Chapter # 6

UNIT # 6

ALGEBRAIC MANIPULATIONS

Ex # 6.1

Highest Common Factor (H.C.F)

The highest number of factors common to all given expressions or polynomials is called Highest Common Factor (H.C.F)

In other words, H.C.F of two or more polynomials is a polynomial of the highest degree, which divides exactly the given polynomials.

There are two methods for finding H.C.F.

(i) H.C.F by Factorization

(ii) H.C.F by Division

H.C.F by Factorization

In this method, first factorize all the given expressions

Then we take all possible common factors which is the H.C.F of the given expression.

Example # 1

Find H.C.F of $x^2-y^2,\ x^2-xy$

Solution:

$x^2-y^2,\ x^2-xy$

$x^2-y^2=(x+y)(x-y)$

And

$x^2-xy=x(x-y)$

Here $x-y$ is a common factor. Thus

$\mathrm{H.C.F}=\mathrm{x}-\mathrm{y}$

Example # 2

Find H.C.F of $ax^2+5ax+6a$,

$ax^3+9ax^2+14ax\ \ and\ \ 15a(x^2-4)$

Solution:

$ax^2+5ax+6a,\ \ ax^3+9ax^2+14ax\ and$

$15a(x^2-4)$

$ax^2+5ax+6a=a(x^2+5x+6)$

$ax^2+5ax+6a=a(x^2+2x+3x+6)$

$ax^2+5ax+6a=a[x(x+2)+3(x+2)]$

$ax^2+5ax+6a=a(x+2)(x+3)$

And

$ax^3+9ax^2+14ax=ax(x^2+9x+14)$

$ax^3+9ax^2+14ax=ax(x^2+2x+7x+14)$

$ax^3+9ax^2+14ax=ax[x(x+2)+7(x+2)]$

$ax^3+9ax^2+14ax=ax(x+2)(x+7)$

Ex # 6.1

Now also

$15a(x^2-4)=3\times5.a[(x)^2-(2)^2]$

$15a(x^2-4)=3\times5.a(x+2)(x-2)$

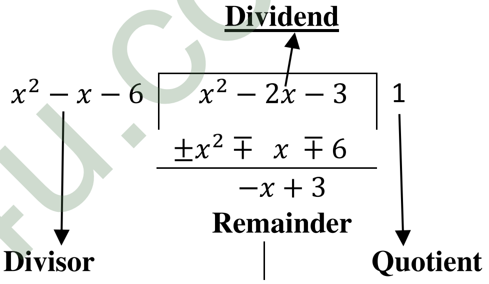
Here $a(x+2)$ is common in given three expressions.

$\mathrm{H.C.F}=\mathrm{a}(x+2)$

Note:

The H.C.F $\mathrm{a}(x+2)$ exactly divides all the given three expression

H.C.F by Division Method



Steps

1 Write the expressions in descending order

2 Take the common from the expressions if any.

3 Divide higher degree polynomial by the polynomial of lower degree

4 Divide to that time till the degree of remainder is less than the degree of divisor.

5 Now bring down the divisor and divide by remainder BUT before this take the common from the remainder if any.

6 Repeat the above steps till the remainder is zero.

7 Last divisor is the H.C.F of the given polynomials.

Note:

1 In H.C.F by division, if required, multiply the expression by a suitable integer to avoid fraction.

2 To find the H.C.F of three polynomials, first find H.C.F of any two of them, then find H.C.F of this H.C.F and the third polynomial.

$(x^2)(6)=6x^2$

Add	Multiply
$+2x$	$+2x$
$+3x$	$+3x$
$\boldsymbol{+5x}$	$\boldsymbol{6x^2}$

$(x^2)(14)=14x^2$

Add	Multiply
$+2x$	$+2x$
$+7x$	$+7x$
$\boldsymbol{+9x}$	$\boldsymbol{14x^2}$

Chapter # 6

Ex # 6.1

H.C.F by Division method in Urdu

1. تمام variables کو descending order میں لکھیں گے۔
2. اگر کوئی common ہو تو پہلے common لینگے۔
3. بڑے expression کو چھوٹے expression پر divide کریں گے۔
4. اس کو اس وقت تک divide کرتے رہیں گے جب تک remainder میں power ہمارے ساتھ divisor کے power سے کم نہ آئے
5. پھر divisor کو نیچے لائیں گے اور remainder پر divide کریں گے لیکن اس سے پہلے remainder میں common لیں گے اگر ہو۔
6. ان steps کو اس وقت تک کرو گے جب تک remainder میں zero نہ آئے۔
6. آخری divisor ہمارے ساتھ H.C.F ہوگا۔

Example # 3

Find H.C.F of $2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

Solution:

$2x^3+7x^2+4x-4$ and $2x^3+9x^2+11x+2$

$$\begin{array}{rrrr}
2x^3+7x^2+4x-4 \,\Big)\, & 2x^3+9x^2+11x+2 \,\Big(\, 1 & & \\
 & \pm 2x^3 \pm 7x^2 \pm 4x \mp 4 & & \\
\hline
 & 2x^2+7x+6 \,\Big)\, & 2x^3+7x^2+4x-4 \,\Big(\, x & \\
 & & \pm 2x^3 \pm 7x^2 \pm 6x & \\
\hline
 & & -2 \,\big|\, -2x-4 & \text{Dividing by } -2 \\
\hline
 & & x+2 \,\Big)\, & 2x^2+7x+6 \,\Big(\, 2x+3 \\
 & & & \pm 2x^2 \pm 4x \\
\hline
 & & & 3x+6 \\
 & & & \pm 3x \pm 6 \\
\hline
 & & & \times
\end{array}$$

Hence H.C.F$= x+2$

Note:

H.C.F by Factorization

H.C.F of 24 and 32

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 24 = 1, 2, 4, 8, 16, 32

Common factors $= 1, 2, 4, 8$

$\text{H.C.F} = 8$

Chapter # 6

Ex # 6.1

Example # 4

Find H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

Solution:

$x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$

$3x^3-5x^2+6x-4\,\big)\,3x^3+5x^2-6x-2\,\big(\,1$

$\pm3x^3\mp5x^2\pm6x\mp4$

$2\,\big)\,10x^2-12x+2$ **Dividing by 2**

$5x^2-6x+1\,\big)\,3x^3-5x^2+6x-4\,\big(\,3x-7$

$\times 5$ **Multiplying by 5**

$15x^3-25x^2+30x-20$

$\pm15x^3\mp18x^2\pm3x$

$-7x^2+27x-20$

$\times 5$ **Multiplying by 5**

$-35x^2+135x-100$

$\mp35x^2\pm42x\mp7$

$93\,\big)\,93x-93$ **Dividing by 93**

$x-1\,\big)\,5x^2-6x+1\,\big(\,5x-1$

$\pm5x^2\mp5x$

$-x+1$

$\mp x\pm1$

$\times$

Hence H.C.F= $x-1$

Now find the H.C.F of $x-1$ and $x^3-6x^2+11x-6$

$x-1\,\big)\,x^3-6x^2+11x-6\,\big(\,x^2-5x+6$

$\pm x^3\mp x^2$

$-5x^2+11x-6$

$\mp5x^2\pm5x$

$6x-6$

$\pm6x\mp6$

$\times$

Hence the required H.C.F of $x^3-6x^2+11x-6$, $3x^3-5x^2+6x-4$ and $2x^3+9x^2+11x+2$ is $\boldsymbol{x-1}$

Least Common Multiple (L.C.M)

The polynomial of least degree which is divisible by the given polynomials.

