UNIT # 7

LINEAR EQUATIONS AND INEQULITIES

Ex # 7.1

Linear equation

An equation the highest degree or exponent of a variable is one is called linear equation.

Linear equation in one variable

A linear equation in which one variable is used is called linear equation in one variable.

General form

$$ax + b = 0$$

Example:

$2x + 3 = 0$

$\frac{5}{2}y - 4 = 0$

$5x - 15 = 2x + 3$

Solution of Linear Equation

To solve the linear equation, follow the following steps.

First solve the brackets if any

Now shift the constant term to other side of equation by adding or subtracting to B.S

Transfer all terms containing variable on one side and simplify them if any.

Divide or multiply both sides of the equation by the co – efficient of the variable.

At last, sing numerical value is obtained.

Verify by putting the value in original equation.

پہلے Brackets کو Solve کریں۔

پھر constant term کو دوسرے طرف Shift کریں یا Add یا Subtract کر کے

Variable والے Term کو بھی ایک طرف Shift کریں

Equation کے دونوں طرف Variable کے Co – efficient کے

ساتھ Multiply یا Divide کریں

Example # 2

Solve $2x + 3 = 1 - (x - 1)$

Solution:

$2x + 3 = 1 - 6(x - 1) \ldots\ldots equ(i)$

$2x + 3 = 1 - 6x + 6$

$2x + 3 = -6x + 1 + 6$

Ex # 7.1

$2x + 3 = -6x + 7$

Subtract 3 from B.S

$2x + 3 - 3 = -6x + 7 - 3$

$2x = -6x + 4$

Add 6x on B.S

$2x + 6x = -6x + 6x + 4$

$8x = 4$

Divide B.S by 8

$\frac{8x}{8} = \frac{4}{8}$

$x = \frac{1}{2}$

Verification

Put $x = \frac{1}{2}$ in equ (i)

$2\left(\frac{1}{2}\right) + 3 = 1 - 6\left(\frac{1}{2} - 1\right)$

$1 + 3 = 1 - 6\left(\frac{1-2}{2}\right)$

$4 = 1 - 6\left(\frac{-1}{2}\right)$

$4 = 1 - 3(-1)$

$4 = 1 + 3$

$4 = 4$

Thus Solution Set $= \left\{\frac{1}{2}\right\}$

Example # 3

Solve $\boldsymbol{3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x}$

Solution:

$3x + \frac{x}{5} - 5 = \frac{1}{5} + 5x \ldots\ldots equ(i)$

Separate the variable and constant

$3x + \frac{x}{5} - 5x = \frac{1}{5} + 5$

$3x - 5x + \frac{x}{5} = \frac{1}{5} + 5$

$\frac{x}{5} + 3x - 5x = \frac{1}{5} + 5$

$\frac{x}{5} - 2x = \frac{1}{5} + 5$

Chapter # 7

Ex # 7.1

$$\frac{x-10x}{5}=\frac{1+25}{5}$$
$$\frac{-9x}{5}=\frac{26}{5}$$
Multiply B.S by 5
$$5\times\frac{-9x}{5}=5\times\frac{26}{5}$$
$$-9x=26$$
Divide B.S by -9
$$\frac{-9x}{-9}=\frac{26}{-9}$$
$$x=-\frac{26}{9}$$

Verification

Put $x=-\frac{26}{9}$ in equ (i)
$$3\left(-\frac{26}{9}\right)+\frac{-\frac{26}{9}}{5}-5=\frac{1}{5}+5\left(-\frac{26}{9}\right)$$
$$-\frac{26}{3}+\left(-\frac{26}{9}\right)\div 5-5=\frac{1}{5}-\frac{130}{9}$$
$$-\frac{26}{3}-\frac{26}{9}\times\frac{1}{5}-5=\frac{1}{5}-\frac{130}{9}$$
$$-\frac{26}{3}-\frac{26}{45}-5=\frac{1}{5}-\frac{130}{9}$$
$$\frac{-390-26-225}{45}=\frac{9-650}{45}$$
$$\frac{-641}{45}=\frac{-641}{45}$$
Thus Solution Set $=\left\{-\frac{26}{9}\right\}$

Example # 4

Age of mother is 13 time the age of her daughter. It will be only five times after four years. Find their present ages.

Solution:

Let the present age of daughter $= x$ years
So the present age of mother $= 13x$ years
After four years
Age of daughter $=(x+4)years$
and age of mother $=(13x+4)years$
According to condition
Age of mother $=5$(Age of daughter)
$$13x+4=5(x+4)$$
$$13x+4=5x+20$$

Ex # 7.1

Now shift the variable and constant
$$13x-5x=20-4$$
$$8x=16$$
Divide B.S by 8
$$\frac{8x}{8}=\frac{16}{8}$$
$$x=2$$
Thus present age of daughter $=x=2years$
And present age of mother $=13\times 2$
$=26years$

Example # 5

A number consist of two digits. The sum of digits is 8. If digits are interchanged, then new number becomes 36 less than the original numbers. Find the number.

Solution:

$Let\ digit\ at\ ones/unit\ place=x$
$And\ digit\ at\ tens\ place=y$
$So\ the\ original\ number\ =10\times y+1\times x$
$=10y+x$
$If\ place\ of\ digits\ are\ interchanged$
$New\ number=10\times x+1\times y$
$=10x+y$
According to given conditions
Sum of digits is 8
So,
$$x+y=8\ldots\ldots equ(i)$$
And
$$New\ number=Original\ number-36$$
$$10x+y=10y+x-36$$
$$10x-x=10y-y-36$$
$$9x=9y-36$$
$$9x=9(y-4)$$
Divide B.S by 9
$$\frac{9x}{9}=\frac{9(y-4)}{9}$$
$$x=y-4\ldots\ldots equ(ii)$$
Put $x=y-4$ in equ (i)
$$y-4+y=8$$
Add 4 on B.S
$$y-4+4+y=8+4$$
$$y+y=12$$
$$2y=12$$

Chapter # 7

Ex # 7.1

Divide B.S by 2

$$\frac{2y}{2}=\frac{12}{2}$$

$y=6$

Put $y=6$in equ (ii)

$x=6-4$

$x=2$

$$As\ the\ Original\ number = 10y+x$$
$$=10(6)+2$$
$$=60+2$$
$$=62$$

Exercise # 7.1

Page # 177

Q1: **Find the solution sets of the following equations and verify the answer.**

(i) $\mathbf{5x+8=23}$

Solution:

$5x+8=23 \dots\dots equ(i)$

Subtract 8 from B.S

$5x+8-8=23-8$

$5x=15$

Divide 5 on B.S

$$\frac{5x}{5}=\frac{15}{5}$$

$x=3$

Verification

Put $x=3$ in equ (i)

$5(3)+8=23$

$15+8=23$

$23=23$

Thus Solution Set = { 3 }

(ii) $\mathbf{\frac{3}{5}x-\frac{2}{3}=2}$

Solution:

$\frac{3}{5}x-\frac{2}{3}=2 \dots\dots equ(i)$

$$\frac{9x-10}{15}=2$$

Multiply 15 on B.S

$$\frac{9x-10}{15}\times 15=2\times 15$$

$9x-10=30$

Ex # 7.1

Add 10 on B.S

$9x-10+10=30+10$

$9x=40$

Divide 9 on B.S

$$\frac{9x}{9}=\frac{40}{9}$$

$$x=\frac{40}{9}$$

Verification

Put $x=\frac{40}{9}$ in equ (i)

$$\frac{3}{5}\times\frac{40}{9}-\frac{2}{3}=2$$

$$\frac{1}{1}\times\frac{8}{3}-\frac{2}{3}=2$$

$$\frac{8}{3}-\frac{2}{3}=2$$

$$\frac{8-2}{3}=2$$

$$\frac{6}{3}=2$$

$2=2$

Thus Solution Set $=\left\{\frac{40}{9}\right\}$

(iii) $\mathbf{6x-5=2x+9}$

Solution:

$6x-5=2x+9 \dots\dots equ(i)$

Add 5 on B.S

$6x-5+5=2x+9+5$

$6x=2x+14$

Subtract $2x$ from B.S

$6x-2x=2x-2x+14$

$4x=14$

Divide B.S by 4

$$\frac{4x}{4}=\frac{14}{4}$$

$$x=\frac{7}{2}$$

Verification

Put $x=\frac{7}{2}$ in equ (i)

$$6\left(\frac{7}{2}\right)-5=2\left(\frac{7}{2}\right)+9$$

$3(7)-5=7+9$

$21-5=16$

$16=16$

Chapter # 7

Ex # 7.1

Thus Solution Set $= \left\{\frac{7}{2}\right\}$

(iv) $\boldsymbol{\frac{2}{x-1} = \frac{1}{x-2}}$

Solution:

$\frac{2}{x-1} = \frac{1}{x-2} \dots\dots equ(i)$

By Cross Multiplication

$2(x-2) = 1(x-1)$

$2x - 4 = x - 1$

Add 4 on B.S

$2x - 4 + 4 = x - 1 + 4$

$2x = x + 3$

Subtract x from B.S

$2x - x = x - x + 3$

$x = 3$

Verification

Put $x = 3$ in equ (i)

$\frac{2}{3-1} = \frac{1}{3-2}$

$\frac{2}{2} = \frac{1}{1}$

$1 = 1$

Solution Set $= \{3\}$

(v) $\boldsymbol{\frac{1}{7x+13} = \frac{2}{9}}$

Solution:

$\frac{1}{7x+13} = \frac{2}{9} \dots\dots equ(i)$

By Cross Multiplication

$1 \times 9 = 2(7x + 13)$

$9 = 14x + 26$

Subtract 26 from B.S

$9 - 26 = 14x - 26$

$-17 = 14x$

Divide B.S by 14

$\frac{-17}{14} = \frac{14x}{14}$

$\frac{-17}{14} = x$

$x = \frac{-17}{14}$

Verification

Ex # 7.1

Put $x = \frac{-17}{14}$ in equ (i)

$\frac{1}{7\left(\frac{-17}{14}\right) + 13} = \frac{2}{9}$

$\frac{1}{\frac{-17}{2} + 13} = \frac{2}{9}$

$\frac{1}{\frac{-17+26}{2}} = \frac{2}{9}$

$\frac{1}{\frac{9}{2}} = \frac{2}{9}$

$1 \div \frac{9}{2} = \frac{2}{9}$

$1 \times \frac{2}{9} = \frac{2}{9}$

$\frac{2}{9} = \frac{2}{9}$

Solution Set $= \left\{\frac{-17}{14}\right\}$

(vi) $\boldsymbol{10(x-4) = 4(2x-1) + 5}$

Solution:

$10(x-4) = 4(2x-1) + 5 \dots\dots equ(i)$

$10x - 40 = 8x - 4 + 5$

$10x - 40 = 8x + 1$

$10x - 40 = 8x + 1$

Add 40 on B.S

$10x - 40 + 40 = 8x + 1 + 40$

$10x = 8x + 41$

Subtract $8x$ from B.S

$10x - 8x = 8x - 8x + 41$

$2x = 41$

Divide B.S by 2

$\frac{2x}{2} = \frac{41}{2}$

$x = \left\{\frac{41}{2}\right\}$

Verification

Put $x = \frac{41}{2}$ in equ (i)

$10\left(\frac{41}{2} - 4\right) = 4\left(2 \times \frac{41}{2} - 1\right) + 5$

$10\left(\frac{41-8}{2}\right) = 4(41-1) + 5$

$10\left(\frac{33}{2}\right) = 4(40) + 5$

Chapter # 7

Ex # 7.1

$5(33) = 160 + 5$
$165 = 165$
$\text{Solution Set} = \frac{41}{2}$

Q2: **Awais thought of a number, add 3 with it. Then he doubled the sum. He got 40. What was the original number?**

Solution:

Let the number $= x$
As the given condition is defined as
Add 3 and double the sum got 40
So, we get
$2(x + 3) = 40$
Divide B.S by 2
$\frac{2(x+3)}{2} = \frac{40}{2}$
$x + 3 = 20$
Subtract 3 from B.S
$x + 3 - 3 = 20 - 3$
$x = 17$
Thus, the original number $= 17$

Q3: **The sum of two numbers is -4 and their difference is 6. What are the numbers?**

Solution:

$\textit{Let the two numbers are } x \textit{ and } y$
According to first condition
The sum of two numbers is -4
So,
$x + y = -4 \ldots\ldots equ(i)$
According to second condition
The difference of two numbers is 6
So,
$x - y = 6 \ldots\ldots equ(ii)$
Now add equ(i) and equ (ii)
$x + y + x - y = -4 + 6$
$x + x + y - y = 2$
$2x = 2$
Divide B.S by 2
$\frac{2x}{2} = \frac{2}{2}$
$x = 1$
Put $x = 1$ in equ (i)
$1 + y = -4$

Ex # 7.1

Subtract 1 from B.S
$1 - 1 + y = -4 - 1$
$y = -5$
$\textit{Thus the two numbers are } 1 \textit{ and } -5$

Q4: **The sum of three consecutive odd integers is 81. Find the numbers.**

Solution:

As the difference is 2 between two consecutive odd integers
$\textit{Let first odd integer} = x$
$\textit{Second odd integer} = x + 2$
$\textit{And third odd integer} = x + 4$
According to given condition
The sum of three consecutive odd integers is 81
So,
$x + x + 2 + x + 4 = 81$
$x + x + x + 2 + 4 = 81$
$3x + 6 = 81$
Subtract 6 from B.S
$3x + 6 - 6 = 81 - 6$
$3x = 75$
Divide B.S by 3
$\frac{3x}{3} = \frac{75}{3}$
$x = 25$
$\textit{Let first odd integer} = x = 25$
$\textit{Second odd integer} = x + 2$
$= 25 + 2$
$= 27$
$\textit{And third odd integer} = x + 4$
$= 25 + 4$
$= 29$
So the consecutive odd integers are 25, 27 and 29

Chapter # 7

Ex # 7.1

Q5: A man is 41 year old and his son is 9 year old. In how many years will the father be three times as old as the son?

Solution:

$let\ father's\ age = 41\ years$

$and\ son'sage = 9\ years$

$Let\ the\ required\ years = x$

$So\ after\ x\ years$

$Father's\ age = 41 + x$

$Son's\ age = 9 + x$

According to given condition

$Age\ of\ father = 3(Age\ of\ son)$

$41 + x = 3(9 + x)$

$41 + x = 27 + 3x$

$41 - 27 = 3x - x$

$14 = 2x$

$2x = 14$

Divide B.S by 2

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

So the required number of years=7

Thus after 7 years father's age will be three times as his son

Q6: The tens digit of a certain two – digitnumber exceeds the unit digit by 4 and is 1 less than twice the ones digit. Find the number.

Solution:

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ two\ digit\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

According to given conditions

Tens digit exceeds the unit digit by 1

So,

$Tens\ digit = Ones\ digit + 4$

$y = x + 4 \dots \dots equ(i)$

And

Tens digit is 1 less than twice the ones digits

So,

$Tens\ digit = twice\ the\ one\ digit - 1$

$y = 2x - 1 \dots \dots equ(ii)$

Ex # 7.1

Compare equ (i) and (ii), we get

$x + 4 = 2x - 1$

$4 + 1 = 2x - x$

$5 = x$

$x = 5$

Put $x = 5$in equ (i)

$y = 5 + 4$

$y = 9$

$Thus\ the\ two\ digit = 10y + x$

$= 10(9) + 5$

$= 90 + 5$

$= 95$

Q7: The sum of two digits is 10. It the place of digits are changed then the new number is decreased by 18. Find the numbers.

