Chapter # 2

Ex # 2.1

Set of Natural Numbers

$$N = \{1, 2, 3, 4, \dots\}$$

Set of Whole Numbers

$$W = \{0, 1, 2, 3, 4, \dots\}$$

Set of Integers

$$Z = \{0, \pm1, \pm2, \pm3, \dots\}$$

OR

$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3\}$$

Rational Numbers

The word Rational means "Ratio".

A rational number is a number that can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Rational numbers is denoted by Q.

Set of Rational Numbers

$$Q = \left\{\frac{p}{q} \mid p, q \in Z, q \neq 0\right\}$$

Irrational Numbers

The word Irrational means "Not Ratio".

Irrational number consists of all those numbers which are not rational. Irrational numbers is denoted by Q'.

Real numbers

The set of rational and irrational numbers is called Real Numbers. Real numbers is denoted by R.

Thus $Q \cup Q' = R$

Note:

All the numbers on the number line are real numbers.

Terminating Decimal Fraction:

A decimal number that contains a finite number of digits after the decimal point.

Non-Terminating Decimal Fraction:

A decimal number that has no end after the decimal point.

Non-Terminating Repeating Decimal Fraction

In non-terminating decimal fraction, some digits are repeated in same order after decimal point.

Non-Terminating Non-Repeating Decimal Fraction.

In non-terminating decimal fraction, the digits are not repeated in same order after decimal point.

Decimal Representation of Rational and Irrational Numbers.

(i) All terminating and repeating decimals are rational numbers.

(ii) Non-terminating recurring (repeating) decimals are rational numbers.

(iii) Never terminating or repeating decimals are irrational numbers.

Non-terminating and non-recurring (repeating) decimals are irrational numbers.

Note:

(i) Repeating decimals are called recurring decimals.

(ii) Non-repeating decimals are called non-recurring decimals.

Chapter # 2

Ex # 2.1
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In Questions 1 – 10, consider the numbers.

$$\mathbf{2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}, 0.333 \ldots}$$

1. Which are whole numbers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad 1$

2. Which are integers?

Ans: $3, \quad 0, \quad \sqrt{36}, \quad -9, \quad 1$

3. Which are irrational numbers?

Ans: $\sqrt{3}, \ \sqrt{7}, -\sqrt{14}, \quad \pi$

4. Which are natural numbers?

Ans: $3, \quad \sqrt{36}, \quad 1$

5. Which are rational numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, -9, 1, \ 4\frac{2}{3},$ $0.333 \ldots$

6. Which are real numbers?

Ans: $2.5, 3, \frac{5}{7}, -1.96, 0, \sqrt{36}, -\frac{7}{6}, \sqrt{3}, -9, 1, \sqrt{7}, -\sqrt{14}, \pi, 4\frac{2}{3}$ $, 0.333.$

7. Which are rational numbers but not integers?

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad 4\frac{2}{3}, \quad 0.333 \ldots$

8. Which are integers but not whole numbers?

Ans: -9

9. Which are integers but not natural numbers?

Ans: $0, -9$

10. Which are real numbers but not integers?

Ans: $2.5, \quad \frac{5}{7}, -1.96, \quad -\frac{7}{6}, \quad \sqrt{3}, \quad \sqrt{7}, -\sqrt{14}, \quad \pi,$ $4\frac{2}{3}, \quad 0.333 \ldots$

11. Write the decimal representation of each of the following numbers.

$\frac{1}{6}, \frac{6}{7}, \frac{2}{9}, \frac{1}{8}$

$\frac{1}{6} = 0.1666 \ldots.$

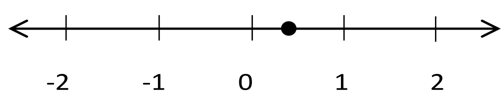
$\frac{6}{7} = 0.8571 \ldots.$

$\frac{2}{9} = 0.222 \ldots.$

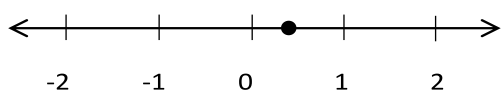
$\frac{1}{8} = 0.125$

12 Depict each number on a number line.

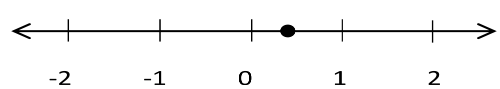
(i) $\mathbf{\frac{1}{3} = 0.333 \ldots}$



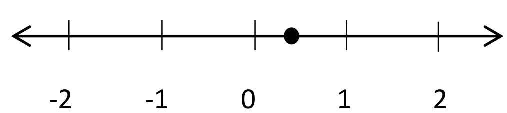
(ii) $\mathbf{\frac{1}{4} = 0.25}$



(ii) $\frac{1}{9} = 0.111 \ldots$



(iv) $\frac{1}{10} = 0.1$



Ex # 2.2

Properties of Real Number

The set R of real number is the union of two disjoint sets. Thus $R = Q \cup Q^{/}$

Note:

$$Q \cap Q^{/} = \emptyset$$

Real Number System

Closure Property w.r.t Addition

The sum of real number is also a real number.

If $a,\ b\ \in R$ then $a + b\ \in R$

Example:

$$7 + 9 = 16$$

Where 16 is a real number.

Closure Property w.r.t Multiplication

The Product of real number is also a real number.

If $a,\ b\ \in R$ then $a\,.\,b\ \in R$

Example:

$$7 \times 9 = 63$$

Where 63 is a real number.

Commutative Property w.r.t Addition

If $a,\ b\ \in R$ then $a + b = b + a$

Example:

$$7 + 9 = 9 + 7$$
$$16 = 16$$

Commutative Property w.r.t Multiplication

If $a,\ b\ \in R$ then $a\ .\ b = b\ .\ a$

Example:

$$7 \times 9 = 9 \times 7$$
$$63 = 63$$

Associative Property w.r.t Addition

If $a,\ b,\ c\ \in R$ then

$$a + (b + c) = (a + b) + c$$

Example:

$$2 + (3 + 5) = (2 + 3) + 5$$
$$2 + 8 = 5 + 5$$
$$10 = 10$$

Associative Property w.r.t Multiplication

If $a,\ b,\ c\ \in R$ then

$$a(bc) = (ab)c$$

Example:

$$2(3 \times 5) = (2 \times 3)5$$
$$2(15) = (6)5$$
$$30 = 30$$

Additive Identity

Zero (0) is called Additive identity because adding "0" to a number does not change that number.

If $a \in R$ there exists $O\ \in R$ then

$$a + 0 = 0 + a = a$$

Example:

$$3 + 0 = 0 + 3 = 3$$

Multiplicative Identity

1 is called Multiplicative identity because multiplying "1" to a number does not change that number.