There are two methods of finding L.C.M

(a) L.C.M by factorization

(b) L.C.M by formula

Ex # 6.1

(a) **L.C.M by factorization**

In this method, first factorize all the given expressions

Then find the L.C.M by given formula.

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

Example # 5

Find L.C.M of $x^2 + 4x + 4$ and $x^2 + 5x + 6$

Solution:

$x^2 + 4x + 4$ and $x^2 + 5x + 6$

$x^2 + 4x + 4 = (x)^2 + 2(x)(2) + (2)^2$

$x^2 + 4x + 4 = (x + 2)^2$

$x^2 + 4x + 4 = (x + 2)(x + 2)$

Now

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

$x^2 + 5x + 6 = x(x + 2) + 3(x + 2)$

$x^2 + 5x + 6 = (x + 2)(x + 3)$

$Common\ Factor = x + 2$

$Non - common\ factor = (x + 2)(x + 3)$

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x + 2)(x + 2)(x + 3)$

$L.C.M = (x + 2)^2(x + 3)$

Example # 6

Find L.C.M of $x^2 - 4x + 3$, $x^2 - 3x + 2$ and $x^2 - 5x + 6$

Solution:

$x^2 - 4x + 3$, $x^2 - 3x + 2$ and $x^2 - 5x + 6$

$x^2 - 4x + 3 = x^2 - x - 3x + 3$

$x^2 - 4x + 3 = x(x - 1) - 3(x - 1)$

$x^2 - 4x + 3 = (x - 1)(x - 3)$….. (i)

Now

$x^2 - 3x + 2 = x^2 - x - 2x + 3$

$x^2 - 3x + 2 = x(x - 1) - 2(x - 1)$

$x^2 - 3x + 2 = (x - 1)(x - 2)$….. (ii)

Now

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$x^2 - 5x + 6 = x(x - 2) - 3(x + 2)$

$x^2 - 5x + 6 = (x - 2)(x - 3)$….. (iii)

$x - 1$ in expression (i)& (ii)

$x - 2$ in expression (ii)& (iii)

$x - 3$ in expression (i)& (iii)

Therefore:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L.C.M = (x - 1)(x - 2)(x - 3) \times 1$

$L.C.M = (x - 1)(x - 2)(x - 3)$

Ex # 6.1

L.C.M Theorem:

If A and B are given polynomials and their H.C.F and L.C.M are represented by H and L respectively, then

$$A \times B = H \times L$$

Proof:

Since H is common factor of polynomial of A and B, then it divides exactly A and B. So

$\frac{A}{H} = a$

$A = Ha\ldots$ equ(i)

and

$\frac{B}{H} = b$

$B = Hb\ldots$ equ(ii)

As a and b have no common factor.

As we know that:

$\boldsymbol{L.C.M = common\ factor \times non - common\ factor}$

$L = H \times a \times b$

Multiply B.S by H

$L \times H = H \times a \times b \times H$

$L \times H = (Ha) \times (Hb)$

Put equ(i) and equ(ii), we get

$L \times H = A \times B$

Or

$H \times L = Product\ of\ two\ polynomials$

Formula for L.C.M

As $L \times H = A \times B$

$L = \frac{A \times B}{H}$

$L.C.M = \frac{Product\ of\ two\ polynomials}{H.C.F}$

Chapter # 6

Ex # 6.1

Example # 7

Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3

$x^3-6x^2+11x-6$ and x^3-4x+3

Solution:

$Let\ A=x^3-6x^2+11x-6$

$and\ B=x^3-4x+3$

As we have:

$$L.C.M=\frac{A\times B}{H.C.F}--equ\ (i)$$

First we find H.C.F

$$x^3-4x+3\ \big)\ x^3-6x^2+11x-6\ \big(\ 1$$
$$\pm x^3 \quad \mp 4x \ \pm 3$$
$$-3\ \big)\ -6x^2+15x-9$$
$$2x^2-5x+3\ \big)\ x^3-4x+3\ \big(\ x+5$$
$$\times 2$$
$$2x^3-8x+6$$
$$\pm 2x^3 \pm 3x \quad \mp 5x^2$$
$$5x^2-11x+6$$
$$\times 2$$
$$10x^2-22x+12$$
$$\pm 10x^2 \mp 25x \pm 15$$
$$3\ \big)\ 3x-3$$
$$x-1\ \big)\ 2x^2-5x+3\ \big(\ 2x-3$$
$$\pm 2x^2 \mp 2x$$
$$-3x+3$$
$$\mp 3x \pm 3$$
$$\times$$

$H.C.F=x-1$

Now put the values in equ (i)

$$L.C.M=\frac{(x^3-6x^2+11x-6)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$x^2-5x+6$$
$$x-1\ \big)\ x^3-6x^2+11x-6$$
$$\pm x^3 \ \mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

$So\ L.C.M=(x^2-5x+6)(x^3-4x+3)$

Chapter # 6

Ex # 6.1

Example # 8

Find H.C.F and L.C.M of $3x^3-2x^2-3x+2$ and $6x^3-7x^2-x+2$

$3x^3-2x^2-3x+2$ and $6x^3-7x^2-x+2$

Solution:

$Let\ A = 3x^3-2x^2-3x+2$

$and\ B = 6x^3-7x^2-x+2$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$
\begin{array}{rrl}
3x^3-2x^2-3x+2 & \overline{)\,6x^3-7x^2-\ x\ +\ 2\,} & 2\\
& \pm 6x^3 \mp 4x^2 \mp\ 6x\ \pm 4 & \\
\hline
-1 & \overline{)\,-3x^2+5x-2} & \\
3x^2-5x+2 & \overline{)\,3x^3-2x^2-3x+2} & x+1\\
& \pm 3x^3 \mp 5x^2 \pm 2x & \\
\hline
& 3x^2-5x+2 & \\
& \pm 3x^2 \mp 5x \pm 2 & \\
\hline
& \times &
\end{array}
$$

$$H.C.F = 3x^2-5x+2$$

Now put the values in equ (i)

$$L.C.M = \frac{(3x^3-2x^2-3x+2)(6x^3-7x^2-x+2)}{3x^2-5x+2}$$

Now by Simple Division

$$
\begin{array}{rl}
& x+1\\
3x^2-5x+2 & \overline{)\,3x^3-2x^2-3x+2}\\
& \pm 3x^3 \mp 5x^2 \pm 2x\\
\hline
& 3x^2-5x+2\\
& \pm 3x^2 \mp 5x \pm 2\\
\hline
& \times
\end{array}
$$

$So\ L.C.M = (x+1)(6x^3-7x^2-x+2)$

Example # 9

If H.C.F and L.C.M of two polynomials are $x-3$ and $x^3-9x^2+26x-24$ respectively. Find the second polynomial when one polynomial is x^2-5x+6.