Solution:

$Let\ digit\ at\ ones/unit\ place = x$

$And\ digit\ at\ tens\ place = y$

$So\ the\ original\ number\ = 10 \times y + 1 \times x$

$= 10y + x$

If place of digits are interchanged

New number $= 10 \times x + 1 \times y$

$= 10x + y$

According to given conditions

Sum of digits is 10

So,

$x + y = 10 \dots \dots equ(i)$

And

$New\ number = Original\ number - 18$

$10x + y = 10y + x - 18$

$10x - x = 10y - y - 18$

$9x = 9y - 18$

$9x = 9(y - 2)$

Divide B.S by 9

$\frac{9x}{9} = \frac{9(y-2)}{9}$

$x = y - 2 \dots \dots equ(ii)$

Put $x = y - 2$in equ (i)

$y - 2 + y = 10$

Add 2 on B.S

$y - 2 + 2 + y = 10 + 2$

$y + y = 12$

$2y = 12$

Chapter # 7

Ex # 7.1

Divide B.S by 2

$$\frac{2y}{2}=\frac{12}{2}$$

$y=6$

Put $y=6$ in equ (ii)

$x=6-2$

$x=4$

$$\begin{aligned} As\ the\ Original\ number &= 10y+x \\ &= 10(6)+4 \\ &= 60+4 \\ &= 64 \end{aligned}$$

Q8: It the breadth of the room is one fourth of its length and the perimeter of the room is 20m. Find length and breadth of the room.

Solution:

$Let\ length\ of\ room = x\ m$

As breadth is one fourth of its length

$Then\ breadth\ of\ room = \frac{x}{4}\ m$

As Perimeter of room=20 m

As we know that

$$P=2(l+2)$$

Put the values

$$20=2\left(x+\frac{x}{4}\right)$$

$$20=2\left(\frac{4x+x}{4}\right)$$

$$20=\frac{5x}{2}$$

$Multiply\ B.S\ by\ \frac{2}{5}$

$$20\times\frac{2}{5}=\frac{5x}{2}\times\frac{2}{5}$$

$4\times 2=x$

$8=x$

$x=8$

Thus

$Let\ length\ of\ room = x\ m = 8m$

$$\begin{aligned} breadth\ of\ room &= \frac{x}{4}\ m \\ &= \frac{8}{4}\ m \\ &= 2\ m \end{aligned}$$

Ex # 7.2

Radical equation

An equation in which the variable occurs under a radical is called radical equation.

Note:

The radicand should be a variable (unknown).

$\sqrt{x}+5=9$ is a radical equation but $2x+\sqrt{5}=9$ is not a radical equation.

The radical equation will be considered as positive numbers.

$\sqrt{x+6}=-11$ has no real solution and is not true for any value of x.

Example # 5

$\boldsymbol{Solve\ \sqrt{2x}+5=9}$

Solution:

$\sqrt{2x}+5=9\ \dots\dots equ(i)$

Subtract 5 from B.S

$\sqrt{2x}+5-5=9-5$

$\sqrt{2x}=4$

Taking square on B.S

$\left(\sqrt{2x}\right)^2=(4)^2$

$2x=16$

Divide B.S by 2

$$\frac{2x}{2}=\frac{16}{2}$$

$x=8$

Verification

Put $x=8$ in equ (i)

$\sqrt{2(8)}+5=9$

$\sqrt{16}+5=9$

$4+5=9$

$9=9$

Thus Solution Set $=\{8\}$

Example # 7

$\boldsymbol{\sqrt{3x-2}=\sqrt{5x+4}}$

Solution:

$\sqrt{3x-2}=\sqrt{5x+4}$

$\sqrt{3x-2}=\sqrt{5x+4}\ \dots\dots equ(i)$

Take square root on B.S

$\left(\sqrt{3x-2}\right)^2=\left(\sqrt{5x+4}\right)^2$

$3x-2=5x+4$

Chapter # 7

Ex # 7.2

Subtract $5x$ from B.S

$3x-5x-2=5x-5x+4$

$-2x-2=4$

Add 2 on B.S

$-2x-2+2=4+2$

$-2x=6$

Divide B.S by -2

$\frac{-2x}{-2}=\frac{6}{-2}$

$x=-3$

Verification

Put $x=-3$ in equ (i)

$\sqrt{3(-3)-2}=\sqrt{5(-3)+4}$

$\sqrt{-9-2}=\sqrt{-15+4}$

$\sqrt{-11}=\sqrt{-11}$

Thus Solution Set $=\{-3\}$

Example # 8

$\mathbf{\sqrt{3x+2}+6=2}$

Solution:

$\sqrt{3x+2}+6=2 \dots\dots equ(i)$

Subtract 6 from B.S

$\sqrt{3x+2}+6-6=2-6$

$\sqrt{3x+2}=-4$

Taking square on B.S

$\left(\sqrt{3x+2}\right)^2=(-4)^2$

$3x+2=16$

Subtract 2 from B.S

$3x+2-2=16-2$

$3x=14$

Divide B.S by 3

$\frac{3x}{3}=\frac{14}{3}$

$x=\frac{14}{3}$

Solution Set $=\{\ \}$

Verification

Put $x=\frac{14}{3}$ in equ (i)

$\sqrt{3\left(\frac{14}{3}\right)+2}+6=2$

$\sqrt{14+2}+6=2$

Ex # 7.2

$\sqrt{16}+6=2$

$4+6=2$

$10=2$

Hence

$10\neq 2$

Thus the given equation has no solution.

Solution Set $=\{\ \}$

Exercise # 7.2

Page # 180

Q: Solve the following radical equation.

1. $\mathbf{2\sqrt{a}-3=7}$

Solution:

$2\sqrt{a}-3=7 \dots\dots equ(i)$

Add 3 on B.S

$2\sqrt{a}-3+3=7+3$

$2\sqrt{a}=10$

Divide B.S by 2

$\frac{2\sqrt{a}}{2}=\frac{10}{2}$

$\sqrt{a}=5$

Taking square on B.S

$\left(\sqrt{a}\right)^2=(5)^2$

$a=25$

Verification

Put $a=25$ in equ (i)

$2\sqrt{25}-3=7$

$2(5)-3=7$

$10-3=7$

$7=7$

Thus Solution Set $=\{25\}$

2. $\mathbf{8+3\sqrt{b}=20}$

Solution:

$8+3\sqrt{b}=20 \dots\dots equ(i)$

Subtract 8 from B.S

$8-8+3\sqrt{b}=20-8$

$3\sqrt{b}=12$

Divide B.S by 3

$\frac{3\sqrt{b}}{3}=\frac{12}{3}$

$\sqrt{b}=4$

Chapter # 7

Ex # 7.2

Taking square on B.S

$\left(\sqrt{b}\right)^2 = (4)^2$

$b = 16$

Verification

Put $b = 16$ in equ (i)

$8 + 3\sqrt{16} = 20$

$8 + 3(4) = 20$

$8 + 12 = 20$

$20 = 20$

Thus Solution Set $= \{16\}$

3. $\mathbf{7 - \sqrt{2b} = 3}$

Solution:

$7 - \sqrt{2b} = 3 \ldots\ldots equ(i)$

Subtract 7 from B.S

$7 - 7 - \sqrt{2b} = 3 - 7$

$-\sqrt{2b} = -4$

$\sqrt{2b} = 4$

Taking square on B.S

$\left(\sqrt{2b}\right)^2 = (4)^2$

$2b = 16$

Divide B.S by 2

$\frac{2b}{2} = \frac{16}{2}$

$b = 8$

Verification

Put $b = 8$ in equ (i)

$7 - \sqrt{2(8)} = 3$

$7 - \sqrt{16} = 3$

$7 - 4 = 3$

$3 = 3$

Thus Solution Set $= \{8\}$

4. $\mathbf{\sqrt{r} - 5 = \sqrt{r} + 9}$

Solution:

$8\sqrt{r} - 5 = \sqrt{r} + 9 \ldots\ldots equ(i)$

Add 5 on B.S

$8\sqrt{r} - 5 + 5 = \sqrt{r} + 9 + 5$

$8\sqrt{r} = \sqrt{r} + 14$

Subtract $\sqrt{r}$ from B.S

$8\sqrt{r} - \sqrt{r} = \sqrt{r} - \sqrt{r} + 14$

$7\sqrt{r} = 14$

Ex # 7.2

Divide B.S by 7

$\frac{7\sqrt{r}}{7} = \frac{14}{7}$

$\sqrt{r} = 2$

Taking square on B.S

$\left(\sqrt{r}\right)^2 = (2)^2$

$r = 4$

Verification

Put r $= 4$ in equ (i)

