hence $\log 0.0256 = \bar{2}.4082$

(vii) $\log 6.753$

Solution:

$\log 6.753$

In Scientific form:

6.753×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table

Mantissa $= .8295$

Hence $\log 6.753 = 0.8295$

$R.W$
$8293 + 2 = 8295$

Q3: Find logarithms of the following numbers.

(i) 2476

Solution:

2476

$Let\ x = 2476$

Taking log on B.S

$\log x = \log 2476$

In Scientific form:

2.476×10^3

Thus Characteristics $= 3$

To find Mantissa, using Log Table

So Mantissa $= .3927 + 11$

Mantissa $= .3938$

Hence $\log 2476 = 3.3938$

$R.W$
$3927 + 11 = 3938$

Ex # 3.3

(ii) 2.4

Solution:

2.4

$Let\ x = 2.4$

Taking log on B.S

$\log x = \log 2.4$

In Scientific form:

2.4×10^0

Thus Characteristics $= 0$

To find Mantissa, using Log Table:

So Mantissa $= .3802$

Hence $\log 2.4 = 0.3802$

(iii) 92.5

Solution:

92.5

$Let\ x = 92.5$

Taking log on B.S

$\log x = \log 92.5$

In Scientific form:

9.25×10^1

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

So Mantissa $= .9661$

Hence $\log 92.5 = 1.9661$

(iv) 482.7

Solution:

482.7

$Let\ x = 482.7$

Taking log on B.S

$\log x = \log 482.7$

In Scientific form:

4.827×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .6836$

Hence $\log 482.7 = 2.6836$

$R.W$
$6830 + 6 = 6836$

Ex # 3.3

(v) 0.783

Solution:

0.783

$Let\ x = 0.783$

Taking log on B.S

$\log x = \log 0.783$

In Scientific form:

7.83×10^{-1}

Thus Characteristics $= -1$

To find Mantissa, using Log Table:

So Mantissa $= .8938$

Hence $\log 0.783 = \bar{1}.8938$

(vi) 0.09566

Solution:

0.09566

$Let\ x = 0.09566$

Taking log on B.S

$\log x = \log 0.09566$

In Scientific form:

9.566×10^{-2}

Thus Characteristics $= -2$

To find Mantissa, using Log Table:

So Mantissa $= .9808$

Hence $\log 0.09566 = \bar{2}.9808$

$R.W$
$9805 + 3 = 9808$

(vii) 0.006753

Solution:

0.006753

$Let\ x = 0.006753$

Taking log on B.S

$\log x = \log 0.006753$

In Scientific form:

6.753×10^{-3}

Thus Characteristics $= -3$

To find Mantissa, using Log Table:

So Mantissa $= .8295$

Hence $\log 0.006735 = \bar{3}.8295$

$R.W$
$8293 + 2 = 8295$

Ex # 3.3

(viii) 700

Solution:

700

$Let\ x = 700$

Taking log on B.S

$\log x = \log 700$

In Scientific form:

7.00×10^2

Thus Characteristics $= 2$

To find Mantissa, using Log Table:

So Mantissa $= .8451$

Hence $\log 700 = 2.8451$

Exercise # 3.4

ANTI-LOGARITHM

If $\log x = y$ then x is the anti-logarithm of y and written as $x = anti - \log y$

Explanation with Example:

2.3456

(i) Here the digit before decimal point is Characteristics i.e. 2

(ii) And Mantissa$= .3456$

To find anti-log, we see Mantissa in Anti-log Table

(i) Take first two digits i.e. $.34$ and proceed along this row until we come to column headed by third digit 5 of the number which is 2213.

(ii) Now take fourth digit i.e. 6 and proceed along this row which is 3.

(iii) Now add $2213 + 3 = 2216$

So to find anti-log, write it in Scientific form like

$anti - \log 2.3456 = 2.2216 \times 10^{char}$

$anti - \log 2.3456 = 2.216 \times 10^2$

$anti - \log 2.3456 = 221.6$

Ex # 3.4

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Q1: Find anti-logarithm of the following numbers.