Solution:

$H.C.F = x-3$

$L.C.M = x^3-9x^2\_26x-24$

$Let\ First\ polynomial = A = x^2-5x+6$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F}$$

$A \times B = L.C.M \times H.C.F$

Ex # 6.1

$$B = \frac{L.C.M \times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3 - 9x^2\_26x - 24)(x-3)}{x^2 - 5x + 6}$$

Now by simple Division

$$
\begin{array}{r|l}
 & x - 4 \\
x^2 - 5x + 6 & x^3 - 9x^2 + 26\,x - 24 \\
 & \pm x^3 \mp 5x^2 \pm 6x \\
\hline
 & -4x^2 + 20x - 24 \\
 & \mp 4x^2 \pm 20x \mp 24 \\
\hline
 & \times
\end{array}
$$

$So\ B = (x-4)(x-3)$

$B = x^2 - 3x - 4x + 12$

$B = x^2 - 7x + 12$

Hence the second polynomial is $x^2 - 7x + 12$

Example # 10

If H.C.F and L.C.M of two polynomials are $x - 1$ and $x^3 + 4x^2 + x - 6$ respectively. Find the polynomials of degree 2.

Solution:

$H.C.F = x - 1$

$L.C.M = x^3 + 4x^2 + x - 6$

$First\ polynomial = A = ?$

$Second\ polynomial = B = ?$

$As\ H.C.F = x - 1$

$then\ it\ is\ also\ the\ factor\ of\ L.C.M$

Now

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
x - 1 & x^3 + 4x^2 + x - 6 \\
 & \pm x^3 \mp x^2 \\
\hline
 & 5x^2 + x - 6 \\
 & \pm 5x^2 \mp 5x \\
\hline
 & 6x - 6 \\
 & \pm 6x \mp 6 \\
\hline
 & \times
\end{array}
$$

$L.C.M = x^3 + 4x^2 + x - 6$

$L.C.M = (x-1)(x^2 + 5x + 6)$

$L.C.M = (x-1)(x^2 + 3x + 2x + 6)$

$L.C.M = (x-1)[x(x+3) + 2(x+3)]$

$L.C.M = (x-1)(x+3)(x+2)$

$As\ x - 1\ is\ common\ factor. So$

$A = (x-1)(x+3)$

Ex # 6.1

$A = x^2 + 2x - 3$

And

$B = (x-1)(x+2)$

$B = x^2 + 2x - 1x - 2$

$B = x^2 + x - 2$

Example # 11

The sum of two numbers is 120 and their H.C.F is 12. Find the numbers.

Solution:

Let x and y be the two numbers.

As H.C.F is 12, means 12 is common factor.

So, it becomes

$12x + 12y = 120$

$12(x + y) = 120$

Divide B.S by 12, we get

$x + y = 12$

As the sum of two numbers is 10, so the possible pairs of numbers are (1,9), (2,8), (3,7), (4,6), (5,5)

As (1,9), (3,7) are non commo factors

Then the required numbers are:

$1 \times 12 = 12$ and $9 \times 12 = 108$

OR

$3 \times 12 = 36$ and $7 \times 12 = 84$

Chapter # 6

Exercise# 6.1

Page # 159-160

Q1: 159 **Find H.C.F of the following expression by factorization method.**

(i) $(\boldsymbol{x+y})^2$ **and** $\boldsymbol{x^2-36}$

Solution:

$(x+y)^2$ and x^2-36

$(x+y)^2 = (x+y)(x+y)$

And

$x^2-36 = (x)^2-(6)^2$

$= (x+6)(x-6)$

$H.C.F = x-6$

(iii) $\boldsymbol{x-3, x^2-9, (x-3)^2}$

Solution:

$x-3, x^2-9, (x-3)^2$

$x-3 = x-3$

And

$x^2-9 = (x)^2-(3)^2$

$= (x+3)(x-3)$

And

$(x-3)^2 = (x-3)(x-3)$

$H.C.F = \ x-3$

(ii) $\boldsymbol{x^4-y^4}$ **and** $\boldsymbol{x^4+2x^2y^2+y^4}$

Solution:

x^4-y^4 and $x^4+2x^2y^2+y^4$

$x^4-y^4 = (x^2)^2-(y^2)^2$

$= (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

And

$x^4+2x^2y^2+y^4 = (x^2)^2+2(x^2)(y^2)+(y^2)^2$

$= (x^2+y^2)^2$

$= (x^2+y^2)(x^2+y^2)$

$H.C.F = x^2+y^2$

(v) $\boldsymbol{2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3}$

Solution:

$2x^4-2y^4, 6x^2+12xy+6y^2, 9x^3+9y^3$

$2x^4-2y^4 = 2[(x^2)^2-(y^2)^2]$

$= 2(x^2+y^2)(x^2-y^2)$

$= 2(x^2+y^2)(x+y)(x-y)$

And

$6x^2+12xy+6y^2 = 6(x^2+2xy+y^2)$

$= 2\times 3(x+y)^2$

$= 2\times 3(x+y)(x+y)$

And

$9x^3+9y^3 = 9(x^3+y^3)$

$= 9(x+y)(x^2-xy+y^2)$

$H.C.F = x+y$

(iv) $\boldsymbol{2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)}$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2 = 2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3 = 2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y) = 3.3(x-y)(x-y)(x+2y)$

$H.C.F = 3.3(x-y)(x-y)(x+2y)$

$H.C.F = 3^2(x-y)^2(x+2y)$

Chapter # 6

Ex # 6.1

Q2: Find H.C.F by division method.

160

(i) $x^2 - x - 6$ **and** $x^2 - 2x - 3$

Solution:

$x^2 - x - 6$ and $x^2 - 2x - 3$

$$
\begin{array}{rll}
x^2 - x - 6 \,\big) & x^2 - 2x - 3 & \big(\, 1 \\
 & \pm x^2 \mp x \mp 6 & \\ \hline
-1 \,\big) & -x + 3 & \\
x - 3 \,\big) & x^2 - x - 6 & \big(\, x + 2 \\
 & \pm x^2 \mp 3x & \\ \hline
 & 2x - 6 & \\
 & \pm 2x \mp 6 & \\ \hline
 & \times &
\end{array}
$$

$H.C.F = x - 3$

(ii) $y^3 - 3y + 2$ **and** $y^3 - 5y^2 + 7y - 3$

Solution:

$y^3 - 3y + 2$ and $y^3 - 5y^2 + 7y - 3$

$$
\begin{array}{rll}
y^3 - 3y + 2 \,\big) & y^3 - 5y^2 + 7y - 3 & \big(\, 1 \\
 & \pm y^3 \qquad \mp 3y \pm 2 & \\ \hline
-5 \,\big) & -5y^2 + 10y - 5 & \\
y^2 - 2y + 1 \,\big) & y^3 - 3y + 2 & \big(\, y + 2 \\
 & \pm y^3 \pm 1y \qquad \mp 2y^2 & \\ \hline
 & 2y^2 - 4y + 2 & \\
 & \pm 2y^2 \mp 4y \pm 2 & \\ \hline
 & \times &
\end{array}
$$

$H.C.F = \ y^2 - 2y + 1$

Chapter # 6

Ex # 6.1

(iii) **$2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$**

Solution:

$2x^5 - 4x^4 - 6x$ and $x^5 + x^4 - 3x^3 - 3x^2$

$2x^5 - 4x^4 - 6x = 2x(x^4 - 2x^3 - 3)$

$x^5 + x^4 - 3x^3 - 3x^2 = x^2(x^3 + x^2 - 3x - 3)$

$= x.x(x^3 + x^2 - 3x - 3)$

$x^3 + x^2 - 3x - 3 \,\overline{)\, x^4 - 2x^3 - 3 \quad}\, x$

$\pm x^4 \pm x^3 \quad \mp 3x^2 \mp 3x$

$-3 \,|\, -3x^3 + 3x^2 + 3x - 3$

$x^3 - x^2 - x + 1 \,\overline{)\, x^3 + x^2 - 3x - 3 \,}\, 1$

$\pm x^3 \mp x^2 \mp x \pm 1$

$2 \,|\, 2x^2 - 2x - 4$

$x^2 - x - 2 \,\overline{)\, x^3 - x^2 - x + 1 \,}\, x$

$\pm x^3 \mp x^2 \mp 2x$

$x + 1 \,\overline{)\, x^2 - x - 2 \,}\, x - 2$

$\pm x^2 \pm x$

$-2x - 2$

$\mp 2x \mp 2$

$\times$

$H.C.F = x(x + 1)$

(iv) **$2x^3 + 10x^2 + 5x + 25$ and $x^3 + 5x^2 - x - 5$**

Solution:

$2x^3 + 10x^2 + 5x + 25$ and $x^3 + 5x^2 - x - 5$

$x^3 + 5x^2 - x - 5 \,\overline{)\, 2x^3 + 10x^2 + 5x + 25 \,}\, 2$

$\pm 2x^3 \pm 10x^2 \mp 2x \mp 10$

$7 \,|\, 7x + 35$

$x + 5 \,\overline{)\, x^3 + 5x^2 - x - 5 \,}\, x^2 - 1$

$\pm x^3 \mp 5x^2$

$-x - 5$

$\mp x \mp 5$

$\times$

$H.C.F = x + 5$

Chapter # 6

Ex # 6.1

Q3: Find L.C.M by factorization.

(i) $\boldsymbol{x+y, \quad x^2-y^2}$

Solution:

$x+y, \quad x^2-y^2$

$x+y=x+y$

And

$x^2-y^2=(x+y)(x-y)$

$Common\ Factor=x+y$

$Non-common\ factor=x-y$

$\boldsymbol{L.C.M=common\ factor\times non-common\ factor}$

$L.C.M=(x+y)(x-y)$

$L.C.M=x^2-y^2$

(ii) $\boldsymbol{x^3-y^3, x-y}$

Solution:

$x^3-y^3, x-y$

$x^3-y^3=(x-y)(x^2+xy+y^2)$

And

$x-y=x-y$

$Common\ Factor=x-y$

$Non-common\ factor=x^2+xy+y^2$

$\boldsymbol{L.C.M=common\ factor\times non-common\ factor}$

$L.C.M=(x-y)(x^2+xy+y^2)$

$L.C.M=x^3-y^3$

Ex # 6.1

(iii) $\boldsymbol{x^5-x, x^5-x^2\ and\ x^5-x^3}$

Solution:

$x^5-x, x^5-x^2\ and\ x^5-x^3$

$x^5-x=x(x^4-1)$

$=x[(x^2)^2-(1)^1]$

$=x(x^2+1)(x^2-1)$

$=x(x^2+1)(x+1)(x-1)$

And

$x^5-x^2=x^2(x^3-1)$

$=x.x[(x)^3-(1)^3]$

$=x.x(x-1)(x^2+(x)(1)+1^2)$

$=x.x(x-1)(x^2+x+1)$

And

$x^5-x^3=x^3(x^2-1)$

$=x.x.x[(x)^2-(1)^2]$

$=x.x.x(x+1)(x-1)$

$Common\ Factor=x(x-1)$

$Non-common\ factor=\ x.x(x^2+1)(x+1)(x^2+x+1)$

$\boldsymbol{L.C.M=common\ factor\times non-common\ factor}$

$L.C.M=x(x-1)\times x.x(x^2+1)(x+1)(x^2+x+1)$

$L.C.M=x^3(x-1)(x+1)(x^2+1)(x^2+x+1)$

(iv) $\boldsymbol{2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)}$

Solution:

$2^3 3^2(x-y)^3(x+2y)^2, 2^3 3^2(x-y)^2(x+2y)^3, 3^2(x-y)^2(x+2y)$

$2^3 3^2(x-y)^3(x+2y)^2=2.2.2.3.3(x-y)(x-y)(x-y)(x+2y)(x+2y)$

$2^3 3^2(x-y)^2(x+2y)^3=2.2.2.3.3(x-y)(x-y)(x+2y)(x+2y)(x+2y)$

$3^2(x-y)^2(x+2y)=3.3(x-y)(x-y)(x+2y)$

$Common\ Factor=3.3(x-y)(x-y)(x+2y)$

$Non-common\ factor=2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M=common\ factor\times non-common\ factor$

$L.C.M=3.3(x-y)(x-y)(x+2y)\times 2.2.2.(x-y)(x+2y)(x+2y)$

$L.C.M=2^3 3^2(x-y)^3(x+2y)^3$

Chapter # 6

Ex # 6.1

Q4: Find H.C.F and L.C.M of the following
160 expression.

(i) $\boldsymbol{x^3-2x^2-13x-10}$ **and** $\boldsymbol{x^3-x^2-10x-8}$

Solution:

$x^3-2x^2-13x-10$ and $x^3-x^2-10x-8$

$Let\ A = x^3-2x^2-13x-10$

$and\ B = x^3-x^2-10x-8$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
x^3-x^2-10x-8 & x^3-2x^2-13x-10 & 1 \\
 & \pm x^3 \mp x^2 \mp 10x \mp 8 & \\
\hline
-1 & -x^2-3x-2 &
\end{array}$$

$$\begin{array}{r|l|l}
x^2+3x+2 & x^3-x^2-10x-8 & x-4 \\
 & \pm x^3 \pm 3x^2 \pm 2x & \\
\hline
 & -4x^2-12x-8 & \\
 & \mp 4x^2 \mp 12x \mp 8 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = x^2+3x+2$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-2x^2-13x-10)(x^3-x^2-10x-8)}{x^2+3x+2}$$

Now by Simple Division

$$\begin{array}{r|l}
 & x-5 \\
\hline
x^2+3x+2 & x^3-2x^2-13x-10 \\
 & \pm x^3 \pm 3x^2 \pm 2x \\
\hline
 & -5x^2-15x-10 \\
 & \mp 5x^2 \mp 15x \mp 10 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (x-5)(x^3-x^2-10x-8)$

Chapter # 6

Ex # 6.1

(ii) $\mathbf{2x^4 - 2x^3 + x^2 + 3x - 6}$ **and** $\mathbf{4x^4 - 2x^3 + 3x - 9}$

Solution:

$2x^4 - 2x^3 + x^2 + 3x - 6 \; and \, 4x^4 - 2x^3 + 3x - 9$

$Let\ A = 2x^4 - 2x^3 + x^2 + 3x - 6$

$and\ B = 4x^4 - 2x^3 + 3x - 9$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$2x^4 - 2x^3 + x^2 + 3x - 6 \; \overline{\big| \; 4x^4 - 2x^3 + 3x - 9 \;}\big| \; 2$$

$$\pm 4x^4 \mp 4x^3 \pm 6x \mp 12 \pm 2x^2$$

$$2x^3 - 2x^2 - 3x + 3 \; \big| \; 2x^4 - 2x^3 + x^2 + 3x - 6 \; \big| \; x$$

$$\pm 2x^4 \mp 2x^3 \mp 3x^2 \pm 3x$$

$$2 \; \big| \; 4x^2 - 6$$

$$2x^2 - 3 \; \big| \; 2x^3 - 2x^2 - 3x + 3 \; \big| \; x - 1$$

$$\pm 2x^3 \qquad \mp 3x$$

$$-2x^2 + 3$$

$$\mp 2x^2 \pm 3$$

$$\times$$

$H.C.F = 2x^2 - 3$

Now put the values in equ (i)

$$L.C.M = \frac{(2x^4 - 2x^3 + x^2 + 3x - 6)(4x^4 - 2x^3 + 3x - 9)}{2x^2 - 3}$$

Now by Simple Division

$$\begin{array}{r l}
 & \quad x^2 - x + 2 \\
2x^2 - 3 & \big) \; 2x^4 - 2x^3 + \; x^2 + 3x - 6 \\
 & \pm 2x^4 \qquad \quad \mp 3x^2 \\
\hline
 & \; -2x^3 + 4x^2 + 3x - 6 \\
 & \; \mp 2x^3 \qquad \quad \pm 3x \\
\hline
 & \qquad 4x^2 - 6 \\
 & \quad \pm 4x^2 \mp 6 \\
\hline
 & \qquad \times
\end{array}$$

$So\ L.C.M = (x^2 - x + 2)(4x^4 - 2x^3 + 3x - 9)$

Chapter # 6

Ex # 6.1

(iii) $\boldsymbol{a^4 - a^3 - a + 1}$ **and** $\boldsymbol{a^4 + a^2 + 1}$

Solution:

$a^4 - a^3 - a + 1$ and $a^4 + a^2 + 1$

$Let\ A = a^4 - a^3 - a + 1$

$and\ B = a^4 + a^2 + 1$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$\begin{array}{r|l|l}
a^4 + a^2 + 1 & a^4 - a^3 - a + 1 & 1 \\
 & \pm a^4 \qquad \pm 1 \pm a^2 & \\
\hline
-a & -a^3 - a^2 - a & \\
\hline
a^2 + a + 1 & a^4 + a^2 + 1 & a^2 - a + 1 \\
 & \pm a^4 \pm a^2 \qquad \pm a^3 & \\
\hline
 & -a^3 + 1 & \\
 & \mp a^3 \qquad \mp a^2 \mp a & \\
\hline
 & a^2 + a + 1 & \\
 & \pm a^2 \pm a \pm 1 & \\
\hline
 & \times &
\end{array}$$

$H.C.F = a^2 + a + 1$

Now put the values in equ (i)

$$L.C.M = \frac{(a^4 - a^3 - a + 1)(a^4 + a^2 + 1)}{a^2 + a + 1}$$

Now by Simple Division

$$\begin{array}{r|l}
 & a^2 - 2a + 1 \\
\hline
a^2 + a + 1 & a^4 - a^3 - a + 1 \\
 & \pm a^4 \pm a^3 \qquad \pm a^2 \\
\hline
 & -2a^3 - a^2 - a + 1 \\
 & \mp 2a^3 \mp 2a^2 \mp 2a \\
\hline
 & a^2 + a + 1 \\
 & \pm a^2 \pm a \pm 1 \\
\hline
 & \times
\end{array}$$

$So\ L.C.M = (a^2 - 2a + 1)(a^4 + a^2 + 1)$

Chapter # 6

Ex # 6.1

(iv) $\mathbf{1-x^2-x^4+x^5}$ **and** $\mathbf{1+2x+x^2-x^4-x^5}$

Solution:

$1-x^2-x^4+x^5$ and $1+2x+x^2-x^4-x^5$

$x^5-x^4-x^2+1$ and $-x^5-x^4+x^2+2x+1$

$Let\ A = x^5-x^4-x^2+1$

$and\ B = -x^5-x^4+x^2+2x+1$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F} - -equ\ (i)$$

First we find H.C.F

$$
\begin{array}{r|l|l}
x^5-x^4-x^2+1 & -x^5-x^4+x^2+2x+1 & -1\\
 & \mp x^5 \pm x^4 \pm x^2 \qquad \mp 1 & \\
\hline
-2 & -2x^4+2x+2 & \\
\hline
\end{array}
$$

$$
\begin{array}{r|l|l}
x^4-x-1 & x^5-x^4-x^2+1 & x-1\\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x & \\
\hline
 & -x^4+x+1 & \\
 & \mp x^4 \pm x \pm 1 & \\
\hline
 & \times &
\end{array}
$$

$H.C.F = \quad x^4-x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^5-x^4-x^2+1)(-x^5-x^4+x^2+2x+1)}{x^4-x-1}$$

Now by Simple Division

$$
\begin{array}{r|l}
 & x-1\\
\hline
x^4-x-1 & x^5-x^4-x^2+1\\
 & \pm x^5 \qquad \mp x^2 \qquad \mp x\\
\hline
 & -x^4+x+1\\
 & \mp x^4 \pm x \pm 1\\
\hline
 & \times
\end{array}
$$

$So\ L.C.M = (x+2)(-x^5-x^4+x^2+2x+1)$

$So\ L.C.M = (x+2)(1+2x+x^2-x^4-x^5)$

Q5: 160 **H.C.F and L.C.M of two polynomials are $x-2$ and x^3+3x^2-6x-8 respectively. If one polynomial is x^2+2x-8, find the second polynomial.**

Solution:

$H.C.F = x-2$

$L.C.M = x^3+3x^2-6x-8$

$First\ polynomial = A = x^2+2x-8$

$Second\ polynomial = B = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

$$L.C.M\times H.C.F = A\times B$$

$$\frac{L.C.M\times H.C.F}{A} = B$$

$$B = \frac{L.C.M\times H.C.F}{A}$$

Put the values

$$B = \frac{(x^3+3x^2-6x-8)(x-2)}{x^2+2x-8}$$

Now by simple Division

$$x+1$$

$$x^2+2x-8,\ \overline{)\ x^3+3x^2-6x-8}$$

$$\pm x^3 \pm 2x^2 \mp 8x$$

$$\overline{x^2+2x-8}$$

$$\pm x^2 \pm 2x \mp 8$$

$$\overline{\qquad\times\qquad}$$

$So\ B = (x+1)(x-2)$

$B = x^2-2x+1x-2$

$B = x^2-x-2$

Q6: 160 **If product of two polynomials is $x^4+5x^3-6x^2-2x-28$ and their H.C.F is $x-2$. Find their L.C.M.**

Solution:

Let Product of two polynomials $= A\times B$

$Then\ A\times B = x^4+5x^3-6x^2-2x-28$

$H.C.F = x-2$

$L.C.M = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

Put the values

$$L.C.M = \frac{x^4+5x^3-6x^2-2x-28}{x-2}$$

$$x^3+7x^2+8x+14$$

$$x-2\ \overline{)\ x^4+5x^3-6x^2-2x-28}$$

$$\pm x^4 \mp 2x^3$$

$$\overline{7x^3-6x^2-2x-28}$$

$$\pm 7x^3 \mp 14x^2$$

$$\overline{8x^2-2x-28}$$

$$\pm 8x^2 \mp 16x$$

$$\overline{14x-28}$$

$$\pm 14 \mp 28$$

$$\overline{\qquad\times\qquad}$$

$L.C.M = x^3+7x^2+8x+14$

Q7: 160 **H.C.F and L.C.M of two polynomials are $x+5$ and $2x^3+11x^2+2x-15$ respectively. Find the polynomials of degree 2.**