$7\sqrt{4} - 5 = \sqrt{4} + 9$

$7(2) - 5 = 2 + 9$

$14 - 5 = 11$

$11 = 11$

Thus Solution Set $= \{4\}$

5. $\mathbf{20 - 3\sqrt{t} = \sqrt{t} - 4}$

Solution:

$20 - 3\sqrt{t} = \sqrt{t} - 4 \ldots\ldots equ(i)$

Subtract 20 from B.S

$20 - 20 - 3\sqrt{t} = \sqrt{t} - 4 - 20$

$-3\sqrt{t} = \sqrt{t} - 24$

Subtract $\sqrt{t}$ from B.S

$-3\sqrt{t} - \sqrt{t} = \sqrt{t} - \sqrt{t} - 24$

$-4\sqrt{t} = -24$

$4\sqrt{t} = 24$

Divide B.S by 4

$\frac{4\sqrt{t}}{4} = \frac{24}{4}$

$\sqrt{t} = 6$

Taking square on B.S

$\left(\sqrt{t}\right)^2 = (6)^2$

$t = 36$

Verification

Put $t = 36$ in equ (i)

$20 - 3\sqrt{36} = \sqrt{36} - 4$

$20 - 3(6) = 6 - 4$

$20 - 18 = 2$

$2 = 2$

Thus Solution Set $= \{36\}$

Chapter # 7

Ex # 7.2

6. $\mathbf{2\sqrt{5x} - 3 = 7}$

Solution:

$2\sqrt{5x} - 3 = 7 \dots\dots equ(i)$

Add 3 on B.S

$2\sqrt{5x} - 3 + 3 = 7 + 3$

$2\sqrt{5x} = 10$

Divide B.S by 2

$\frac{2\sqrt{5x}}{2} = \frac{10}{2}$

$\sqrt{5x} = 5$

Taking square on B.S

$\left(\sqrt{5x}\right)^2 = (5)^2$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

Verification

Put $x = 5$ in equ (i)

$2\sqrt{5(5)} - 3 = 7$

$2\sqrt{25} - 3 = 7$

$2(5) - 3 = 7$

$10 - 3 = 7$

$7 = 7$

Thus Solution Set $= \{5\}$

7. $\mathbf{\sqrt{2x - 7} + 8 = 11}$

Solution:

$\sqrt{2x - 7} + 8 = 11 \dots\dots equ(i)$

Subtract 8 from B.S

$\sqrt{2x - 7} + 8 - 8 = 11 - 8$

$\sqrt{2x - 7} = 3$

Taking square on B.S

$\left(\sqrt{2x - 7}\right)^2 = (3)^2$

$2x - 7 = 9$

Add 7 on B.S

$2x - 7 + 7 = 9 + 7$

$2x = 16$

Divide B.S by 2

$\frac{2x}{2} = \frac{16}{2}$

$x = 8$

Ex # 7.2

Verification

Put $x = 8$ in equ (i)

$\sqrt{2(8) - 7} + 8 = 11$

$\sqrt{16 - 7} + 8 = 11$

$\sqrt{9} + 8 = 11$

$3 + 8 = 11$

$11 = 11$

Thus Solution Set $= \{8\}$

8. $\mathbf{22 = 17 + \sqrt{40 - 3y}}$

Solution:

$22 = 17 + \sqrt{40 - 3y} \dots\dots equ(i)$

Subtract 17 from B.S

$22 - 17 = 17 - 17 + \sqrt{40 - 3y}$

$5 = \sqrt{40 - 3y}$

$\sqrt{40 - 3y} = 5$

Taking square on B.S

$\left(\sqrt{40 - 3y}\right)^2 = (5)^2$

$40 - 3y = 25$

Subtract 40 from B.S

$40 - 40 - 3y = 25 - 40$

$-3y = -15$

$3y = 15$

Divide B.S by 3

$\frac{3y}{3} = \frac{15}{3}$

$y = 5$

Verification

Put $x = 5$ in equ (i)

$22 = 17 + \sqrt{40 - 3(5)}$

$22 = 17 + \sqrt{40 - 15}$

$22 = 17 + \sqrt{25}$

$22 = 17 + 5$

$22 = 22$

Thus Solution Set $= \{5\}$

Chapter # 7

Ex # 7.3

Absolute value

The absolute value of a number is always be non–negative.

Example

$$|5| = 5$$

And also

$$|-5| = 5$$

Note:

It should be noted that $|x|$ can never be negative, that is $|x| \geq 0$

$|0| = 0$

Solution of Absolute value equation

To solve equations involving absolute value in one variable, we have to consider both the possible values of the variable.

Example

$$|x| = 2$$

Then there is two possibilities

$$x = 2$$

Or

$$x = -2$$

Example # 9

$\boldsymbol{|x - 1| = 7}$

Solution:

$|x - 1| = 7$

$\textit{There are two possibilities}$

$\textit{Either}$

$x - 1 = 7 \;\dots\dots equ(i)$

or

$x - 1 = -7 \;\dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$x - 1 = 7$

Add 1 on B.S

$x - 1 + 1 = 7 + 1$

$x = 8$

$Now\ equ(ii) \Longrightarrow$

$x - 1 = -7$

Add 1 on B.S

$x - 1 + 1 = -7 + 1$

$x = -6$

$\text{Solution Set} = \{8, -6\}$

Ex # 7.3

Example # 10

$\boldsymbol{|3x - 5| + 7 = 11}$

Solution:

$|3x - 5| + 7 = 11$

Subtract 7 from B.S

$|3x - 5| + 7 - 7 = 11 - 7$

$|3x - 5| = 4$

$\textit{There are two possibilities}$

$\textit{Either}$

$3x - 5 = 4 \;\dots\dots equ(i)$

or

$3x - 5 = -4 \;\dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$3x - 5 = 4$

Add 5 on B.S

$3x - 5 + 5 = 4 + 5$

$3x = 9$

$Divide\ B.S\ by\ 3$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

$Now\ equ(ii) \Longrightarrow$

$3x - 5 = -4$

Add 5 on B.S

$3x - 5 + 5 = -4 + 5$

$3x = 1$

$Divide\ B.S\ by\ 3$

$\frac{3x}{3} = \frac{1}{3}$

$x = \frac{1}{3}$

$\text{Solution Set} = \left\{3, \frac{1}{3}\right\}$

Exercise # 7.3

Page # 182

Q: 1. **Solve for $\boldsymbol{x}$**

$\boldsymbol{|x + 3| = 5}$

Solution:

$|x + 3| = 5$

$\textit{There are two possibilities}$

$\textit{Either}$

$x + 3 = 5 \;\dots\dots equ(i)$

or

$x + 3 = -5 \;\dots\dots equ(ii)$

Chapter # 7

Ex # 7.3

$Now\ equ(i) \Longrightarrow$

$x + 3 = 5$

Subtract 3 from B.S

$x + 3 - 3 -= 5 - 3$

$x = 2$

$Now\ equ(ii) \Longrightarrow$

$x + 3 = -5$

Subtract 3 from B.S

$x + 3 - 3 -= -5 - 3$

$x = -8$

Solution Set $= \{2, -8\}$

2. $|-5x + 1| = 6$

Solution:

$|-5x + 1| = 6$

$There\ are\ two\ possibilities$

$Either$

$-5x + 1 = 6\ \ \dots\dots equ(i)$

or

$-5x + 1 = -6\ \ \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$-5x + 1 = 6$