(i) 1.2508

Solution:

1.2508

$Let\ \log x = 1.2508$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 1.2508$

$x = \text{Anti} - \log 1.2508$

Characteristics $= 1$

Mantissa $= .2508$

So

$x = 1.781 \times 10^1$

$x = 17.81$

$R.W$
$1778+3$ $= 1781$

(ii) 0.8401

Solution:

0.8401

$Let\ \log x = 0.8401$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 0.8401$

$x = \text{Anti} - \log 0.8401$

Characteristics $= 0$

Mantissa $= .8401$

So

$x = 6.920 \times 10^0$

$x = 6.920$

$R.W$
$6918+2$ $= 6920$

(iii) 2.540

Solution:

2.540

$Let\ \log x = 2.540$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 2.540$

$x = \text{Anti} - \log 2.540$

Characteristics $= 2$

Mantissa $= .540$

So

$x = 3.467 \times 10^2$

$x = 346.7$

Ex # 3.4

(iv) $\bar{2}.2508$

Solution:

$\bar{2}.2508$

$Let\ \log x = \bar{2}.2508$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log \bar{2}.2508$

$x = \text{Anti} - \log \bar{2}.2508$

Characteristics $= -2$

Mantissa $= .2508$

So

$x = 1.781 \times 10^{-2}$

$x = 0.01781$

$R.W$
1778+3 = 1781

(v) $\bar{1}.5463$

Solution:

$\bar{1}.5463$

$Let\ \log x = \bar{1}.5463$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log \bar{1}.5463$

$x = \text{Anti} - \log \bar{1}.5463$

Characteristics $= -1$

Mantissa $= .5463$

So

$x = 3.518 \times 10^{-1}$

$x = 0.3518$

$R.W$
3516+2 = 3518

(vi) **3.5526**

Solution:

3.5526

$Let\ \log x = 3.5526$

Taking anti-log on B.S

$Anti - \log(\log x) = \text{Anti} - \log 3.5526$

$x = \text{Anti} - \log 3.5526$

Characteristics $= 3$

Mantissa $= .5526$

So

$x = 3.570 \times 10^{3}$

$x = 3570$

$R.W$
3565+5 = 3570

Ex # 3.4

Q2: Find the values of x from the following equations:

(i) $\log x = \bar{1}.8401$

Solution:

$\log x = \bar{1}.8401$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \bar{1}.8401$

$x = \text{Anti} - \log \bar{1}.8401$

Characteristics $= -1$

Mantissa $= .8401$

So

$x = 6.920 \times 10^{-1}$

$x = 0.6920$

$R.W$
6918 + 2 = 6920

(ii) $\log x = 2.1931$

Solution:

$\log x = 2.1931$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 2.1931$

$x = \text{Anti} - \log 2.1931$

Characteristics $= 2$

Mantissa $= .1931$

So

$x = 1.560 \times 10^{2}$

$x = 156.0$

$R.W$
1560 + 0 = 1560

(iii) $\log x = 4.5911$

Solution:

$\log x = 4.5911$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 4.5911$

$x = \text{Anti} - \log 4.5911$

Characteristics $= 4$

Mantissa $= .5911$

So

$x = 3.900 \times 10^{4}$

$x = 39000.0$

$R.W$
3899 + 1 = 3900

Ex # 3.4

(i) $\log x = \overline{3}.0253$

Solution:

$\log x = \overline{3}.0253$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \overline{3}.0253$

$x = \text{Anti} - \log \overline{3}.0253$

Characteristics $= -3$

Mantissa $= .0253$

So

$x = 1.060 \times 10^{-3}$

$x = 0.001060$

$R.W$
$1059 + 1 = 1060$

(ii) $\log x = 1.8716$

Solution:

$\log x = 1.8716$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log 1.8716$

$x = \text{Anti} - \log 1.8716$

Characteristics $= 1$

Mantissa $= .8716$

So

$x = 7.440 \times 10^{1}$

$x = 74.40$

$R.W$
$7430 + 10 = 7440$

(iii) $\log x = \overline{2}.8370$

Solution:

$\log x = \overline{2}.8370$

Taking anti − log on B.S

$\text{Anti} - \log(\log x) = \text{Anti} - \log \overline{2}.8370$

$x = \text{Anti} - \log \overline{2}.8370$

Characteristics $= -2$

Mantissa $= .8370$

So

$x = 6.871 \times 10^{-2}$

$x = 0.06781$

Ex # 3.5

LAWS OF LOGARITHM

(i) $\log_a mn = \log_a m + \log_a n$

or $\log mn = \log m + \log n$

Example:

$\log 2 \times 3 = \log 2 + \log 3$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

or $\log \frac{m}{n} = \log m - \log n$

Example:

$\log \frac{3}{5} = \log 3 - \log 5$

$\log 6 - \log 3 = \log \frac{6}{3} = \log 2$

(iii) $\log_a m^n = n \log_a m$

or $\log m^n = n \log m$

Example:

$\log 2^3 = 3 \log 2$

$\log_a m \log_m n = \log_a n$

$\log_2 3 \log_3 5 = \log_3 5$

$\log_m n = \frac{\log_a n}{\log_a m}$

Example:

(iv) $\frac{\log_7 r}{\log_7 t} = \log_t r$

Note:

(i) $\log_a a = 1$

(ii) $\log_{10} 10 = 1$

(iii) $\log 10 = 1$

(iv) $\log_{10} 1 = 0$

(v) $\log 1 = 0$

(vi) $\log_m n = \frac{\log_a n}{\log_a m}$

This is called Change of Base Law

Ex # 3.5

Proof of Laws of Logarithm one by one

(i) $\log_a mn = \log_a m + \log_a n$

Proof:

$Let \ \log_a m = x \ and \ \log_a n = y$

Write them in Exponential form:

$a^x = m \ and \ a^y = n$

Now multiply these:

$a^x \times a^y = mn$

Or $mn = a^x \times a^y$

$mn = a^{x+y}$

Taking $\log_a$ on B.S

$\log_a mn = \log_a a^{x+y}$

$\log_a mn = (x+y)\log_a a$

$\log_a mn = (x+y)(1) \qquad \therefore \log_a a = 1$

$\log_a mn = x + y$

$\log_a mn = \log_a m + \log_a n$

(ii) $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof:

$Let \ \log_a m = x \ and \ \log_a n = y$

Write them in Exponential form:

$a^x = m \ and \ a^y = n$

Now Divide these:

$\frac{a^x}{a^y} = \frac{m}{n}$

Or

$\frac{m}{n} = \frac{a^x}{a^y}$

$\frac{m}{n} = a^{x-y}$

Taking $\log_a$ on B.S

$\log_a \frac{m}{n} = \log_a a^{x-y}$

$\log_a \frac{m}{n} = (x-y)\log_a a$

$\log_a \frac{m}{n} = (x-y)(1) \qquad \therefore \log_a a = 1$

$\log_a \frac{m}{n} = x - y$

$Hence \ \log_a \frac{m}{n} = \log_a m - \log_a n$

Ex # 3.5

(iii) $\log_a m^n = n\log_a m$

Proof:

$Let \ \log_a m = x$

In Exponential form:

$a^x = m$

Or

$m = a^x$

Taking power 'n' on B.S

$m^n = (a^x)^n$

$m^n = a^{nx}$

Taking $\log_a$ on B.S

$\log_a m^n = \log_a a^{nx}$

$\log_a m^n = nx\log_a a$

$\log_a m^n = nx(1) \qquad \therefore \log_a a = 1$

$\log_a m^n = nx$

$\log_a m^n = n\log_a m$

(iv) $\log_a m \log_m n = \log_a n$

Proof:

$Let \ \log_a m = x \ and \ \log_m n = y$

Write them in Exponential form:

$a^x = m \ and \ m^y = n$

Now multiply these:

$As \ a^{xy} = (a^x)^y$

$But \ (a^x)^y = m$

$So \ a^{xy} = (m)^y = n$

$So \ a^{xy} = n$

Taking $\log_a$ on B.S

$\log_a a^{xy} = \log_a n$

$(xy)\log_a a = \log_a n$

$xy(1) = \log_a n \qquad \therefore \log_a a = 1$

Now

$\log_a m \log_m n = \log_a n$

Example # 14 page # 90

$-1 + \log y$

Solution:

$= -1 + \log y$

$= -\log 10 + \log y$

$= \log 10^{-1} + \log y$

$= \log \frac{1}{10} + \log y$

$= \log 0.1 + \log y$

$= \log 0.1\, y$

Ex # 3.5

Page # 91

Q1: Use logarithm properties to simplify the expression.

(i) $\log_7 \sqrt{7}$

Solution:

$\log_7 \sqrt{7}$

$Let\ x = \log_7 \sqrt{7}$

$x = \log_7 (7)^{\frac{1}{2}}$

$\boldsymbol{As\ \log_a m^n = n \log_a m}$

$x = \frac{1}{2} \log_7 7$

$x = \frac{1}{2}(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = \frac{1}{2}$

(ii) $\boldsymbol{\log_8 \frac{1}{2}}$ **Trick**

Solution:

$\log_8 \frac{1}{2}$

$\log_8 \frac{1}{2}$

$Let\ \log_8 \frac{1}{2} = x$

In exponential form:

$8^x = \frac{1}{2}$

$(2^3)^x = 2^{-1}$

$2^{3x} = 2^{-1}$

Now

$3x = -1$

Divide B.S by 3, we get

$x = \frac{-1}{3}$

(iii) $\boldsymbol{\log_{10} \sqrt{1000}}$

Solution:

$\log_{10} \sqrt{1000}$

$Let\ x = \log_{10} (10^3)^{\frac{1}{2}}$

$x = \log_{10} (10)^{\frac{3}{2}}$

$\boldsymbol{As\ \log_a m^n = n \log_a m}$

$x = \frac{3}{2} \log_{10} 10$

$x = \frac{3}{2}(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = \frac{3}{2}$

(iv) $\boldsymbol{\log_9 3 + \log_9 27}$

Solution:

$\log_9 3 + \log_9 27$

$Let\ x = \log_9 3 + \log_9 27$

$\boldsymbol{As\ \log_a mn = \log_a m + \log_a n}$

$x = \log_9 3 \times 27$

$x = \log_9 81$

$x = \log_9 9^2$

$\boldsymbol{As\ \log_a m^n = n \log_a m}$

$x = 2 \log_9 9$

$x = 2(1) \qquad \therefore \boldsymbol{\log_a a = 1}$

$x = 2$

(v) $\boldsymbol{\log \frac{1}{(0.0035)^{-4}}}$

Solution:

$\log \frac{1}{(0.0035)^{-4}}$

$Let\ x = \log \frac{1}{(0.0035)^{-4}}$

$\boldsymbol{As\ \log_a \frac{m}{n} = \log_a m - \log_a n}$

$x = \log 1 - \log(0.0035)^{-4}$

$\boldsymbol{As\ \log 1 = 0\ and\ \log_a m^n = n \log_a m}$

Thus

$x = 0 - (-4) \log 0.0035$

Here $Ch = -3$

And $M = .5441$

So

$x = 4(-3 + .5441)$

$x = 4(-2.4559)$

$x = -9.8236$

$\boldsymbol{R.W}$
3.5×10^{-3}

Ex # 3.5

(vi) $\log 45$

Solution:

$\log 45$

$Let\ x = \log 45$

$x = \log 3 \times 3 \times 5$

$x = \log 3^2 \times 5$

$\log_a mn = \log_a m + \log_a n$

$\text{and } \log_a m^n = n \log_a m$

$x = 2\log 3 + \log 5$

$x = 2\log 3.00 + \log 5.00$

$x = 2(0 + .4771) + (0 + .6990)$

$x = 2(0.4771) + (0.6990)$

$x = 0.9542 + 0.6990$

$x = 1.6532$

Q2: Express each of the following as a single logarithm.

(i) $3\log 2 - 4\log 3$

Solution:

$3\log 2 - 4\log 3$

$As\ \log_a m^n = n\log_a m$

$3\log 2 - 4\log 3 = \log 2^2 - \log 3^4$

$3\log 2 - 4\log 3 = \log 8 - \log 81$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$3\log 2 - 4\log 3 = \log \frac{8}{81}$

(ii) $2\log 3 + 4\log 2 - 3$

Solution:

$2\log 3 + 4\log 2 - 3$

$As\ \log_a m^n = n\log_a m$

$2\log 3 + 4\log 2 - 3 = \log 3^2 + \log 2^4 - 3(1)$

$As\ \log 10 = 1$

So

$2\log 3 + 4\log 2 - 3 = \log 9 + \log 16 - 3(\log 10)$

$As\ \log_a mn = \log_a m + \log_a n$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 10^3$

$2\log 3 + 4\log 2 - 3 = \log 9 \times 16 - \log 1000$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$2\log 3 + 4\log 2 - 3 = \log \frac{144}{1000}$

$2\log 3 + 4\log 2 - 3 = \log 0.144$

(iii) $\log 5 - 1$

Solution:

$\log 5 - 1$

$As\ \log 10 = 1$

$\log 5 - 1 = \log 5 - \log 10$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 5 - 1 = \log \frac{5}{10}$

$\log 5 - 1 = \log 0.5$

(iv) $\frac{1}{2}\log x - 2\log 3y + 3\log z$

Solution:

$\frac{1}{2}\log x - 2\log 3y + 3\log z$

$As\ \log_a m^n = n\log_a m$

$= \log x^{\frac{1}{2}} - \log(3y)^2 + \log z^3$

$= \log \sqrt{x} - \log 9y^2 + \log z^3$

$As\ \log_a \frac{m}{n} = \log_a m - \log_a n$

$And\ \log_a mn = \log_a m + \log_a n$

$\frac{1}{2}\log x - 2\log 3y + 3\log z = \log \frac{\sqrt{x}z^3}{9y^2}$

Q3: Find the value of $'a'$ from the following equations.