If $a \in R$ there exists $1\ \in R$ then

$$a\,.\,1 = 1\,.\,a = a$$

Example:

$$3 \times 1 = 1 \times 3 = 3$$

Additive Inverse

When the sum of two numbers is zero (0)

If $a \in R$ there exists an element $a^{/}$ then $a + a^{/} = a^{/} + a = 0$ then $a^{/}$ is called additive inverse of a

Or

$$a + (-a) = -a + a = 0$$

Example:

$$3 + (-3) = 3 - 3 = 0$$
$$-3 + 3 = 0$$

Chapter # 2

Ex # 2.2

Multiplicative Inverse

When the Product of two numbers is "1".

If $a \in R$ and $a \neq 0$ there exists an element $a^{-1} \in R$ then

$a\,.\,a^{-1} = a^{-1}\,.\,a = 1$ then a^{-1} is called multiplicative inverse of a

Or

$$a\,.\,\frac{1}{a} = \frac{1}{a}.a = 1$$

Example:

$$3 \times \frac{1}{3} = \frac{1}{3} \times 3 = 1$$

Distributive Property of Multiplication over Addition

If $a,\ b,\ c\ \in R$ then

$$a(b+c) = ab + ac$$
$$(b+c)a = ba + ca$$

Example:

$$2(3+5) = 2 \times 3 + 2 \times 5$$
$$2(8) = 6 + 10$$
$$16 = 16$$

Properties of Equality of Real Numbers

Reflexive Property of equality

Every number is equal to itself.

$$a = a$$

Example:

$$3 = 3$$

Symmetric Property of Equality

If $a = b$ then also $b = a$

Examples:

$$x = 5$$
$$or\ 5 = x$$
$$x^2 = y$$
$$or\ y = x^2$$

Transitive Property of Equality

If $a = b$ and $b = c$ then $a = c$

Example:

$if\ x + y = z$ and $z = a + b$

Then $x + y = a + b$

Ex # 2.2

Additive Property of Equality

If $a = b$ then also $a + c = b + c$

Examples:

$$x - 3 = 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

$$x + 3 = 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property of Equality

If $a = b$ then also $a.\ c = b\ .\ c$

Or

$a = b$ then $\frac{a}{c} = \frac{b}{c}$

Examples:

$$\frac{x}{3} = 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

$$2x = 24$$

Divide B.S by 2

$$\frac{2x}{2} = \frac{24}{2}$$
$$x = 12$$

Cancellation Property w.r.t Addition

If $a + c = b + c$ then $a = b$

Examples:

$$2x + 5 = y + 5$$
$$2x = y$$
$$2x - 5 = y - 5$$
$$2x = y$$

Ex # 2.2

Cancellation Property w.r.t Multiplication

If $a\,.\,c = b\,.\,c$ then $a = b$

OR

If $\frac{a}{c} = \frac{b}{c}$ then $a = b$

Examples:

$$2x \times 5 = y \times 5$$
$$2x = y$$
$$\frac{2x}{5} = \frac{y}{5}$$
$$2x = y$$

Properties of Inequality of Real Numbers

Trichotomy Property

Trichotomy property means when comparing two numbers, one of the following must be true:

$$a = b$$
$$a < b$$
$$a > b$$

Examples:

$$5 = 5$$
$$3 < 5$$
$$3 > 5$$

Transitive Property

(i) **If $a > b$ and $b > c$ then $a > c$**

Example:

If $7 > 5$ and $5 > 3$ then $\mathbf{7 > 3}$

(ii) **If $a < b$ and $b < c$ then $a < c$**

Example:

If $3 < 5$ and $5 < 7$ then $\mathbf{3 < 7}$

Additive Property

(i) **If $a < b$ then $a + c < b + c$**

Example:

$3 < 5$ then $3 + 2 < 5 + 2$

$$x - 3 > 5$$

Add 3 on B.S

$$x - 3 + 3 = 5 + 3$$
$$x = 8$$

Ex # 2.2

(ii) **If $a > b$ then $a + c > b + c$**

Example:

(a) $5 > 3$ then $5 - 2 > 3 - 2$

(b) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(c) $$x + 3 > 5$$

Subtract 3 from B.S

$$x + 3 - 3 = 5 - 3$$
$$x = 2$$

Multiplicative Property

When $c > 0$:

(i) If $a < b$ then $ac < bc$

(ii) If $a > b$ then $ac > bc$

Example:

(a) $5 > 3$ then $5 \times 2 > 3 \times 2$

(b) $$\frac{x}{3} > 5$$

Multiply B.S by 3

$$\frac{x}{3} \times 3 > 5 \times 3$$
$$x > 15$$

$$2x > 24$$

Divide B.S by 2

$$\frac{2x}{2} > \frac{24}{2}$$
$$x > 12$$

When $c < 0$:

(i) If $a < b$ then $ac > bc$

(ii) If $a > b$ then $ac < bc$

Example:

(a) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

(b) $$\frac{x}{-3} < 5$$

Multiply B.S by -3

$$\frac{x}{-3} \times -3 > 5 \times -3$$
$$x > -15$$

Example: 4

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Solve the following equation using properties of real numbers.

$$2x - 5 = 3x + 4$$

Solution:

$2x - 5 = 3x + 4$	
$2x - 5 + 5 = 3x + 4 + 5$	$\therefore$ $a = b$ *then* $a + c = b + c$
$2x - 5 + 5 = 3x + 9$	$\therefore$ *Closure Property w.r.t Additon*
$2x + 0 = 3x + 9$	$\therefore$ -5 & 5 *are additive inverse*
$2x = 3x + 9$	$\therefore$ 0 *is the additive identity*
$3x + 9 = 2x$	$\therefore$ *Symmetric Property*
$3x + 9 - 2x = 2x - 2x$	$\therefore$ $a = b$ *then* $a - c = b - c$
$3x - 2x + 9 = 0$	$\therefore$ $2x$ & $-2x$ *are additive inverse*
$(3 - 2)x + 9 = 0$	$\therefore$ *Distributive Property*
$1.x + 9 = 0$	
$x + 9 = 0$	$\therefore$ 1 *is Multiplicative Identity*
$x + 9 - 9 = 0 - 9$	$\therefore$ $a = b$ *then* $a - c = b - c$
$x + 9 - 9 = -9$	$\therefore$ 0 *is the Additive Identity*
$x + 0 = -9$	$\therefore$ 9 & -9 *are additive inverse*
$x = -9$	$\therefore$ 0 *is the Additive Identity*

Ex # 2.2

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Q1: Name the properties used in following equations.