Subtract 1 from B.S

$-5x + 1 - 1 = 6 - 1$

$-5x = 5$

$Divide\ B.S\ by\ -5$

$$\frac{-5x}{-5} = \frac{5}{-5}$$

$x = -1$

$Now\ equ(ii) \Longrightarrow$

$-5x + 1 = -6$

Subtract 1 from B.S

$-5x + 1 - 1 = -6 - 1$

$-5x = -7$

$5x = 7$

$Divide\ B.S\ by\ 5$

$$\frac{5x}{5} = \frac{7}{5}$$

$$x = \frac{7}{5}$$

Solution Set $= \left\{-1, \frac{7}{5}\right\}$

Ex # 7.3

3. $\left|\frac{3}{4}x - 8\right| = 1$

Solution:

$\left|\frac{3}{4}x - 8\right| = 1$

$There\ are\ two\ possibilities$

$Either$

$\frac{3}{4}x - 8 = 1\ \ \dots\dots equ(i)$

or

$\frac{3}{4}x - 8 = -16\ \ \dots\dots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$\frac{3}{4}x - 8 = 1$

Add 8 on B.S

$\frac{3}{4}x - 8 + 8 = 1 + 8$

$\frac{3}{4}x = 9$

$Multiply\ B.S\ by\ \frac{4}{3}$

$$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times 9$$

$x = 4 \times 3$

$x = 12$

$Now\ equ(ii) \Longrightarrow$

$\frac{3}{4}x - 8 = -1$

Add 8 on B.S

$\frac{3}{4}x - 8 + 8 = -1 + 8$

$\frac{3}{4}x = 7$

$Multiply\ B.S\ by\ \frac{4}{3}$

$$\frac{4}{3} \times \frac{3}{4}x = \frac{4}{3} \times 7$$

$$x = \frac{28}{3}$$

Solution Set $= \left\{12, \frac{28}{3}\right\}$

Chapter # 7

Ex # 7.3

4. $|x-4|=3$

Solution:

$|x-4|=3$

There are two possibilities

Either

$x-4=3$ ……equ(i)

or

$x-4=-3$ ……equ(ii)

Now equ(i) $\Longrightarrow$

$x-4=3$

Add 4 on B.S

$x-4+4=3+4$

$x=7$

Now equ(ii) $\Longrightarrow$

$x-4=-3$

Add 4 on B.S

$x-4+4=-3+4$

$x=1$

Solution Set $=\{7,1\}$

5. $|3x+4|=-2$

Solution:

$|3x+4|=-2$

As there is no such a number whose absolute value is negative

Thus Solution Set $=\{\ \}$

6. $|2x-9|=0$

Solution:

$|2x-9|=0$

$|x|=0 \Longrightarrow x=0$

So

$2x-9=0$

Add 9 on B.S

$2x-9+9=0+9$

$2x=9$

Divide B.S by 2

$\frac{2x}{2}=\frac{9}{2}$

$x=\frac{9}{2}$

Solution Set $=\left\{\frac{9}{2}\right\}$

Ex # 7.3

7. $\left|\frac{3x-2}{5}\right|=7$

Solution:

$\left|\frac{3x-2}{5}\right|=7$

There are two possibilities

Either

$\frac{3x-2}{5}=7$ ……equ(i)

or

$\frac{3x-2}{5}=-7$ ……equ(ii)

Now equ(i) $\Longrightarrow$

$\frac{3x-2}{5}=7$

Multiply B.S by 5

$5\times\frac{3x-2}{5}=5\times 7$

$3x-2=35$

Add 2 on B.S

$3x-2+2=35+2$

$3x=37$

Divide B.S by 3

$\frac{3x}{3}=\frac{37}{3}$

$x=\frac{37}{3}$

Now equ(ii) $\Longrightarrow$

$\frac{3x-2}{5}=-7$

Multiply B.S by 5

$5\times\frac{3x-2}{5}=-7\times 5$

$3x-2=-35$

Add 2 on B.S

$3x-2+2=-35+2$

$3x=-33$

Divide B.S by 3

$\frac{3x}{3}=\frac{-33}{3}$

$x=-11$

Solution Set $=\left\{\frac{37}{3},-11\right\}$

Chapter # 7

Ex # 7.3

8. $\mathbf{4|5x-2|+3=11}$

Solution:

$4|5x-2|+3=11$

Subtract 3 from B.S

$4|5x-2|+3-3=11-3$

$4|5x-2|=8$

Divide B.S by 4

$\frac{4|5x-2|}{4}=\frac{8}{4}$

$|5x-2|=2$

There are two possibilities

Either

$5x-2=2 \quad \ldots\ldots equ(i)$

or

$5x-2=-2 \quad \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=2+2$

$5x=4$

Divide B.S by 5

$\frac{5x}{5}=\frac{4}{5}$

$x=\frac{4}{5}$

$Now\ equ(ii) \Longrightarrow$

$5x-2=2$

Add 2 on B.S

$5x-2+2=-2+2$

$5x=0$

Divide B.S by 5

$\frac{5x}{5}=\frac{0}{5}$

$x=0$

Solution Set $=\left\{\frac{4}{5},0\right\}$

9. $\mathbf{\frac{2}{5}|4x-3|-9=-1}$

Solution:

$\frac{2}{5}|4x-3|-9=-1$

Add 9 on B.S

$\frac{2}{5}|4x-3|-9+9=-1+9$

Ex # 7.3

$\frac{2}{5}|4x-3|=8$

Multiply B.S by $\frac{5}{2}$

$\frac{5}{2}\times\frac{2}{5}|4x-3|=\frac{5}{2}\times 8$

$|4x-3|=5\times 4$

$|4x-3|=20$

There are two possibilities

Either

$4x-3=20 \quad \ldots\ldots equ(i)$

or

$4x-3=-20 \quad \ldots\ldots equ(ii)$

$Now\ equ(i) \Longrightarrow$

$4x-3=20$

Add 3 on B.S

$4x-3+3=20+3$

$4x=23$

Divide B.S by 4

$\frac{4x}{4}=\frac{23}{4}$

$x=\frac{23}{4}$

$Now\ equ(ii) \Longrightarrow$

$4x-3=-20$

Add 3 on B.S

$4x-3+3=-20+3$

$4x=-17$

Divide B.S by 4

$\frac{4x}{4}=\frac{-17}{4}$

$x=\frac{-17}{4}$

Solution Set $=\left\{\frac{23}{4},\frac{-17}{4}\right\}$

Chapter # 7

Ex # 7.4

Linear Inequality

Inequality

The relation which compares two real numbers e.g. x and y but $x \neq y$.

Following symbols of inequality as under:

$<$ less than

$>$ greater than

$\leq$ less than or equal to

$\geq$ greater than or equal to

We have the following possibilities

$x < y$ *means that x less than y*

$x > y$ *means that x greater than y*

$x \leq y$ *means that x less than or equal to y*

$x \geq y$ *means that x is grater than or equal to y*

Solution of Linear Inequalities

The set of all possible values of the variable which makes the inequality a true statement is called solution set of the inequality.

It is simple to represent the solution of an inequality with the help on real number line.

Real Number Line

A line whose points are represented by real number is called real number line.

Geometrical representation with examples

Example: $\boldsymbol{x < 4}$

$x < 4$, it means that all real numbers less than 4. Geometrically all real numbers lying to the left of 4 but 4 is not included.

This is represented by using hollow circle around 4.

Example: $\boldsymbol{x \leq 4}$

$x \leq 4$, it means that all real numbers less than or equal to 4. Geometrically all real numbers lying to the left of 4 and also including 4.

This is represented by using thick, filled or solid circle around 4.

Ex # 7.4

Example # 11

Show $-2 < x < 5$ on a number line.

Solution:

$$-2 < x < 5$$

$-2 < x < 5$ means the set of real numbers which are greater than -2 but less than 5.

$-2 < x < 5$ means the set of real numbers which are between -2 and 5

Geometrical $-2 < x < 5$ means the set of real numbers lying to the right of -2 and left to 5.

Note:

Here -2 and 5 are not included.