(i) $\log_2 6 + \log_2 7 = \log_2 a$

Solution:

$\log_2 6 + \log_2 7 = \log_2 a$

$As\ \log_a mn = \log_a m + \log_a n$

$\log_2 6 \times 7 = \log_2 a$

$\log_2 42 = \log_2 a$

Thus

$a = 42$

(ii) $\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

Solution:

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 5 + \log_{\sqrt{3}} 8 - \log_{\sqrt{3}} 2$

As $\log_a mn = \log_a m + \log_a n$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{5 \times 8}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} \frac{40}{2}$

$\log_{\sqrt{3}} a = \log_{\sqrt{3}} 20$

Thus

$a = 20$

(iii) $\frac{\log_7 r}{\log_7 t} = \log_a r$

Solution:

$\frac{\log_7 r}{\log_7 t} = \log_a r$

As $\log_m n = \frac{\log_a n}{\log_a m}$

$\log_t r = \log_a r$

Thus

$a = t$

(iv) $\log_6 25 - \log_6 5 = \log_6 a$

Solution:

$\log_6 25 - \log_6 5 = \log_6 a$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log_6 \frac{25}{5} = \log_6 a$

$\log_6 5 = \log_6 a$

Thus

$a = 5$

Q4: **Find $\log_2 3 \,.\, \log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$**

Solution:

Let $x = \log_2 3 \,.\, \log_3 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

As $\log_a m^n = n \log_a m$

So

$x = \log_2 4 \,.\, \log_4 5 \,.\, \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

$x = \log_2 5 \;.\; \log_5 6 \,.\, \log_6 7 \,.\, \log_7 8$

$x = \log_2 6 \;.\log_6 7 \,.\, \log_7 8$

$x = \log_2 7 \;.\log_7 8$

$x = \log_2 8$

$x = \log_2 2^3$

$x = 3 \log_2 2$

As $\log_a a = 1$

$x = 3(1)$

$x = 3$

Ex # 3.6

Page # 93

Q1: **Simplify 3.81×43.4 with the help of logarithm.**

Solution:

(i) 3.81×43.4

Let $x = 3.81 \times 43.4$

Taking log on B.S

$\log x = \log 3.81 \times 43.4$

As $\log mn = \log m + \log n$

$\log x = \log 3.81 + \log 43.4$

$\log x = (0 + .5809) + (1 + .6375)$

$\log x = 0.5809 + 1.6375$

$\log x = 2.2184$

Taking anti $-$ log on B.S

Anti $-$ log $(\log x)$ = Anti $-$ log 2.2184

x = Anti $-$ log 2.2184

Here

Characteristics = 2

Mantissa = .2184

So

$x = 1.654 \times 10^2$

$x = 16.54$

log 3.81 $Ch = 0$ $M = .5809$
log 43.4 $Ch = 1$ $M = .6375$

$1652 + 2$ $= 1654$

Ex # 3.6

(ii) $\mathbf{73.42 \times 0.00462 \times 0.5143}$

Solution:

$73.42 \times 0.00462 \times 0.5143$

Let $x = 73.42 \times 0.00462 \times 0.5143$

Taking log on B.S

$\log x = 73.42 \times 0.00462 \times 0.5143$

As $\log mn = \log m + \log n$

$\log x = \log 73.42 + \log 0.00462 + \log 0.5143$

$\log x = (1 + .8658) + (-3 + .6646) + (-1 + .7113)$

$\log x = 1.8658 + (-2.3354) + (-0.2887)$

$\log x = 1.8658 - 2.3354 - 0.2887$

$\log x = -0.7583$

Add and Subtract -1

$\log x = -1 + 1 - 0.7583$

$\log x = -1 + .2417$

$\log x = \bar{1}.2417$

Taking anti $-$ log on B. S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \bar{1}.2417$

$x = \text{anti} - \log \bar{1}.2417$

Here

Characteristics $= -1$

Mantissa $= .2417$

So

$x = 1.745 \times 10^{-1}$

$x = 0.1745$

$\log 73.42$ $Ch = 1$ $8657 + 1$ $M = .8658$
$\log 0.00462$ $Ch = -3$ $M = .6646$
$\log 0.5143$ $Ch = -1$ $7110 + 3$ $M = .7113$
$1742 + 3$ $= 1745$