(i) $\mathbf{1 + (4 + 3) = (1 + 4) + 3}$

Ans: Associative law of addition

(ii) $\mathbf{5(a + b) = 5a + 5b}$

Ans: Distributive law of multiplication over addition

(iii) $\mathbf{a + 0 = 0 + a = a}$

Ans: Additive identity

(iv) $\mathbf{5 \times \frac{1}{5} = \frac{1}{5} \times 5 = 1}$

Ans: Multiplicative inverse

Q2: Write the missing number.

(i) $\mathbf{2 + (\_\_\_ + 4) = (2 + 6) + 4}$

Answer: 6

(ii) $\mathbf{7 + (4 + 2) = 13}$**, so** $\mathbf{(7 + 4) + 2 = \_\_\_\_}$

Answer: 13

(iii) $\mathbf{9 \times (3 \times 4) = 108}$**, so** $\mathbf{(9 \times 3) \times 4 = \_\_\_\_\_}$

Answer: 108

(iv) $\mathbf{5 \times (8 \times 9) = (5 \times \_\_\_\_) \times 9}$

Answer: 8

Q3: Chose the correct option

(i) $\mathbf{8 \times (6 \times 7)}$ **is equal to:**

(a) $8 \times 6 - 7$ **(b)** $8 - (6 - 7)$

(c) 8×12 **(d)** $(8 \times 6) \times 7$

Answer: d. $(8 \times 6) \times 7$

(ii) Which one of the following illustrates the Associative Law of Addition?

(a) $3 + (2 + 4) = (4 + 4) + 1$

(b) $3 + (2 + 4) = (3 + 2) + 4$

(c) $3 + (2 + 4) = (5 + 2) + 2$

(d) $3 + (2 + 4) = (2 + 6) + 1$

Answer: b. $3 + (2 + 4) = (3 + 2) + 4$

Ex # 2.2

(iii) Which one of the following illustrates the Associative Law of Multiplication?

(a) $4 \times (3 \times 6) = (6 \times 6) \times 2$

(b) $4 \times (3 \times 6) = (3 \times 12) \times 2$

(c) $4 \times (3 \times 6) = (4 \times 3) \times 6$

(d) $4 \times (3 \times 6) = (3 \times 8) \times 3$

Answer: c. $4 \times (3 \times 6) = (4 \times 3) \times 6$

Q4: Do this without using distributive property.

(i) $\mathbf{39 \times 63 + 39 \times 37}$

Solution:

$39 \times 63 + 39 \times 37$
$= 2457 + 1443$
$= 3900$

(ii) $\mathbf{81 \times 450 + 81 \times 550}$

Solution:

$81 \times 450 + 81 \times 550$
$= 36450 + 44550$
$= 81000$

(iii) $\mathbf{50 \times 161 - 50 \times 81}$

Solution:

$50 \times 161 - 50 \times 81$
$= 8050 - 4050$
$= 4000$

(iv) $\mathbf{827 \times 60 - 327 \times 60}$

Solution:

$827 \times 60 - 327 \times 60$
$= 49620 - 19620$
$= 30000$

Ex # 2.3

RADICALS AND RADICANDS

$\sqrt[n]{a}$ is the radical form of the nth root of a.

$a^{\frac{1}{n}}$ is the exponential form of the nth root of a.

If $n = 2$ then it becomes square root and write $\sqrt{a}$ instead of $\sqrt[2]{a}$

If $n = 3$ then it is called cube root like $\sqrt[3]{a}$

If $n = 5$ then it is called 5th root like $\sqrt[5]{625}$

Important Notes

(i) If a is positive, then the nth root of a is also positive.

Example:

$$\sqrt[3]{64} = \sqrt[3]{(4)^3} = 4$$

(ii) If a is negative, then n must be odd for the nth root of a to be a real number.

Example:

$$\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$$

(iii) If a is zero, then $\sqrt[n]{0} = 0$

Properties of Radicals:

Product Rule of Radicals:

$\sqrt[n]{ab} = \sqrt[n]{a} . \sqrt[n]{b}$

Example:

$\sqrt{6x}\sqrt{6y^2}$
$\sqrt{(6x)(6y^2)} = \sqrt{36y^2x} = \sqrt{36}\sqrt{y^2}\sqrt{x}$
$= 6y\sqrt{x}$

$\sqrt{6x}\sqrt{6x^2}$
$\sqrt{(6x)(6x^2)} = \sqrt{36x^2x} = \sqrt{36}\sqrt{x^2}\sqrt{x}$
$= 6y\sqrt{x}$

Ex # 2.3

Quotient Rule of Radicals:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Example:

Simplify: $2\sqrt{\dfrac{150xy}{3x}}$

Solution:

$$2\sqrt{\frac{150xy}{3x}} = 2\sqrt{50y} = 2\sqrt{5 \times 5 \times 2y}$$

$$= 2\sqrt{5^2}\sqrt{2y} = 2(5)\sqrt{2y} = 10\sqrt{2y}$$

Radical form of an Expression:

The number or quantity that is written under a radical sign ($\sqrt{\ }$ or $\sqrt[n]{\ }$) is called radical form of an expression.

Example:

$\sqrt{9}$ is the radical form of 3.

Exponential form of an Expression:

The number or quantity that is written in the form of exponent is called exponential form of an expression.

Example:

3^2 is the exponential form of 9.

Radical Form	Exponential Form
$\sqrt[n]{a}$	$a^{\frac{1}{n}}$
$\sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$	$a^{\frac{m}{n}}$
$\sqrt[n]{a^n}$	a

Some frequently used radicals are given in the following table

Square Root	Cube Root	Fourth Root
$\sqrt{1} = 1$	$\sqrt[3]{1} = 1$	$\sqrt[4]{1} = 1$
$\sqrt{4} = 2$	$\sqrt[3]{8} = 2$	$\sqrt[4]{16} = 2$
$\sqrt{9} = 3$	$\sqrt[3]{27} = 3$	$\sqrt[4]{81} = 3$
$\sqrt{16} = 4$	$\sqrt[3]{64} = 4$	$\sqrt[4]{256} = 4$
$\sqrt{25} = 5$	$\sqrt[3]{125} = 5$	$\sqrt[4]{625} = 5$
$\sqrt{36} = 6$	$\sqrt[3]{216} = 6$	$\sqrt[4]{1296} = 6$

Example 5 Page # 61

What is the difference between (i) $x^2 = 16$ (ii) $x = \sqrt{16}$?

(i) $$x^2 = 16$$

Solution:

$$x^2 = 16$$

This means what numbers squared becomes 16. Thus x can be 4 or -4 like $(4)^2 = 16$ and also $(-4)^2 = 16$.

Hence the value of $x = \pm 4$.

(ii) $$x = \sqrt{16}$$

Solution:

$$x = \sqrt{16}$$

Here x is the principal square root of 16, which has always a positive value such is $x = 4$.