Properties of Inequality of Real Numbers

Trichotomy Property

Trichotomy property means when comparing two numbers, one of the following must be true:

(a) $a = b$

(b) $a < b$

(c) $a > b$

Examples:

(i) $5 = 5$

(ii) $3 < 5$

(iii) $3 > 5$

Transitive Property

(a) If $a > b$ and $b > c$ then $a > c$

Example:

If $7 > 5$ and $5 > 3$ then $\mathbf{7} > 3$

(b) If $a < b$ and $b < c$ then $a < c$

Example:

If $3 < 5$ and $5 < 7$ then $\mathbf{3} < 7$

Additive Property

(a) If $a < b$ then $a + c < b + c$

(b) If $a < b$ then $a - c < b - c$

Examples:

(i) $3 < 5$ then $3 + 2 < 5 + 2$

(ii) $3 < 5$ then $3 - 2 < 5 - 2$

(iii) $\boldsymbol{x - 3 > 5}$

Add 3 on B.S

$x - 3 + 3 = 5 + 3$

$x = 8$

Chapter # 7

Ex # 7.4

(c) If $a > b$ then $a + c > b + c$

(d) If $a > b$ then $a - c > b - c$

Example:

(i) $5 > 3$ then $5 + 2 > 3 + 2$

(ii) $5 > 3$ then $5 - 7 > 3 - 7$ So $-2 > -4$

(iii) $\mathbf{x + 3 > 5}$

Subtract 3 from B.S

$x + 3 - 3 = 5 - 3$

$x = 2$

Multiplicative Property

1. When $c > 0$:

(a) If $a < b$ then $ac < bc$

(b) If $a > b$ then $ac > bc$

Example:

(i) $5 > 3$ then $5 \times 2 > 3 \times 2$

(ii) $\mathbf{\frac{x}{3} > 5}$

Multiply B.S by 3

$\frac{x}{3} \times 3 > 5 \times 3$

$x > 15$

(iii) $\mathbf{2x > 24}$

$\frac{2x}{2} > \frac{24}{2}$

Divide B.S by 2

$x > 12$

2. When $c < 0$:

(a) If $a < b$ then $ac > bc$

(b) If $a > b$ then $ac < bc$

Example:

(i) $5 > 3$ then $5 \times -2 < 3 \times -2$ So $-10 < -6$

(ii) $\frac{x}{-3} < 5$

Multiply B.S by -3

$\frac{x}{-3} \times -3 > 5 \times -3$

$x > -15$

Example # 12

Write the names of properties used in the following statements.

$21 < 31 \quad \Rightarrow \quad 31 < 41$

$21 < 31 \quad \Rightarrow \quad 21 + 10 < 31 + 10$

Hence $21 < 31 \quad \Rightarrow \quad 31 < 41$

Additive Property

Ex # 7.4

$\mathbf{15 > 8 \quad \Rightarrow \quad 22 > 15}$

Solution:

$15 > 8 \quad \Rightarrow \quad 15 + 7 > 8 + 7$

Hence $15 > 8 \quad \Rightarrow \quad 22 > 15$

Additive Property

$\mathbf{10 < 20 \quad \Rightarrow \quad 30 < 60}$

Solution:

$10 < 20 \quad \Rightarrow \quad 10 \times 3 < 20 \times 3$

Hence $10 < 20 \quad \Rightarrow \quad 30 < 60$

Multiplicative Property

$\mathbf{-12 > -15 \quad \Rightarrow \quad 24 < 30}$

Solution:

$-12 > -15 \quad \Rightarrow \quad -12 \times -2 < -15 \times -2$

Hence $-12 > -15 \quad \Rightarrow \quad 24 < 30$

Multiplicative Property

If $x > 4$ and $4 > z$ then $x > z$

Solution:

$x > 4$ and $4 > z \Rightarrow x > z$

Transitive Property

Solution of Linear Inequalities

Linear inequalities are solved in almost the same way as linear equations.

Principles in Inequalities

(i) If $a > b$, then

$a + c > b + c$, $a - c > b - c$, $a - b > 0$

(ii) If $a > b$ and $k > 0$, then

$ka > kb$ and $\frac{a}{k} > \frac{b}{k}$

(iii) If $a > b$ and $k < 0$, then

$ka < kb$ and $\frac{a}{k} < \frac{b}{k}$

Example # 13

You are checking a bag at an airport. Bags can weigh no more than 50 Kgs. Your bag weighs 16.8 kg. Find the possible weight w (in Kg) that you can add to the bag.

Solution:

$Bag's\ weight + weight\ you\ can\ add \leq weight\ limit$

$16.8 + W \leq 50$

Subtract 16.8 from B.S

$16.8 - 16.8 + W \leq 50 - 16.8$

$W \leq 33.2$

So we can add upto 33.2 Kg

Chapter # 7

Ex # 7.4

Example # 14 (i)

Solve the inequality $2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

where x is a natural number

Solution:

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$

$\frac{x+4}{2}<\frac{3}{2}$

Multiply B.S by 2

$2\times\frac{x+4}{2}<2\times\frac{3}{2}$

$x+4<3$

Subtract 4 from B.S

$x+4-4<3-4$

$x<-1$

As natural number cannot be less than -1,
then it has no solution

Thus, Solution Set $=\{\ \}$

Example # 14 (ii)

Solve the inequality $2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

where x is a real number

Solution:

$2\left(\frac{x}{4}+1\right)<\frac{3}{2}$

$2\left(\frac{x+4}{4}\right)<\frac{3}{2}$

$\frac{x+4}{2}<\frac{3}{2}$

Multiply B.S by 2

$2\times\frac{x+4}{2}<2\times\frac{3}{2}$

$x+4<3$

Subtract 4 from B.S

$x+4-4<3-4$

$x<-1$

Thus it consists of all real numbers less than -1

Thus Solution Set $=\{x : x \epsilon R \wedge x<-1\}$

Ex # 7.4

Example # 15 (i)

Solve the inequality $x-\frac{5}{7}\leq\frac{15+2x}{7}$

where x is a natural number

Solution:

$x-\frac{5}{7}\leq\frac{15+2x}{7}$

$\frac{7x-5}{7}\leq\frac{15+2x}{7}$

Multiply B.S by 7

$7\times\frac{7x-5}{7}\leq 7\times\frac{15+2x}{7}$

$7x-5\leq 15+2x$

Add 5 on B.S

$7x-5+5\leq 15+5+2x$

$7x\leq 20+2x$

Subtract $2x$ from B.S

$7x-2x\leq 20+2x-2x$

$5x\leq 20$

Divide B.S by 5

$\frac{5x}{5}\leq\frac{20}{5}$

$x\leq 4$

As x is natural number and less than or equal to 4

Thus Solution Set $=\{1,2,3,4\}$

Example # 15 (ii)

Solve the inequality $x-\frac{5}{7}\leq\frac{15+2x}{7}$

where x is a real number

Solution:

$x-\frac{5}{7}\leq\frac{15+2x}{7}$

$\frac{7x-5}{7}\leq\frac{15+2x}{7}$

Multiply B.S by 7

$7\times\frac{7x-5}{7}\leq 7\times\frac{15+2x}{7}$

$7x-5\leq 15+2x$

Add 5 on B.S

$7x-5+5\leq 15+5+2x$

$7x\leq 20+2x$

Chapter # 7

Ex # 7.4

Subtract $2x$ from B.S

$7x - 2x \leq 20 + 2x - 2x$

$5x \leq 20$

Divide B.S by 5

$\frac{5x}{5} \leq \frac{20}{5}$

$x \leq 4$

Thus it consists of all real numbers less than or equal to 4

Thus Solution Set $= \{x : x \epsilon R \wedge x \leq 4\}$

Example # 16

Solve the inequality $\frac{x+3}{2} \leq \frac{x-5}{3}$ ***where*** $x \in R$

Solution:

$\frac{x+3}{2} \leq \frac{x-5}{3}$

Multiply B.S by 6

$6 \times \frac{x+3}{2} \leq 6 \times \frac{x-5}{3}$

$3(x+3) \leq 2(x-5)$

$3x + 9 \leq 2x - 10$

Subtract 9 from B.S

$3x + 9 - 9 \leq 2x - 10 - 9$

$3x \leq 2x - 19$

Subtract $2x$ from B.S

$3x - 2x \leq 2x - 2x - 19$

$x \leq -19$

Thus it consists of all real numbers less than or equal to -19

Thus Solution Set $= \{x : x \epsilon R \wedge x \leq -19\}$

Exercise # 7.4

Page # 188

Q1: Show the following inequalities on number line.