(iii) $\dfrac{\mathbf{784.6 \times 0.0431}}{\mathbf{28.23}}$

Solution:

$\dfrac{784.6 \times 0.0431}{28.23}$

$Let\ x = \dfrac{784.6 \times 0.0431}{28.23}$

Taking log on B.S

$\log x = \log \dfrac{784.6 \times 0.0431}{28.23}$

$\boldsymbol{As\ \log \dfrac{m}{n} = \log m - \log n}$

$\log x = \log 784.6 \times 0.0431 - \log 28.23$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 784.6 + \log 0.0431 - \log 28.23$

Ex # 3.6

$\log x = (2 + .8946) + (-2 + .6345) + (1 + .4507)$

$\log x = 2.8946 + (-1.3655) + (1.4507)$

$\log x = 2.8946 - 1.3655 - 1.4507$

$\log x = 0\,.0784$

Taking anti $-$ log on B. S

anti $-$ log $(\log x) =$ anti $-$ log $0\,.0784$

$x =$ anti $-$ log $0\,.0784$

Here

Characteristics $= 0$

Mantissa $= .0784$

So

$x = 1.198 \times 10^0$

$x = 1.198$

log 784.6 $Ch = 2$ $8943 + 3$ $M = .8946$
log 0.0431 $Ch = -2$ $M = .6345$
log 28.23 $Ch = 1$ $4502 + 5$ $M = .4507$
$1197 + 1$ $= 1198$

(iv) $\dfrac{\mathbf{0.4932 \times 653.7}}{\mathbf{0.07213 \times 8456}}$

Solution:

$\dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

$Let\ x = \dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

Taking log on B.S

$\log x = \log \dfrac{0.4932 \times 653.7}{0.07213 \times 8456}$

$\boldsymbol{As\ \ \log \dfrac{m}{n} = \log m - \log n}$

$\log x = \log(0.4932 \times 653.7) - \log(0.07213 \times 8456)$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log 0.4932 + \log 653.7 - (\log 0.07213 + \log 8456)$

$\log x = \log 0.4932 + \log 653.7 - \log 0.07213 - \log 8456$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-1 + .6930) + (2 + .8154) - (-2 + .8581) - (3 + .9271)$

$\log x = (-0.3070) + (2.8154) - (-1.1419) - (3.9271)$

$\log x = -0.3070 + 2.8154 + 1.1419 - 3.9271$

$\log x = -0.2768$

log 0.4932 $Ch = -1$ $6928 + 2$ $M = .6930$
log 653.7 $Ch = 2$ $8149 + 5$ $M = .8154$
log 0.07213 $Ch = -2$ $8579 + 2$ $M = .8581$
log 8456 $Ch = 3$ $9269 + 3$ $M = .9272$

Ex # 3.6

Add and Subtract -1

$\log x = -1 + 1 - 0.2768$

$\log x = -1 + .7232$

$\log x = \overline{1}\,.7232$

Taking anti $-$ log on B. S

$\text{anti} - \log\ (\log x) = \text{anti} - \log \overline{1}\,.7232$

$x = \text{anti} - \log \overline{1}\,..7232$

Here

Characteristics $= -1$

Mantissa $= .7232$

So

$x = 5.286 \times 10^{-1}$

$x = 0.5286$

$5284 + 2$ $= 5286$

(v) $\dfrac{(\mathbf{78.41})^{\mathbf{3}}\sqrt{\mathbf{142.3}}}{\sqrt[\mathbf{4}]{\mathbf{0.1562}}}$

Solution:

$\dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$Let\ x = \dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

Taking log on B.S

$\log x = \log\dfrac{(78.41)^3\sqrt{142.3}}{\sqrt[4]{0.1562}}$

$\boldsymbol{As\ \ \log\dfrac{m}{n} = \log m - \log n}$

$\log x = \log(78.41)^3\sqrt{142.3} - \log\sqrt[4]{0.1562}$

As $\boldsymbol{\log mn = \log m + \log n}$

$\log x = \log(78.41)^3 + \log\sqrt{142.3} - \log\sqrt[4]{0.1562}$

$\log x = \log(78.41)^3 + \log(142.3)^{\frac{1}{2}} - \log(0.1562)^{\frac{1}{4}}$