Ex # 2.3

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Q1: 64 **Write down the index and radicand for each of the following expressions.**

(i) $\sqrt{\frac{11}{y}}$

$index = 2, radicand = \frac{11}{y}$

(ii) $\sqrt[3]{\frac{13}{3x}}$

$index = 3, radicand = \frac{13}{3x}$

(iii) $\sqrt[5]{ab^2}$

$index = 5, radicand = ab^2$

Q2: 64 **Transform the following radical forms into exponential forms. Do not simplify.**

(i) $\sqrt{36}$

Exponential form= $(36)^{\frac{1}{2}}$

(ii) $\sqrt{1000}$

Exponential form= $(1000)^{\frac{1}{2}}$

(iii) $\sqrt[3]{8}$

Exponential form= $(8)^{\frac{1}{3}}$

(iv) $\sqrt[n]{q}$

Exponential form= $(q)^{\frac{1}{n}}$

(v) $\sqrt{(5-6a^2)^3}$

$((5-6a^2)^3)^{\frac{1}{2}}$

Exponential form= $(5-6a^2)^{\frac{3}{2}}$

(vi) $\sqrt[3]{-64}$

Exponential form= $(-64)^{\frac{1}{3}}$

Ex # 2.3

Q3: 64 **Transform the following exponential form of an expression into radical form.**

(i) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(ii) $x^{-\frac{3}{2}}$

$(x^{-3})^{\frac{1}{2}}$

$\sqrt{x^{-3}}$

(iii) $(-8)^{\frac{1}{5}}$

$\sqrt[5]{-8}$

(iv) $y^{\frac{3}{4}}$

$(y^3)^{\frac{1}{4}}$

$\sqrt[4]{y^3}$

(v) $b^{\frac{4}{5}}$

$(b^4)^{\frac{1}{5}}$

$\sqrt[5]{b^4}$

(vi) $(3x)^{\frac{1}{q}}$

$\sqrt[q]{3x}$

Q4: **Simplify:**

(i) $\sqrt[3]{125x}$

Solution:

$\sqrt[3]{125x}$

$= (125x)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5 \times 5 \times 5)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= (5^3)^{\frac{1}{3}}(x)^{\frac{1}{3}}$

$= 5(x)^{\frac{1}{3}}$

$= 5\sqrt[3]{x}$

Ex # 2.3

(ii) $\sqrt[3]{\frac{8}{27}}$

$$= \left(\frac{8}{27}\right)^{\frac{1}{3}}$$

$$= \left(\frac{2 \times 2 \times 2}{3 \times 3 \times 3}\right)^{\frac{1}{3}}$$

$$= \left(\frac{2^3}{3^3}\right)^{\frac{1}{3}}$$

$$\frac{= (2^3)^{\frac{1}{3}}}{= (3^3)^{\frac{1}{3}}}$$

$$= \frac{2}{3}$$

(iii) $\sqrt{\frac{\mathbf{625x^3y^4}}{\mathbf{25xy^2}}}$

Solution:

$$\sqrt{\frac{625x^3y^4}{25xy^2}}$$

$$= \sqrt{25x^2y^2}$$

$$= (25x^2y^2)^{\frac{1}{2}}$$

$$= (25)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}(y^2)^{\frac{1}{2}}$$

$$= 5xy$$

(iv) $\sqrt{(\mathbf{3y - 5})^2}$

Solution:

$$\sqrt{(3y - 5)^2}$$

$$= [(3y - 5)^2]^{\frac{1}{2}}$$

$$= 3y - 5$$

Ex # 2.3

(v) $\mathbf{6\sqrt{18}}$

Solution:

$$6\sqrt{18}$$

$$= 6(18)^{\frac{1}{2}}$$

$$= 6(3 \times 3 \times 2)^{\frac{1}{2}}$$

$$= 6(3^2 \times 2)^{\frac{1}{2}}$$

$$= 6(3^2)^{\frac{1}{2}}(2)^{\frac{1}{2}}$$

$$= 6(3)\sqrt{2}$$

$$= 18\sqrt{2}$$

(vi) $\sqrt[3]{\mathbf{54x^3y^3z^2}}$

Solution:

$$\sqrt[3]{54x^3y^3z^2}$$

$$= (54x^3y^3z^2)^{\frac{1}{3}}$$

$$= (54)^{\frac{1}{3}}(x^3)^{\frac{1}{3}}(y^3)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$$

$$= (3 \times 3 \times 3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$$

$$= (3^3 \times 2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$$

$$= (3^3)^{\frac{1}{3}}(2)^{\frac{1}{3}}(x)(y)(z^2)^{\frac{1}{3}}$$

$$= (3)(x)(y)(2)^{\frac{1}{3}}(z^2)^{\frac{1}{3}}$$

$$= 3xy(2z^2)^{\frac{1}{3}}$$

$$= 3xy\sqrt[3]{2z^2}$$

Ex # 2.4

Base

جس کے اوپر power ہو اسے Base کہتے ہیں۔

Exponent /Power

Base کے اوپر جو چھوٹا سا نمبر ہوتا ہے اسے power کہتے ہیں۔ اس کو index بھی کہتے ہیں۔

Co-efficient

Base کے Left طرف جو نمبر ہوتا ہے اسے Co-efficient کہتے ہیں۔

Base اور Co-efficient آپس میں Multiply ہوتے ہیں

$4x^2$ Base: x Power: 2 Co-efficient: 4	$5y^{-3}$ Base: y Power: −3 Co-efficient: 5	$-2y^3$ Base: y Power: 3 Co-efficient: −2
x Base: x Power: 1 Co-efficient: 1	x^3 Base: x Power: 3 Co-efficient: 1	$5z$ Base: z Power: 1 Co-efficient: 5

Note:

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16} \qquad \frac{1}{3^{-3}} = 3^3 = 27$$

$$-4x^{-2} = \frac{-4}{x^2} \qquad (a+b)^{-1} = \frac{1}{(a+b)}$$

Laws of Exponents

Multiplication of Same Bases

To multiply powers of the same base, keep the same base and add the exponents.

اگر ایک جیسے bases آپس میں multiply ہوتے ہیں تو:

- ❖ **Co-efficient کو multiply کریں گے**
- ❖ **Base ایک لکھیں گے**
- ❖ **Powers کو Add کریں گے**

Example:

$$a^m . a^n = a^{m+n}$$

Ex # 2.4

Multiplication of Different Bases

When different bases are multiplied just multiply the co-efficient or constant.

اگر مختلف bases آپس میں multiply ہوتے ہیں تو صرف Co-efficient کو multiply کریں گے

Law of Quotient

To divide two expressions with the same bases and different exponents, keep the same base and subtract the exponents.