(i) $x > 0$

Solution:

$x > 0$

(ii) $x < 0$

Solution:

$x < 0$

(iii) $\frac{x-3}{2} \leq -1$

Solution:

$\frac{x-3}{2} \leq -1$

Multiply B.S by 2

$2 \times \frac{x-3}{2} \leq -1 \times 2$

$x - 3 \leq -2$

Add 3 on B.S

$x - 3 + 3 \leq -2 + 3$

$x \leq 1$

(v) $x \leq -5$

Solution:

$x \leq -5$

$x \geq -3$

Solution:

$x \geq -3$

Chapter # 7

Ex # 7.4

(vi) $\frac{3x-2}{6} > \frac{5}{2}$

Solution:

$\frac{3x-2}{6} > \frac{5}{2}$

Multiply B.S by 6

$6 \times \frac{3x-2}{6} > 6 \times \frac{5}{2}$

$3x - 2 > 3 \times 5$

$3x - 2 > 15$

Add 2 on B.S

$3x - 2 + 2 > 15 + 2$

$3x > 17$

Divide B.S by 3

$\frac{3x}{3} > \frac{17}{3}$

$x > 5.67$

(vii) $-5 \leq x \leq 6$

Solution:

$-5 \leq x \leq 6$

(viii) $3 \geq x \geq -2$

Solution:

$3 \geq x \geq -2$

(ix) $0 < \frac{x}{4} - 1 < \frac{1}{2}$

Solution:

$0 < \frac{x}{4} - 1 < \frac{1}{2}$

Multiply by 4

$4 \times 0 < 4\left(\frac{x}{4} - 1\right) < 4 \times \frac{1}{2}$

$0 < 4 \times \frac{x}{4} - 4 \times 1 < 2 \times 1$

$0 < x - 4 < 2$

Ex # 7.4

Add 4

$0 + 4 < x - 4 + 4 < 2 + 4$

$4 < x < 6$

(x) $0 < \frac{x+3}{2} < \frac{3}{2}$

Solution:

$0 < \frac{x+3}{2} < \frac{3}{2}$

Multiply by 2

$2 \times 0 < 2 \times \frac{x+3}{2} < 2 \times \frac{3}{2}$

$0 < x + 3 < 3$

Subtract 3

$0 - 3 < x + 3 - 3 < 3 - 3$

$-3 < x < 0$

Q2: Find the solution set of the following inequalities.

(i) $7 - 2x \geq 1, \quad x \in N$

Solution:

$7 - 2x \geq 1, \quad x \in N$

Now

$7 - 2x \geq 1$

Subtract 7 from B.S

$7 - 7 - 2x \geq 1 - 7$

$-2x \geq -6$

Divide B.S by -2

$\frac{-2x}{-2} \leq \frac{-6}{-2}$

$x \leq 3$

As $x \in N$ *and* $x \leq 3$

Thus Solution Set $= \{1, 2, 3\}$

(ii) $5x + 4 < 34, \quad x \in N$

Solution:

$5x + 4 < 34, \quad x \in N$

Now

$5x + 4 < 34$

Subtract 4 from B.S

$5x + 4 - 4 < 34 - 4$

$5x < 30$

Chapter # 7

Ex # 7.4

Divide B.S by 5

$\frac{5x}{5} < \frac{30}{5}$

$x < 6$

$As \ \ x \, \epsilon \, N \ and \ x < 6$

Thus Solution Set $= \{1, 2, 3, 4, 5\}$

(iii) $\frac{\mathbf{8x+1}}{\mathbf{2}} < 2\boldsymbol{x} - \mathbf{1.5}\,, \quad \boldsymbol{x \, \epsilon \, R}$

Solution:

$\frac{8x+1}{2} < 2x - 1.5\,, \quad x \, \epsilon \, R$

Now

$\frac{8x+1}{2} < 2x - 1.5$

Multiply B.S by 2

$2 \times \frac{8x+1}{2} < 2(2x - 1.5)$

$8x + 1 < 4x - 3$

Now

$8x - 4x < -3 - 1$

$4x < -4$

Divide B.S by 4

$\frac{4x}{4} < \frac{-4}{4}$

$x < -1$

$As \ \ x \, \epsilon \, R \ and \ x < -1$

Thus Solution Set $= \{x : x \, \epsilon \, R \land x < -1\}$

(iv) $(\mathbf{4}\boldsymbol{x} + \mathbf{3}) \geq \mathbf{23}\,, \quad \boldsymbol{x \, \epsilon} \{\mathbf{1, 2, 3, 4, 5, 6}\}$

Solution:

$(4x + 3) \geq 23\,, \quad x \, \epsilon \, \{1, 2, 3, 4, 5, 6\}$

Now

$4x + 3 \geq 23$

Subtract 3 from B.S

$4x + 3 - 3 \geq 23 - 3$

$4x \geq 20$

Divide B.S by 4

$\frac{4x}{4} \geq \frac{20}{4}$

$x \geq 5$

$As \ \ x \, \epsilon \, \{1, 2, 3, 4, 5, 6\} \ and \ x \geq 5$

Thus Solution Set $= \{5, 6\}$

Ex # 7.4

(v) $\mathbf{5}\boldsymbol{x} + \mathbf{1}$
$\geq \mathbf{13} - \boldsymbol{x}\,, \quad \boldsymbol{x \, \epsilon} \{\mathbf{-2, -1, 0, 1, 2, 3, 4, 5}\}$

Solution:

$5x + 1 \geq 13 - x\,, \quad x \, \epsilon \, \{-2, -1, 0, 1, 2, 3, 4, 5\}$

Now

$5x + 1 \geq 13 - x$

Now

$5x + x \geq 13 - 1$

$6x \geq 12$

Divide B.S by 6

$\frac{6x}{6} \geq \frac{12}{6}$

$x \geq 2$

$As \ \ x \, \epsilon \ \{-2, -1, 0, 1, 2, 3, 4, 5\} \ and \ x \geq 2$

Thus Solution Set $= \{2, 3, 4, 5\}$

(vi) $\frac{\mathbf{2x+6}}{\mathbf{2}} \leq \frac{\boldsymbol{x} - \mathbf{9}}{\mathbf{3}}\,, \quad \boldsymbol{x \, \epsilon \, R}$

Solution:

$\frac{2x+6}{2} \leq \frac{x-9}{3}\,, \quad x \, \epsilon \, R$

Now

$\frac{2x+6}{2} \leq \frac{x-9}{3}$

Multiply B.S by 6

$6 \times \frac{2x+6}{2} \leq 6 \times \frac{x-9}{3}$

$3(2x + 6) \leq 2(x - 9)$

$6x + 18 \leq 2x - 18$

Now

$6x - 2x \leq -18 - 18$

$4x \leq -36$

Divide B.S by 4

$\frac{4x}{4} \leq \frac{-36}{4}$

$x \leq -9$

$As \ \ x \, \epsilon \, R \ and \ x \leq -9$

Thus Solution Set $= \{x : x \, \epsilon \, R \land x \leq -9\}$

(vii) $\frac{\boldsymbol{x} - \mathbf{1}}{\mathbf{3}} \leq \frac{\mathbf{1} - \boldsymbol{x}}{\mathbf{2}}\,, \quad \boldsymbol{x \, \epsilon \, Z}$

Solution:

$\frac{x-1}{3} \leq \frac{1-x}{2}\,, \quad x \, \epsilon \, Z$

Now

$\frac{x-1}{3} \leq \frac{1-x}{2}$

Chapter # 7

Ex # 7.4

Multiply B.S by 6

$6\times\frac{x-1}{3}\le 6\times\frac{1-x}{2}$

$2(x-1)\le 3(1-x)$

$2x-2\le 3-3x$

Now

$2x+3x\le 3+2$

$5x\le 5$

Divide B.S by 5

$\frac{5x}{5}\le\frac{5}{5}$

$x\le 1$

$As\ \ x\,\epsilon\, Z\ and\ x\le 1$

$\text{Thus Solution Set}=\{1,0,-1,-2,-3\ldots\ldots\}$

Q3: **Solve the following inequalities and plot the solution on the number line.**

(i) $\boldsymbol{\frac{x}{12}\le\frac{1}{4}}$

Solution:

$\frac{x}{12}\le\frac{1}{4}$

Multiply B.S by 12

$12\times\frac{x}{12}\le 12\times\frac{1}{4}$

$x\le 3\times 1$

$x\le 3$

(ii) $\boldsymbol{x+7\ge 2}$

Solution:

$x+7\ge 2$

Subtract 7 from B.S

$x+7-7\ge 2-7$

$x\ge -5$

Ex # 7.4

(iii) $\boldsymbol{3(x-2)}>15$

Solution:

$3(x-2)>15$

$3x-6>15$

Add 6 on B.S

$3x-6+6>15+6$

$3x>21$

Divide B.S by 3

$\frac{3x}{3}>\frac{21}{3}$

$x>7$

(iv) $\boldsymbol{\frac{1}{2}>\frac{x}{4}}>-2$

Solution:

$\frac{1}{2}>\frac{x}{4}>-2$

Multiply by 4

$4\times\frac{1}{2}>4\times\frac{x}{4}>-2\times 4$

$2\times 1>x>-8$

$2>x>-8$

(v) $\boldsymbol{2.5\le\frac{x}{2}+1\le 4.5}$

Solution:

$2.5\le\frac{x}{2}+1\le 4.5$

Multiply B.S by 2

$2\times 2.5\le 2\left(\frac{x}{2}+1\right)\le 2\times 4.5$

$5\le x+2\le 9$

Subtract 2 from them

$5-2\le x+2-2\le 9-2$

$3\le x\le 7$

(vi) $-2 \le x < 2$

Solution:

$-2 \le x < 2$

Review Ex # 7

Page # 190

Q2: **Solve the following equation for x**

(i) $5(3x+1) = 2(x-4)$

Solution:

$5(3x+1) = 2(x-4) \dots\dots equ(i)$

$15x + 5 = 2x - 8$

Subtract 5 from B.S

$15x + 5 - 5 = 2x - 8 - 5$

$15x = 2x - 13$

Subtract $2x$ from B.S

$15x - 2x = 2x - 2x - 13$

$13x = -13$

Divide B.S by 13

$\frac{13x}{13} = \frac{-13}{13}$

$x = -1$

Verification

Put $x = -1$ in equ (i)

$5(3(-1)+1) = 2(-1-4)$

$5(-3+1) = 2(-5)$

$5(-2) = -10$

$-10 = -10$

Solution Set $= \{-1\}$

(ii) $\frac{x-8}{3} + \frac{x-3}{2} = 0$

Solution:

$\frac{x-8}{3} + \frac{x-3}{2} = 0 \dots\dots equ(i)$

Multiply all terms by 6

$6 \times \frac{x-8}{3} + 6 \times \frac{x-3}{2} = 6 \times 0$

$2(x-8) + 3(x-3) = 0$

$2x - 16 + 3x - 9 = 0$

$2x + 3x - 16 - 9 = 0$

$5x - 25 = 0$

Add 25 on B.S

$5x - 25 + 25 = 0 + 25$

$5x = 25$

Divide B.S by 5

$\frac{5x}{5} = \frac{25}{5}$

$x = 5$

Verification

Put $x = 5$ in equ (i)

$\frac{5-8}{3} + \frac{5-3}{2} = 0$

$\frac{-3}{3} + \frac{2}{2} = 0$

$-1 + 1 = 0$

$0 = 0$

Solution Set $= \{5\}$

(iii) $\sqrt{2(5x-1)} = \sqrt{2x+14}$

Solution:

$\sqrt{2(5x-1)} = \sqrt{2x+14}$

$\sqrt{2(5x-1)} = \sqrt{2x+14} \dots\dots equ(i)$

Take square root on B.S

$\left(\sqrt{2(5x-1)}\right)^2 = \left(\sqrt{2x+14}\right)^2$

$2(5x-1) = 2x + 14$

$10x - 2 = 2x + 14$

Now

$10x - 2x = 14 + 2$

$8x = 16$

Divide B.S by 4

$\frac{8\sqrt{x}}{8} = \frac{16}{8}$

$\sqrt{x} = 2$

Taking square on B.S

$\left(\sqrt{x}\right)^2 = (2)^2$

$x = 4$

Verification

Put $x = 2$ in equ (i)

$\sqrt{2(5(2)-1)} = \sqrt{2(2)+14}$

$\sqrt{2(10-1)} = \sqrt{4+14}$

$\sqrt{2(9)} = \sqrt{18}$

$\sqrt{18} = \sqrt{18}$

$\sqrt{9 \times 2} = \sqrt{9 \times 2}$

$3\sqrt{2} = 3\sqrt{2}$

Solution Set $= \{36\}$

Chapter # 7

Review Ex # 7

(iv) $|2x + 7| = 9$

Solution:

$|2x + 7| = 9$

There are two possibilities

Either

$2x + 7 = 9 \quad \dots \dots equ(i)$

or

$2x + 7 = -9 \quad \dots \dots equ(ii)$

Now $equ(i) \Longrightarrow$

$2x + 7 = 9$

Subtract 7 from B.S

$2x + 7 - 7 = 9 - 7$

$2x = 2$

Divide B.S by 2

$$\frac{2x}{2} = \frac{2}{2}$$

$x = 1$

Now $equ(ii) \Longrightarrow$

$2x + 7 = -9$

Subtract 7 from B.S

$2x + 7 - 7 = -9 - 7$

$2x = -16$

Divide B.S by 2

$$\frac{2x}{2} = \frac{-16}{2}$$

$x = -8$

Solution Set $= \{1, -8\}$

Q3: Solve the following inequalities and graph the solution on the number line.

(i) $-1 < \dfrac{x-3}{2} < 0$

Solution:

$$-1 < \frac{x-3}{2} < 0$$

Multiply by 2

$$-1 \times 2 < 2 \times \frac{x-3}{2} < 2 \times 0$$

$-2 < x - 3 < 0$

Add 3

$-2 + 3 < x - 3 + 3 < 0 + 3$

$1 < x < 3$

Review Ex # 7

(ii) $-1 < \dfrac{x-4}{5} < 0$

Solution:

$$-1 < \frac{x-4}{5} < 0$$

Multiply by 5

$$-1 \times 5 < 5 \times \frac{x-4}{5} < 5 \times 0$$

$-5 < x - 4 < 0$

Add 4

$-5 + 4 < x - 4 + 4 < 0 + 4$

$-1 < x < 4$

(iii) $7 < -3x + 1 \le 13$

Solution:

$7 < -3x + 1 \le 13$

Subtract 1

$7 - 1 < -3x + 1 - 1 \le 13 - 1$

$6 < -3x \le 12$

Divide B.S by 3

$$\frac{6}{-3} > \frac{-3x}{-3} \ge \frac{12}{-3}$$

$-2 > x \le -4$

Chapter # 7

Review Ex # 7

Q4: A father is 4 times older than his son. In 20 years, he will be twice as old as his son. What ages have they now?

Solution:

Let the present age of son $= x$ years

So the present age of father $= 4x$ years

After twenty years

Age of son $= (x + 20) years$

and age of son $= (4x + 20) years$

According to condition

Age of father $= 2$(Age of son)

$4x + 20 = 2(x + 20)$

$4x + 20 = 2x + 40$

Now shift the variable and constant

$4x - 2x = 40 - 20$

$2x = 20$

Divide B.S by 2

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

Thus present age of son $= x = 10\ years$

And present age of father $= 4x\ years$

$= 4 \times 10\ years$

$= 40\ years$