Solution:

$H.C.F = x+5$

$L.C.M = 2x^3+11x^2+2x-15$

$First\ polynomial = A = ?$

$Second\ polynomial = B = ?$

$As\ H.C.F = x+5$

$then\ it\ is\ also\ the\ factor\ of\ L.C.M$

Now

$$2x^2+x-3$$

$$x+5\ \overline{)\ 2x^3+11x^2+2x-15}$$

$$\pm 2x^3 \pm 10x^2$$

$$\overline{x^2+2x-15}$$

$$\pm x^2 \pm 5x$$

$$\overline{-3x-15}$$

$$\mp 3x \mp 15$$

$$\overline{\qquad\times\qquad}$$

$L.C.M = 2x^3+11x^2+2x-15$

$L.C.M = (x+5)(2x^2+x-3)$

$L.C.M = (x+5)(2x^2+3x-2x-3)$

$L.C.M = (x+5)[x(2x+3)-1(2x+3)]$

$L.C.M = (x+5)(2x+3)(x-1)$

$As\ x+5\ is\ common\ factor. So$

$A = (x+5)(2x+3)$

$A = 2x^2+3x+10x+15$

$A = 2x^2+13x+15$

And

$B = (x+5)(x-1)$

$B = x^2-1x+5x-5$

$B = x^2+4x-5$

Chapter # 6

Ex # 6.1

Q8: 160 **If product of two polynomials is $x^4+6x^3-3x^2-56x-48$ and their L.C.M is $x^3+2x^2-11x-12$. Find their H.C.F.**

Solution:

Let Product of two polynomials $= A\times B$

$Then\ A\times B = x^4+6x^3-3x^2-56x-48$

$L.C.M = x^3+2x^2-11x-12$

$H.C.F = ?$

As we have:

$$L.C.M = \frac{A\times B}{H.C.F}$$

$$H.C.F = \frac{A\times B}{L.C.M}$$

Put the values

$$H.C.F = \frac{x^4+6x^3-3x^2-56x-48}{x^3+2x^2-11x-12}$$

Now by Simple Division

$$\begin{array}{r|l}
 & x+4 \\
x^3+2x^2-11x-12 & x^4+6x^3-3x^2-56x-48 \\
 & \pm x^4 \pm 2x^3 \mp 11x^2 \mp 12x \\
\hline
 & 4x^3+8x^2-44x-48 \\
 & \pm 4x^3 \pm 8x^2 \mp 44x \mp 48 \\
\hline
 & \times
\end{array}$$

$So\ H.C.F = x+4$

Q9: 160 **Waqar wishes to distribute 128 bananas and also 176 apples equally among certain number of children. Find the highest number of children who can get the fruit in this way.**

Solution:

Bananas = 128

Apples = 176

Highest number of children = ?

Now

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	176
2	88
2	44
2	22
11	11
	1

$128 = 2\times2\times2\times2\times2\times2\times2$

$176 = 2\times2\times2\times2\times11$

$H.C.F = 2\times2\times2\times2$

$= 16$

So highest number of children = 16

Ex # 6.2

Algebraic fractions

An algebraic fraction is the quotient of two algebraic expressions.

Example:

$$\frac{x-y}{y^2-4x^2}$$

Example # 12

Simplify $\boldsymbol{\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}}$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$

$$=\frac{x+y+x-y}{3x+2y}$$

$$=\frac{x+x+y-y}{3x+2y}$$

$$=\frac{2x}{3x+2y}$$

Example # 13

Simplify $\boldsymbol{\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}}$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$

$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$

$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$

$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$

$$=\frac{3y^2-2xy}{x^2-y^2}$$

Ex # 6.2

Example # 14

Simplify $\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3}+\frac{x^2+xy+y^2}{x^3-y^3}-\frac{1}{x^2-y^2}$$

$$=\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}+\frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$=\frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$=\frac{x+x-y+y-1}{x^2-y^2}$$

$$=\frac{2x-1}{x^2-y^2}$$

Example # 15

Simplify $\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$

Solution:

$$\frac{y}{y^2-y-2}-\frac{1}{y^2+5y-14}-\frac{2}{y^2+8y+7}$$

$$=\frac{y}{y^2-2y+y-2}-\frac{1}{y^2-2y+7y-14}-\frac{2}{y^2+1y+7y+7}$$

$$=\frac{y}{y(y-2)+1(y-2)}-\frac{1}{y(y-2)+7(y-2)}-\frac{2}{y(y+1)+7(y+1)}$$

$$=\frac{y}{(y-2)(y+1)}-\frac{1}{(y-2)(y+7)}-\frac{2}{(y+1)(y+7)}$$

$$=\frac{y(y+7)-1(y+1)-2(y-2)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+7y-1y-1-2y+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+6y-2y-1+4}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+4y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y^2+1y+3y+3}{(y-2)(y+1)(y+7)}$$

$$=\frac{y(y+1)+3(y+1)}{(y-2)(y+1)(y+7)}$$

$$=\frac{(y+1)(y+3)}{(y-2)(y+1)(y+7)}$$

$$=\frac{y+3}{(y-2)(y+7)}$$

Ex # 6.2

Example # 16

Simplify $\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$

Solution:

$$\frac{x+4}{x-3}\times\frac{x^2-9}{x^2-x-2}$$

$$=\frac{x+4}{x-3}\times\frac{x^2-3^2}{x^2-2x+1x-2}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{x(x-2)+1(x-2)}$$

$$=\frac{x+4}{x-3}\times\frac{(x+3)(x-3)}{(x-2)(x+1)}$$

$$=\frac{x+4}{1}\times\frac{(x+3)}{(x-2)(x+1)}$$

$$=\frac{(x+4)(x+3)}{(x-2)(x+1)}$$

Example # 17

Multiply $\frac{x^2-2x}{2x^2+5x+3}$ **by** $\frac{2x^2-3x-9}{x^2-9}$

Solution:

$$\frac{x^2-2x}{2x^2+5x+3}\times\frac{2x^2-3x-9}{x^2-9}$$

$$=\frac{x(x-2)}{2x^2+2x+3x+3}\times\frac{2x^2+3x-6x-9}{x^2-9^2}$$

$$=\frac{x(x-2)}{2x(x+1)+3(x+1)}\times\frac{x(2x+3)-3(2x+3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)(2x+3)}\times\frac{(2x+3)(x-3)}{(x+3)(x-3)}$$

$$=\frac{x(x-2)}{(x+1)}\times\frac{1}{(x+3)}$$

$$=\frac{x(x-2)}{(x+1)(x+3)}$$

Example # 18

$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$

Solution:

$$\left(\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\right)\div\frac{x^2+xy+y^2}{y^2}$$

$$=\frac{x^3-y^3}{y^3}\times\frac{y}{x-y}\times\frac{y^2}{x^2+xy+y^2}$$

$$=\frac{(x-y)(x^2+xy+y^2)}{y.y.y}\times\frac{y}{x-y}\times\frac{y.y}{x^2+xy+y^2}$$

$$=1$$

Chapter # 6

Ex # 6.2

Q1: Simplify:

$(i)\ \dfrac{\boldsymbol{x}}{\boldsymbol{x+y}}+\dfrac{\boldsymbol{2y}}{\boldsymbol{x+y}}$

Solution:

$$\frac{x}{x+y}+\frac{2y}{x+y}$$
$$=\frac{x+2y}{x+y}$$

$(ii)\ \dfrac{\boldsymbol{x+y}}{\boldsymbol{3x+2y}}+\dfrac{\boldsymbol{x-y}}{\boldsymbol{3x+2y}}$

Solution:

$$\frac{x+y}{3x+2y}+\frac{x-y}{3x+2y}$$
$$=\frac{x+y+x-y}{3x+2y}$$
$$=\frac{x+x+y-y}{3x+2y}$$
$$=\frac{2x}{3x+2y}$$

$(iii)\ \dfrac{\boldsymbol{3}}{\boldsymbol{y-2}}-\dfrac{\boldsymbol{2}}{\boldsymbol{y+2}}-\dfrac{\boldsymbol{y}}{\boldsymbol{y^2-4}}$

Solution:

$$\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-4}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{y^2-(2)^2}$$
$$=\frac{3}{y-2}-\frac{2}{y+2}-\frac{y}{(y+2)(y-2)}$$
$$=\frac{3(y+2)-2(y-2)-y}{(y+2)(y-2)}$$
$$=\frac{3y+6-2y+4-y}{(y+2)(y-2)}$$
$$=\frac{3y-2y-y+6+4}{(y+2)(y-2)}$$
$$=\frac{3y-3y+10}{y^2-(2)^2}$$
$$=\frac{10}{y^2-4}$$

$(iv)\ \dfrac{\boldsymbol{x-y}}{\boldsymbol{x+y}}-\dfrac{\boldsymbol{x^2-2y^2}}{\boldsymbol{x^2-y^2}}$

Solution:

$$\frac{x-y}{x+y}-\frac{x^2-2y^2}{x^2-y^2}$$

Ex # 6.2

$$=\frac{x-y}{x+y}-\frac{x^2-2y^2}{(x+y)(x-y)}$$
$$=\frac{(x-y)(x-y)-(x^2-2y^2)}{(x+y)(x-y)}$$
$$=\frac{(x-y)^2-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2+y^2-2xy-x^2+2y^2}{(x+y)(x-y)}$$
$$=\frac{x^2-x^2+2y^2+y^2-2xy}{(x+y)(x-y)}$$
$$=\frac{3y^2-2xy}{x^2-y^2}$$

$(v)\ \dfrac{\boldsymbol{x}}{\boldsymbol{2x^2+3xy+y^2}}-\dfrac{\boldsymbol{x-y}}{\boldsymbol{y^2-4x^2}}+\dfrac{\boldsymbol{y}}{\boldsymbol{2x^2+xy-y^2}}$

Solution:

$$\frac{x}{2x^2+3xy+y^2}-\frac{x-y}{y^2-4x^2}+\frac{y}{2x^2+xy-y^2}$$
$$=\frac{x}{2x^2+2xy+1xy+y^2}-\frac{x-y}{-4x^2+y^2}+\frac{y}{2x^2+2xy-1xy-y^2}$$
$$=\frac{x}{2x(x+y)+y(x+y)}-\frac{x-y}{-(4x^2-y^2)}+\frac{y}{2x(x+y)-y(x+y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x)^2-y^2}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x}{(x+y)(2x+y)}+\frac{x-y}{(2x+y)(2x-y)}+\frac{y}{(x+y)(2x-y)}$$
$$=\frac{x(2x-y)+(x-y)(x+y)+y(2x+y)}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2-xy+x^2-y^2+2xy+y^2}{(x+y)(2x+y)(2x-y)}$$
$$=\frac{2x^2+x^2-xy+2xy-y^2+y^2}{(x+y)((2x)^2-y^2)}$$
$$=\frac{3x^2+xy}{(x+y)(4x^2-y^2)}$$

$(vi)\ \dfrac{\boldsymbol{a}}{\boldsymbol{3x-y}}+\dfrac{\boldsymbol{a}}{\boldsymbol{3x+y}}-\dfrac{\boldsymbol{6ax}}{\boldsymbol{9x^2-y^2}}$

Solution:

$$\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{9x^2-y^2}$$
$$=\frac{a}{3x-y}+\frac{a}{3x+y}-\frac{6ax}{(3x)^2-y^2}$$

Ex # 6.2

$$= \frac{a}{3x-y} + \frac{a}{3x+y} - \frac{6ax}{(3x+y)(3x-y)}$$

$$= \frac{a(3x+y) + a(3x-y) - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + ay + 3ax - ay - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{3ax + 3ax - 6ax + ay - ay}{(3x+y)(3x-y)}$$

$$= \frac{6ax - 6ax}{(3x+y)(3x-y)}$$

$$= \frac{0}{(3x+y)(3x-y)}$$

$$= 0$$

(vii) $\boldsymbol{\frac{y}{x-y} + \frac{y}{x+y} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}}$

Solution:

$$\frac{y}{x-y} + \frac{y}{x+y} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{y(x+y) + y(x-y)}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + y^2 + xy - y^2}{(x-y)(x+y)} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{xy + xy + y^2 - y^2}{x^2 - y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy}{x^2-y^2} + \frac{2xy}{x^2+y^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2xy(x^2+y^2) + 2xy(x^2-y^2)}{(x^2-y^2)(x^2+y^2)} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2xy^3 + 2x^3y - 2xy^3}{(x^2)^2 - (y^2)^2} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{2x^3y + 2x^3y + 2xy^3 - 2xy^3}{x^4 - y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y}{x^4-y^4} + \frac{4x^3y}{x^4+y^4}$$

$$= \frac{4x^3y(x^4+y^4) + 4x^3y(x^4-y^4)}{(x^4-y^4)(x^4+y^4)}$$

Ex # 6.2

$$= \frac{4x^7y + 4x^3y^5 + 4x^7y - 4x^3y^5}{(x^4)^2 - (y^4)^2}$$

$$= \frac{4x^7y + 4x^7y + 4x^3y^5 - 4x^3y^5}{x^8 - y^8}$$

$$= \frac{8x^7y}{x^8 - y^8}$$

(viii) $\boldsymbol{\frac{1}{a^2+7a+10} + \frac{1}{a^2+10a+16}}$

Solution:

$$\frac{1}{a^2+7a+10} + \frac{1}{a^2+10a+16}$$

$$= \frac{1}{a^2+2a+5a+10} + \frac{1}{a^2+2a+8a+16}$$

$$= \frac{1}{a(a+2)+5(a+2)} + \frac{1}{a(a+2)+8(a+2)}$$

$$= \frac{1}{(a+2)(a+5)} + \frac{1}{(a+2)(a+8)}$$

$$= \frac{1(a+8) + 1(a+5)}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+8+a+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{a+a+8+5}{(a+2)(a+5)(a+8)}$$

$$= \frac{2a+13}{(a+2)(a+5)(a+8)}$$

(ix) $\boldsymbol{\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}}$

Solution:

$$\frac{1}{a-b} + \frac{1}{a+b} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{1(a+b) + 1(a-b)}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+b+a-b}{(a-b)(a+b)} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{a+a+b-b}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a}{a^2-b^2} + \frac{2a}{a^2+b^2} + \frac{4a^3}{a^4+b^4}$$