جب fraction میں ایک جیسے bases ہو تو اس base کو اوپر لے جائیں گے لیکن اس کے power کا sign تبدیل ہو جائے گا۔

- ❖ اگر plus ہو گا تو minus ہو جائے گا
- ❖ اگر minus ہو گا تو plus ہو جائے گا

Law of Power of Power

To raise an exponential expression to a power, keep the same base multiply the exponents.

جب کسی بریکٹ کے اوپر Power آجائیں تو اس کو تمام Bases کے Powers کے ساتھ Multiply کریں گے۔

اگر Base یا Co-efficient کے ساتھ minus کا sign ہو تو:

1) جب power میں even نمبر ہو تو expression کے ساتھ plus کا sign لگائیں گے۔

$$(-x)^{22} = x^{22} \qquad (-4y)^2 = 16y^2$$

2) جب power میں Odd نمبر ہو تو expression کے ساتھ plus کا minus لگائیں گے۔

$$(-x)^{25} = -x^{25} \qquad (-2y)^3 = -8y^3$$

Zero Exponent Rule

Any non-zero number raised to the zero power equals one.

کسی بھی Base کا Power اگر Zero ہو تو 1 کے برابر ہو گا۔

$100^0 = 1$ and $(xy)^0 = 1$

Ex # 2.4

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Q1: Write the base, exponent and value of the following.

(i) $(2)^{-9} = \frac{1}{1024}$

$base = 2,\ Exponent = -9, value = \frac{1}{1024}$

(ii) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

$base = \frac{a}{b},\ Exponent = p,\ \ value = \frac{a^p}{b^p}$

(iii) $(-4)^2 = 16$

$base = -4,\ \ Exponent = 2,\ \ value = 16$

Q2: 67 If a, b denote the real numbers then simplify the following.

(i) $a^3 \times a^5$

Solution:

$a^3 \times a^5$

$= a^{3+5}$

$= a^8$

(ii) $\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{2}{3}}$

Solution:

$\left(\frac{b}{a}\right)^{\frac{3}{2}}\left(\frac{b}{a}\right)^{-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{3}{2}-\frac{2}{3}}$

$= \left(\frac{b}{a}\right)^{\frac{9-4}{6}}$

$= \left(\frac{b}{a}\right)^{\frac{5}{6}}$

(iii) $(-a)^4 \times (-a)^3$

Solution:

$(-a)^4 \times (-a)^3$

$= (-a)^{4+3}$

$= (-a)^7$

$= -a^7$

(iv) $(-2a^2b^3)^3$

Solution:

$(-2a^2b^3)^3$

$= (-2)^3a^{2\times3}b^{3\times3}$

$= -8a^6b^9$

(v) $a^3(-2b)^2$

Solution:

$= a^3(-2b)^2$

$= a^3(-2)^2(b)^2$

$= a^3 \times 4b^2$

$= 4a^3b^2$

(vi) $(a^2b)(a^2b)$

Solution:

$(a^2b)(a^2b)$

$= a^{2+2}b^{1+1}$

$= a^4b^2$

(vii) $\frac{a^0.b^0}{2}$

Solution:

$\frac{a^0.b^0}{2}$

$= \frac{1 \times 1}{2}$

$= \frac{1}{2}$

(viii) $(-3a^2b^2)^2$

Solution:

$(-3a^2b^2)^2$

$= (-3)^2a^{2\times2}b^{2\times2}$

$= 9a^4b^4$

Ex # 2.4

(ix) $\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

Solution:

$\left(\frac{a^2}{b^4}\right)^{\frac{3}{2}}$

$= \frac{a^{2\times\frac{3}{2}}}{b^{4\times\frac{3}{2}}}$

$= \frac{a^{1\times3}}{b^{2\times3}}$

$= \frac{a^3}{b^6}$

Q3: Simplify the following.

(i) $\frac{7^6}{7^4}$

Solution:

$\frac{7^6}{7^4}$

$= 7^6.7^{-4}$

$= 7^{6-4}$

$= 7^2$

(ii) $\frac{2^4.5^3}{10^2}$

Solution:

$\frac{2^4.5^3}{10^2}$

$= \frac{2^4.5^3}{(2\times5)^2}$

$= \frac{2^4.5^3}{2^2.5^2}$

$= 2^4.5^3.2^{-2}.5^{-2}$

$= 2^{4-2}.5^{3-2}$

$= 2^2.5^1$

$= 4\times5$

$= 20$

Ex # 2.4

(iii) $\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

Solution:

$\left\{\frac{(a+b)^2.(c+d)^3}{(a+b).(c+d)^2}\right\}^3$

$= \frac{(a+b)^{2\times3}.(c+d)^{3\times3}}{(a+b)^{1\times3}.(c+d)^{2\times3}}$

$= \frac{(a+b)^6.(c+d)^9}{(a+b)^3.(c+d)^6}$

$= (a+b)^6.(c+d)^9.(a+b)^{-3}.(c+d)^{-6}$

$= (a+b)^{6-3}.(c+d)^{9-6}$

$= (a+b)^3.(c+d)^3$

(iv) $(\sqrt[3]{a})^{\frac{1}{2}}$

Solution:

$(\sqrt[3]{a})^{\frac{1}{2}}$

$= \left(a^{\frac{1}{3}}\right)^{\frac{1}{2}}$

$= a^{\frac{1}{3}\times\frac{1}{2}}$

$= a^{\frac{1}{6}}$

(v) $\sqrt[5]{x^5}.\sqrt[4]{x^4}$

Solution:

$\sqrt[5]{x^5}.\sqrt[4]{x^4}$

$= (x^5)^{\frac{1}{5}}(x^4)^{\frac{1}{4}}$

$= (x)^{5\times\frac{1}{5}}.(x)^{4\times\frac{1}{4}}$

$= x.x$

$= x^2$

Ex # 2.4

Q4: 67 **Simplify the following in such a way that no answers should contain fractional or negative exponent.**

(i) $\left(\frac{25}{81}\right)^{\frac{1}{2}}$

Solution:

$$\left(\frac{25}{81}\right)^{\frac{1}{2}}$$
$$=\left(\frac{5\times 5}{9\times 9}\right)^{\frac{1}{2}}$$
$$=\left(\frac{5^2}{9^2}\right)^{\frac{1}{2}}$$
$$=\frac{5^{2\times\frac{1}{2}}}{9^{2\times\frac{1}{2}}}$$
$$=\frac{5}{9}$$