$\log x = 3\log 78.41 + \dfrac{1}{2}\log 142.3 - \dfrac{1}{4}\log 0.1562$

$\log x = 3\log(78.41) + \dfrac{1}{2}\log(142.3) - \dfrac{1}{4}\log(0.1562)$

$\log x = 3\,(1 + .8944) + \dfrac{1}{2}(\,2 + .1532) - \dfrac{1}{4}(-1 + .1937)$

$\log 78.41$ $Ch = 1$ $8943 + 1$ $M = .8944$
$\log 142.3$ $Ch = 2$ $1523 + 9$ $M = .1523$
$\log 0.1562$ $Ch = -1$ $1931 + 6$ $M = .1937$

$$\log x = 3\,(1.8944) + \frac{1}{2}(\,2.1532) - \frac{1}{4}(-0.8063)$$

Ex # 3.6

$\log x = 5.6832 + 1.0766 + 0.2016$

$\log x = 6.9614$

Taking anti $-$ log on B. S

anti $-$ log $(\log x)$ = anti $-$ log 6.9614

x = anti $-$ log 6.9614

Here

Characteristics = 6

Mantissa = .9614

9141 + 8 = 9149

So

$x = 9.149 \times 10^6$

$x = 9149000$

Q2: Find the following if $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, $\log 7 = 0.8451$

(i) $\log 105$

Solution:

$\log 105$

$\log 105 = \log 3 \times 5 \times 7$

As $\log mn = \log m + \log n$

$\log 105 = \log 3 + \log 5 + \log 7$

$\log 105 = 0.4771 + 0.6990 + 0.8451$

$\log 105 = 2.0211$

(ii) $\log 108$

$\log 108$

$\log 108 = \log 2 \times 2 \times 3 \times 3 \times 3$

$\log 108 = \log 2^2 \times 3^3$

As $\log mn = \log m + \log n$

$\log 108 = \log 2^2 + \log 3^3$

As $\log_a m^n = n \log_a m$

$\log 108 = 2 \log 2 + 3 \log 3$

$\log 108 = 2(0.3010) + 3(0.4771)$

$\log 108 = 0.6020 + 1.4313$

$\log 108 = 2.0333$

(iii) $\log \sqrt[3]{72}$

Solution:

$\log \sqrt[3]{72}$

$\log \sqrt[3]{72} = \log(72)^{\frac{1}{3}}$

Review Ex # 3

As $\log_a m^n = n \log_a m$

$\log \sqrt[3]{72} = \frac{1}{3} \log 72$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2 \times 2 \times 2 \times 3 \times 3)$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3 \times 3^2)$

As $\log mn = \log m + \log n$

$log \sqrt[3]{72} = \frac{1}{3}(log\, 2^3 + log\, 3^2)$

$log \sqrt[3]{72} = \frac{1}{3}(3\, log\, 2 + 2\, log\, 3)$

$log \sqrt[3]{72} = \frac{1}{3}[3(0.3010) + 2(0.4771)]$

$log \sqrt[3]{72} = \frac{1}{3}[0.9030 + 0.9542]$

$log \sqrt[3]{72} = \frac{1}{3}[1.8572]$

$log \sqrt[3]{72} = 0.6191$

(iv) $\log 2.4$

Solution:

$\log 2.4$

$\log 2.4 = \log \frac{24}{10}$

As $\log_a \frac{m}{n} = \log_a m - \log_a n$

$\log 2.4 = \log 24 - \log 10$

$\log 2.4 = \log 2 \times 2 \times 2 \times 3 - \log 10$

$\log 2.4 = \log 2^3 \times 3 - \log 10$

As $\log mn = \log m + \log n$

$\log 2.4 = \log 2^3 + \log 3 - \log 10$

As $\log_a m^n = n \log_a m$

$\log 2.4 = 3 \log 2 + \log 3 - \log 10$

$\log 2.4 = 3(0.3010) + 0.4771 - \log 10$

$\log 2.4 = 0.9030 + 0.4771 - 1 \therefore \log 10 = 1$

$\log 2.4 = 1.3801 - 1$

$\log 2.4 = 0.3801$

Ex # 3.6

(v) $\log 0.0081$

Solution:

$\log 0.0081$

$$\log 0.0081 = \log \frac{81}{10000}$$

$$\log 0.0081 = \log \frac{3^4}{10^4}$$

$$\log 0.0081 = \log \left(\frac{3}{10}\right)^4$$

$$As \quad \log_a m^n = n \log_a m$$

$$\log 0.0081 = 4 \log \frac{3}{10}$$

$$As \quad \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$\log 0.0081 = 4(\log 3 - \log 10)$$

$$\log 0.0081 = 4(0.4771 - 1) \quad \therefore \log 10 = 1$$

$$\log 0.0081 = 4(-0.5229)$$

$$\log 0.0081 = -2.0916$$

REVIEW EXERCISE # 3

Page # 95

Q2: **Write 9473.2 in scientific notation.**

9473.2

In scientific notation:

$$9.4732 \times 10^3$$

Q3: **Write 5.4×10^6 in standard notation.**

$$5.4 \times 10^6$$

In standard form:

5400000

Q4: **Write in logarithm form: $3^{-3} = \frac{1}{27}$**

$$3^{-3} = \frac{1}{27}$$

In logarithm form:

$$\log_3 \frac{1}{27} = -3$$

Review Ex # 3

Q5: **Write in exponential form: $\log_5 1 = 0$**

$$\log_5 1 = 0$$

In exponential form:

$$5^0 = 1$$

Q6: **Solve for x: $\log_4 16 = x$**

$$\log_4 16 = x$$

In exponential form:

$$4^x = 16$$

$$4^x = 4^2$$

So

$$x = 2$$

Q7: **Find the characteristic of the common logarithm 0.0083.**

0.0083

In scientific notation:

$$8.3 \times 10^{-3}$$

So Characteristics -3

Q8: **Find $\log 12.4$**

$$\log 12.4$$

In Scientific form:

$$1.24 \times 10^1$$

Thus Characteristics $= 1$

To find Mantissa, using Log Table:

Mantissa $= .0934$

Hence $\log 12.4 = 0.0934$

Q9: **Find the value of $'a'$,**

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

Solution:

$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 9 + \log_{\sqrt{5}} 2 - \log_{\sqrt{5}} 3$

$$As \quad \log_a mn = \log_a m + \log_a n$$

$$As \quad \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} \frac{9 \times 2}{3}$$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 3 \times 2$$

$$\log_{\sqrt{5}} 3a = \log_{\sqrt{5}} 6$$

Thus $3a = 6$

$$a = \frac{6}{2}$$

$$a = 3$$

Q10 $\dfrac{(\mathbf{63.28})^{\mathbf{3}}(\mathbf{0.00843})^{\mathbf{2}}(\mathbf{0.4623})}{(\mathbf{412.3})(\mathbf{2.184})^{\mathbf{5}}}$

Solution:

$$\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$Let\ x = \frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

Taking log on B.S

$$\log x = \log\frac{(63.28)^3(0.00843)^2(0.4623)}{(412.3)(2.184)^5}$$

$$As\ \ \log\frac{m}{n} = \log m - \log n$$

$$\log x = \log((63.28)^3(0.00843)^2(0.4623)) - \log((412.3)(2.184)^5)$$

As $\log mn = \log m + \log n$

$$\log x = \log(63.28)^3 + \log(0.00843)^2 + \log 0.4623 - (\log 412.3 + \log(2.184)^5)$$

$$\log x = 3\log 63.28 + 2\log 0.00843 + \log 0.4623 - (\log 412.3 + 5\log 2.184)$$

$$\log x = 3\log 63.28 + 2\log 0.00843 + \log 0.4623 - \log 412.3 - 5\log 2.184$$

$$\log x = 3(1 + .8012) + 2(-3 + .9258) + (-1 + .6649) - (2 + .6152) - 5(0 + .3393)$$

$$\log x = 3(1.8012) + 2(-2.0742) + (-0.3351) - (2.6152) - 5(0.3393)$$

$$\log x = 5.4036 - 4.1484 - 0.3351 - 2.6152 - 1.6965$$

$$\log x = -3.3916$$

Add and Subtract -4

$$\log x = -4 + 4 - 3.3916$$

$$\log x = -4 + .6084$$

$$\log x = \overline{4}.6084$$

Taking $anti - \log$ on B. S

$$anti - \log\ (\log x) = anti - \log \overline{4}.6084$$

$$x = anti - \log \overline{4}.6084$$

Here

Characteristics $= -4$

Mantissa $= .6084$

So

$$x = 4.059 \times 10^{-4}$$

$$x = 0.000405$$

$\log 63.28$ $Ch = 1$ $8007 + 5$ $M = .8012$
$\log 0.00843$ $Ch = -3$ $M = .9258$
$\log 0.4623$ $Ch = -1$ $6646 + 3$ $M = .6649$
$\log 412.3$ $Ch = 2$ $6149 + 3$ $M = .6152$
$\log 2.184$ $Ch = 0$ $3385 + 8$ $M = .3393$

$4055 + 4$ $= 4059$