(ii) $\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$

Solution:

$$\frac{(ab)^{\frac{1}{b}}}{\left(\frac{1}{ab}\right)^{\frac{1}{a}}}$$
$$=\frac{(ab)^{\frac{1}{b}}}{((ab)^{-1})^{\frac{1}{a}}}$$
$$=\frac{(ab)^{\frac{1}{b}}}{(ab)^{-\frac{1}{a}}}$$
$$=(ab)^{\frac{1}{b}}.(ab)^{\frac{1}{a}}$$
$$=(ab)^{\frac{1}{b}+\frac{1}{a}}$$
$$=(ab)^{\frac{a+b}{ba}}$$
$$=(ab)^{\frac{a+b}{ab}}$$
$$=a^{\frac{a+b}{ab}}.b^{\frac{a+b}{ab}}$$

Ex # 2.4

(iii) $\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$

Solution:

$$\frac{2^{p+1}.3^{2p-q}.5^{p+q}.6^q}{6^p.10^{q+2}.15^p}$$
$$=\frac{2^{p+1}.3^{2p-q}.5^{p+q}.(2\times 3)^q}{(2\times 3)^p.(2\times 5)^{q+2}.(3\times 5)^p}$$
$$=\frac{2^{p+1}.3^{2p-q}.5^{p+q}.2^q.3^q}{2^p.3^p.2^{q+2}.5^{q+2}.3^p.5^p}$$
$$=\frac{2^{p+1+q}.3^{2p-q+q}.5^{p+q}}{2^{p+q+2}.3^{p+p}.5^{q+2+p}}$$
$$=\frac{2^{p+1+q}.3^{2p}.5^{p+q}}{2^{p+q+2}.3^{2p}.5^{q+2+p}}$$
$$=2^{p+1+q}.3^{2p}.5^{p+q}.2^{-p-q-2}.3^{-2p}.5^{-q-2-p}$$
$$=2^{p+1+q-p-q-2}.3^{2p-2p}.5^{p+q-q-2-p}$$
$$=2^{1-2}.3^0.5^{-2}$$
$$=2^{-1}.3^0.5^{-2}$$
$$=\frac{1}{2}\times 1\times\frac{1}{5^2}$$
$$=\frac{1}{2}\times 1\times\frac{1}{25}$$
$$=\frac{1}{50}$$

(iv) $\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$

Solution:

$$\left(\frac{x^p}{x^q}\right)^{p+q}\left(\frac{x^q}{x^r}\right)^{q+r}\left(\frac{x^r}{x^p}\right)^{r+p}$$
$$=(x^p.x^{-q})^{p+q}(x^q.x^{-r})^{q+r}(x^r.x^{-p})^{r+p}$$
$$=(x^{p-q})^{p+q}(x^{q-r})^{q+r}(x^{r-p})^{r+p}$$
$$=(x)^{(p-q)(p+q)}.(x)^{(q-r)(q+r)}.(x)^{(r-p)(r+p)}$$
$$=(x)^{p^2-q^2}.(x)^{q^2-r^2}.(x)^{r^2-p^2}$$
$$=x^{p^2-q^2+q^2-r^2+r^2-p^2}$$
$$=x^0$$
$$=1$$

Ex # 2.4

Q5: 67 **Prove that** $\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$

Solution:

$$\left(\frac{4^5.64^3.2^3}{8^5.(128)^2}\right)^{\frac{1}{2}} = 2$$

L.H.S

$$= \left(\frac{(2^2)^5.(2^6)^3.2^3}{(2^3)^5.(2^7)^2}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10}.2^{18}.2^3}{2^{15}.2^{14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{10+18+3}}{2^{15+14}}\right)^{\frac{1}{2}}$$

$$= \left(\frac{2^{31}}{2^{29}}\right)^{\frac{1}{2}}$$

$$= (2^{31-29})^{\frac{1}{2}}$$

$$= (2^2)^{\frac{1}{2}}$$

$$= 2^{2\times\frac{1}{2}}$$

$$= 2$$

=R.H.S

Ex # 2.5

Complex Number

A number of the form $a + bi$ where a and b are real numbers is called complex number where "a" is called real part and "b" is called imaginary part.

Conjugate of a Complex Numbers

A conjugate of a complex number is obtained by changing the sign of imaginary part. The conjugate of $a + bi$ is $a - bi$ or the conjugate of $a + bi$ is denoted by $\overline{a + bi} = a - bi$.

Ex # 2.5

Equality of Two Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then $Z_1 = Z_2$ if real parts are equal i.e. $a = c$ and imaginary parts are equal i.e. $b = d$.

Operation on Complex Numbers

Addition of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 + Z_2 = (a + bi) + (c + di)$

$Z_1 + Z_2 = a + bi + c + di$

$Z_1 + Z_2 = a + c + bi + di$

$Z_1 + Z_2 = (a + c) + (b + d)i$

Subtraction of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1 - Z_2 = (a + bi) - (c + di)$

$Z_1 - Z_2 = a + bi - c - di$

$Z_1 - Z_2 = a - c + bi - di$

$Z_1 - Z_2 = (a - c) + (b - d)i$

Multiplication of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$Z_1.Z_2 = (a + bi)(c + di)$

$Z_1.Z_2 = ac + adi + bci + bdi^2$

$Z_1.Z_2 = ac + (ad + bc)i + bd(-1)$ as $i^2 = -1$

$Z_1.Z_2 = ac + (ad + bc)i - bd$

$Z_1.Z_2 = (ac - bd) + (ad + bc)i$

Division of Complex Numbers

Let $Z_1 = a + bi$ and $Z_2 = c + di$ then

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di}$$

Multiply and Divide by $c - di$

$$\frac{Z_1}{Z_2} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$$

$$\frac{Z_1}{Z_2} = \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$\frac{Z_1}{Z_2} = \frac{ac - adi + bci - bdi^2}{c^2 - (di)^2}$$

$$\frac{Z_1}{Z_2}=\frac{ac+bci-adi-bd(-1)}{c^2-d^2i^2} \quad As\ i^2=-1$$

$$\frac{Z_1}{Z_2}=\frac{ac+(bc-ad)i+bd}{c^2-d^2(-1)}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$\frac{Z_1}{Z_2}=\frac{(ac+bd)}{c^2+d^2}+\frac{(bc-ad)i}{c^2+d^2}$$

Ex # 2.5

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Q1: Add the following complex number

(i) $8+9i, 5+2i$

Solution:

$8+9i, 5+2i$

Let $Z_1=8+9i$

And $Z_2=5+2i$

Now

$Z_1+Z_2=(8+9i)+(5+2i)$

$Z_1+Z_2=8+9i+5+2i$

$Z_1+Z_2=8+5+9i+2i$

$Z_1+Z_2=13+11i$

(ii) $6+3i, 3-5i$

Solution:

$6+3i, 3-5i$

Let $Z_1=6+3i$

And $Z_2=3-5i$

Now

$Z_1+Z_2=(6+3i)+(3-5i)$

$Z_1+Z_2=6+3i+3-5i$

$Z_1+Z_2=6+3+3i-5i$

$Z_1+Z_2=9-2i$

(iii) $2i+3, 8-5\sqrt{-1}$

Solution:

$2i+3, 8-5\sqrt{-1}$

Let $Z_1=2i+3$

And $Z_2=8-5\sqrt{-1}$

$8-5i \quad \because \sqrt{-1}=i$

Now

$Z_1+Z_2=(2i+3)+(8-5i)$

$Z_1+Z_2=2i+3+8-5i$

$Z_1+Z_2=3+8+2i-5i$

$Z_1+Z_2=11-3i$

(iv) $\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Solution:

$\sqrt{3}+\sqrt{2}i, 3\sqrt{3}-2\sqrt{2}i$

Let $Z_1=\sqrt{3}+\sqrt{2}i$

And $Z_2=3\sqrt{3}-2\sqrt{2}i$

Now

$Z_1+Z_2=\left(\sqrt{3}+\sqrt{2}i\right)+\left(3\sqrt{3}-2\sqrt{2}i\right)$

$Z_1+Z_2=\sqrt{3}+\sqrt{2}i+3\sqrt{3}-2\sqrt{2}i$

$Z_1+Z_2=\sqrt{3}+3\sqrt{3}+\sqrt{2}i-2\sqrt{2}i$

$Z_1+Z_2=4\sqrt{3}-\sqrt{2}i$

Q2: Subtract:

(i) $-2+3i$ from $6-3i$

Solution:

$-2+3i$ from $6-3i$

Let $Z_1=-2+3i$

And $Z_2=6-3i$

Now

$Z_2-Z_1=(6-3i)-(-2+3i)$

$Z_2-Z_1=6-3i+2-3i$

$Z_2-Z_1=6+2-3i-3i$

$Z_2-Z_1=8-6i$

(ii) $9+4i$ from $9-8i$

Solution:

$9+4i$ from $9-8i$

Let $Z_1=9+4i$

And $Z_2=9-8i$

Now

$Z_2-Z_1=(9-8i)-(9+4i)$

$Z_2-Z_1=9-8i-9-4i$

$Z_2-Z_1=9-9-8i-4i$

$Z_2-Z_1=0-12i$

$Z_2-Z_1=-12i$

Ex # 2.5

(iii) $\mathbf{1-3i}$ **from** $\mathbf{8-i}$

Solution:

$1-3i$ from $8-i$

Let $Z_1 = 1-3i$

And $Z_2 = 8-i$

Now

$Z_2 - Z_1 = (8-i)-(1-3i)$

$Z_2 - Z_1 = 8-i-1+3i$

$Z_2 - Z_1 = 8-1-i+3i$

$Z_2 - Z_1 = 7+2i$

(iv) $6-7i$ from $6+7i$

Solution:

$6-7i$ from $6+7i$

Let $Z_1 = 6-7i$

And $Z_2 = 6+7i$

Now

$Z_2 - Z_1 = (6+7i)-(6-7i)$

$Z_2 - Z_1 = 6+7i-6+7i$

$Z_2 - Z_1 = 6-6+7i+7i$

$Z_2 - Z_1 = 0+14i$

$Z_2 - Z_1 = 14i$

Q3: Multiply the following complex numbers

(i) $\mathbf{1+2i, 3-8i}$

Solution:

$1+2i, 3-8i$

Let $Z_1 = 1+2i$

And $Z_2 = 3-8i$

Now

$Z_1.Z_2 = (1+2i)(3-8i)$

$Z_1.Z_2 = 1(3-8i)+2i(3-8i)$

$Z_1.Z_2 = 3-8i+6i-16i^2$

$Z_1.Z_2 = 3-2i-16(-1)$

$Z_1.Z_2 = 3-2i+16$

$Z_1.Z_2 = 3+16-2i$

$Z_1.Z_2 = 19-2i$

(ii) $\mathbf{2i, 4-7i}$

Solution:

$2i, 4-7i$

Let $Z_1 = 2i$

And $Z_2 = 4-7i$

Ex # 2.5

Now

$Z_1.Z_2 = (2i)(4-7i)$

$Z_1.Z_2 = 2i(4-7i)$

$Z_1.Z_2 = 8i-14i^2$

$Z_1.Z_2 = 8i-14(-1)$

$Z_1.Z_2 = 8i+14$

$Z_1.Z_2 = 14+8i$

(iii) $\mathbf{5-3i, 2-4i}$

Solution:

$5-3i, 2-4i$

Let $Z_1 = 5-3i$

And $Z_2 = 2-4i$

Now

$Z_1.Z_2 = (5-3i)(2-4i)$

$Z_1.Z_2 = 5(2-4i)-3i(2-4i)$

$Z_1.Z_2 = 10-20i-6i+12i^2$

$Z_1.Z_2 = 10-26i+12(-1)$

$Z_1.Z_2 = 10-26i-12$

$Z_1.Z_2 = 10-12-26i$

$Z_1.Z_2 = -2-26i$

(iv) $\mathbf{\sqrt{2}+i, 1-\sqrt{2}i}$

Solution:

$\sqrt{2}+i, 1-\sqrt{2}i$

Let $Z_1 = \sqrt{2}+i$

And $Z_2 = 1-\sqrt{2}i$

Now

$Z_1.Z_2 = (\sqrt{2}+i)(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}(1-\sqrt{2}i)+i(1-\sqrt{2}i)$

$Z_1.Z_2 = \sqrt{2}-\sqrt{2\times 2}i+1i-\sqrt{2}i^2$

$Z_1.Z_2 = \sqrt{2}-2i+1i-\sqrt{2}(-1)$

$Z_1.Z_2 = \sqrt{2}-i+\sqrt{2}$

$Z_1.Z_2 = \sqrt{2}+\sqrt{2}-i$

$Z_1.Z_2 = 2\sqrt{2}-i$

Ex # 2.5

Q4: Divide the first complex number by the second.

(i) $\boldsymbol{Z_1 = 2 + i, Z_2 = 5 - i}$

Solution:

$Z_1 = 2 + i, Z_2 = 5 - i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i}$$

Multiply and divide by $5 + i$

$$\frac{Z_1}{Z_2} = \frac{2+i}{5-i} \times \frac{5+i}{5+i}$$

$$\frac{Z_1}{Z_2} = \frac{(2+i)(5+i)}{(5-i)(5+i)}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 2i + 5i + i^2}{(5)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i + (-1)}{25 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{10 + 7i - 1}{25 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{10 - 1 + 7i}{25 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{9 + 7i}{26}$$

$$\frac{Z_1}{Z_2} = \frac{9}{26} + \frac{7}{26}i$$

(ii) $\boldsymbol{Z_1 = 3i + 4, Z_2 = 1 - i}$

Solution:

$Z_1 = 3i + 4$

$4 + 3i$

$Z_2 = 1 - i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i}$$

Multiply and divide by $1 + i$

$$\frac{Z_1}{Z_2} = \frac{4+3i}{1-i} \times \frac{1+i}{1+i}$$

Ex # 2.5

$$\frac{Z_1}{Z_2} = \frac{(4+3i)(1+i)}{(1-i)(1+i)}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 4i + 3i + 3i^2}{(1)^2 - (i)^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i + 3(-1)}{1 - i^2}$$

$$\frac{Z_1}{Z_2} = \frac{4 + 7i - 3}{1 - (-1)}$$

$$\frac{Z_1}{Z_2} = \frac{4 - 3 + 7i}{1 + 1}$$

$$\frac{Z_1}{Z_2} = \frac{1 + 7i}{2}$$

$$\frac{Z_1}{Z_2} = \frac{1}{2} + \frac{7}{2}i$$

Q5: Perform the indicated operations and reduce to the form $\boldsymbol{a + bi}$

(i) $\boldsymbol{(4 - 3i) + (2 - 3i)}$

Solution:

$(4 - 3i) + (2 - 3i)$
$= 4 - 3i + 2 - 3i$
$= 4 + 2 - 3i - 3i$
$= 6 - 6i$

(ii) $\boldsymbol{(5 - 2i) - (4 - 7i)}$

Solution:

$(5 - 2i) - (4 - 7i)$
$= 5 - 2i - 4 + 7i$
$= 5 - 4 - 2i + 7i$
$= 1 + 5i$

(iii) $\boldsymbol{2i(4 - 5i)}$

Solution:

$2i(4 - 5i)$
$= 2i - 10i^2$
$= 2i - 10(-1)$
$= 2i + 10$
$= 10 + 2i$

Ex # 2.5

(iv) $(\mathbf{2-3i}) \div (\mathbf{4-5i})$

Solution:

$(2-3i) \div (4-5i)$

$=\dfrac{2-3i}{4-5i}$

Multiply and divide by $4+5i$

$=\dfrac{2-3i}{4-5i}\times\dfrac{4+5i}{4+5i}$

$=\dfrac{(2-3i)(4+5i)}{(4-5i)(4+5i)}$

$=\dfrac{8+10i-12i-15i^2}{(4)^2-(5i)^2}$

$=\dfrac{8-2i-15(-1)}{16-25i^2}$

$=\dfrac{8-2i+15}{16-25(-1)}$

$=\dfrac{8+15-2i}{16+25}$

$=\dfrac{23-2i}{41}$

$=\dfrac{23}{41}-\dfrac{2}{41}i$

Q6: Find the complex conjugate of the following complex numbers.

(i) $-8-3i$

The complex conjugate of $-8-3i$ is $-8+3i$

(i) $\mathbf{-4+9i}$

The complex conjugate of $-4+9i$ is $-4-9i$

(iii) $\mathbf{7+6i}$

The complex conjugate of $7+6i$ is $7-6i$

(iv) $\mathbf{\sqrt{5}-i}$

The complex conjugate of $\sqrt{5}-i$ is $\sqrt{5}+i$

Review Ex # 2

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Q3: Simplify each of the following.

(i) $\left(\dfrac{-2}{3}\right)^3$

Solution:

$\left(\dfrac{-2}{3}\right)^3$

$=\dfrac{(-2)^3}{(3)^3}$

$=\dfrac{-8}{27}$

(ii) $(-2)^3.(3)^2$

Solution:

$(-2)^3.(3)^2$

$=-8\times 9$

-72

(iii) $-3\sqrt{48}$

Solution:

$-3\sqrt{48}$

$-3\sqrt{4\times 4\times 3}$

$-3\sqrt{4\times 4}\times\sqrt{3}$

$-3\times 4\sqrt{3}$

$-12\sqrt{3}$

(iv) $\dfrac{5}{\sqrt[3]{9}}$

Solution:

$\dfrac{5}{\sqrt[3]{9}}$

$=\dfrac{5}{(9)^{\frac{1}{3}}}$

$=\dfrac{5}{(3^2)^{\frac{1}{3}}}$

$=\dfrac{5}{(3)^{\frac{2}{3}}}$

Review Ex # 2

Multiply and Divide by $\sqrt[3]{3}$

$$\frac{5}{(3)^{\frac{2}{3}}}\times\frac{\sqrt[3]{3}}{\sqrt[3]{3}}$$

$$\frac{5\times\sqrt[3]{3}}{(3)^{\frac{2}{3}}\times(3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{2}{3}+\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{(3)^{\frac{3}{3}}}$$

$$\frac{5\sqrt[3]{3}}{3}$$

Q4: Multiply $8i, \quad -8i$

Solution:

$8i, \quad -8i$

Now

$(8i)(-8i) = -64i^2$

$= -64(-1)$

$= 64$

Q5: Divide $2-5i$ by $1-6i$

Solution:

$$\frac{2-5i}{1-6i}$$

i

Multiply and divide by $1+6i$

$$=\frac{2-5i}{1-6}\times\frac{1+6i}{1+6i}$$

$$=\frac{(2-5i)(1+6i)}{(1-6i)(1+6i)}$$

$$=\frac{2+12i-5i-30i^2}{(1)^2-(6i)^2}$$

$$=\frac{2+7i-30(-1)}{1-36i^2}$$

$$=\frac{2+7i+30}{1-36(-1)}$$

Review Ex # 2

$$=\frac{2+30+7i}{1+36}$$

$$=\frac{32+7i}{37}$$

$$=\frac{32}{37}-\frac{7}{37}i$$

Q7: Use laws of exponents to simplify:

$$\frac{(81)^n.3^5+(3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

Solution:

$$\frac{(81)^n.3^5+(3)^{4n-1}(243)}{(9^{2n})(3^3)}$$

$$=\frac{(3^4)^n.3^5+3^{4n-1}.(3^5)}{(3^2)^{2n}(3^3)}$$

$$=\frac{3^{4n}.3^5+3^{4n}.3^{-1}.3^5}{3^{4n}.3^3}$$

$$=\frac{3^{4n}.3^5(1+3^{-1})}{3^{4n}.3^3}$$

$$=\frac{3^{4n}.3^3.3^2(1+3^{-1})}{3^{4n}.3^3}$$

$= 3^2(1+3^{-1})$

$$=9\left(1+\frac{1}{3}\right)$$

$$=9\left(\frac{3+1}{3}\right)$$

$$=9\left(\frac{4}{3}\right)$$

$= 3\times 4$

$= 12$

Q6: Name the property used

$$7\times\frac{1}{7}=\frac{1}{7}\times 7=1$$

Answer:

Multiplicative Property