Ex # 6.2

$$= \frac{2a(a^2+b^2)+2a(a^2-b^2)}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2ab^2+2a^3-2ab^2}{(a^2)^2-(b^2)^2} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{2a^3+2a^3+2ab^2-2ab^2}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3}{a^4-b^4} + \frac{4a^3}{a^4+b^4}$$

$$= \frac{4a^3(a^4+b^4)+4a^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}$$

$$= \frac{4a^7+4a^3b^4+4a^7-4a^3b^4}{(a^4)^2-(b^4)^2}$$

$$= \frac{4a^7+4a^7+4a^3b^4-4a^3b^4}{a^8-b^8}$$

$$= \frac{8a^7}{a^8-b^8}$$

(x) $\boldsymbol{\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}}$

Solution:

$$\frac{x^2-xy+y^2}{x^3+y^3} + \frac{x^2+xy+y^2}{x^3-y^3} - \frac{1}{x^2-y^2}$$

$$= \frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)} + \frac{x^2+xy+y^2}{(x-y)(x^2+xy+y^2)} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1}{x+y} + \frac{1}{x-y} - \frac{1}{(x+y)(x-y)}$$

$$= \frac{1(x-y)+1(x+y)-1}{(x+y)(x-y)}$$

$$= \frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$= \frac{x+x-y+y-1}{x^2-y^2}$$

$$= \frac{2x-1}{x^2-y^2}$$

Ex # 6.2

Q2: Simplify

(i) $\boldsymbol{\frac{x^2-25}{5-x}}$

Solution:

$$\frac{x^2-25}{5-x}$$

$$= \frac{x^2-(5)^2}{-x+5}$$

$$= \frac{(x+5)(x-5)}{-(x-5)}$$

$$= -(x+5)$$

(ii) $\boldsymbol{\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}}$

Solution:

$$\frac{x^2+5x+4}{4y^3} \times \frac{2y^2}{x^2+3x+2}$$

$$= \frac{x^2+4x+1x+4}{4y.y.y} \times \frac{2y.y}{x^2+2x+1x+2}$$

$$= \frac{x(x+4)+1(x+4)}{2y} \times \frac{1}{x(x+2)+1(x+2)}$$

$$= \frac{(x+4)(x+1)}{2y} \times \frac{1}{(x+2)(x+1)}$$

$$= \frac{x+4}{2y} \times \frac{1}{x+2}$$

$$= \frac{x+4}{2y(x+2)}$$

(iii) $\boldsymbol{\frac{x^2-5x+4}{x^3-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}}$

Solution:

$$\frac{x^2-5x+4}{x^2-3x-4} \div \frac{x^3-4x^2+x-4}{2x-1}$$

$$= \frac{x^2-5x+4}{x^2-3x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x^2-4x-1x+4}{x^2-4x+1x-4} \times \frac{2x-1}{x^3-4x^2+x-4}$$

$$= \frac{x(x-4)-1(x-4)}{x(x-4)+1(x-4)} \times \frac{2x-1}{x^2(x-4)+1(x-4)}$$

$$= \frac{(x-4)(x-1)}{(x-4)(x+1)} \times \frac{2x-1}{(x-4)(x^2+1)}$$

Ex # 6.2

$$=\frac{(x-1)}{(x+1)}\times\frac{2x-1}{(x-4)(x^2+1)}$$

$$=\frac{(x-1)(2x-1)}{(x+1)(x-4)(x^2+1)}$$

$$(\boldsymbol{iv})\quad \frac{\boldsymbol{a(a+b)}}{\boldsymbol{a^3-b^3}}\times\frac{\boldsymbol{a^2+ab+b^2}}{\boldsymbol{a^2+b^2}}$$

Solution:

$$\frac{a(a+b)}{a^3-b^3}\times\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)(a^2+ab+b^2)}\times\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)}\times\frac{1}{a^2+b^2}$$

$$=\frac{a(a+b)}{(a-b)(a^2+b^2)}$$

$$(\boldsymbol{v})\quad \frac{\boldsymbol{7}}{\boldsymbol{x^2-4}}\div\frac{\boldsymbol{xy}}{\boldsymbol{x+2}}$$

Solution:

$$\frac{7}{x^2-4}\div\frac{xy}{x+2}$$

$$=\frac{7}{x^2-2^2}\times\frac{x+2}{xy}$$

$$=\frac{7}{(x+2)(x-2)}\times\frac{x+2}{xy}$$

$$=\frac{7}{x-2}\times\frac{1}{xy}$$

$$=\frac{7}{xy(x-2)}$$

$$(\boldsymbol{vi})\quad \frac{\boldsymbol{a^3-b^3}}{\boldsymbol{a^4-b^4}}\div\frac{\boldsymbol{a^2+ab+b^2}}{\boldsymbol{a^2+b^2}}$$

Solution:

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

Ex # 6.2

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{(a+b)}\times\frac{1}{1}$$

$$=\frac{1}{(a+b)}$$

$$(\boldsymbol{vii})\quad \frac{\boldsymbol{2x}}{\boldsymbol{3x-12}}\div\frac{\boldsymbol{x^2-2x}}{\boldsymbol{x^2-6x+8}}$$

Solution:

$$\frac{2x}{3x-12}\div\frac{x^2-2x}{x^2-6x+8}$$

$$=\frac{2x}{3x-12}\times\frac{x^2-6x+8}{x^2-2x}$$

$$=\frac{2x}{3(x-4)}\times\frac{x^2-2x-4x+8}{x(x-2)}$$

$$=\frac{2x}{3(x-4)}\times\frac{x(x-2)-4(x-2)}{x(x-2)}$$

$$=\frac{2x}{3(x-4)}\times\frac{(x-2)(x-4)}{x(x-2)}$$

$$=\frac{2}{3}\times\frac{1}{1}$$

$$=\frac{2}{3}$$

$$(\boldsymbol{viii})\quad \frac{\boldsymbol{a^4-8a}}{\boldsymbol{2a^2+5a-3}}\times\frac{\boldsymbol{2a-1}}{\boldsymbol{a^2+2a+4}}\div\frac{\boldsymbol{a^2-2a}}{\boldsymbol{a+3}}$$

Solution:

$$\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}\div\frac{a^2-2a}{a+3}$$

$$=\frac{a^4-8a}{2a^2+5a-3}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a^2-2a}$$

$$=\frac{a(a^3-8)}{2a^2+6a-1a-3}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=\frac{a(a^3-2^3)}{2a(a+3)-1(a+3)}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=\frac{a(a-2)(a^2+2a+4)}{(a+3)(2a-1)}\times\frac{2a-1}{a^2+2a+4}\times\frac{a+3}{a(a-2)}$$

$$=1$$

Chapter # 6

Ex # 6.2

(ix) $\dfrac{9-x^2}{x^4+6x^3} \div \dfrac{x^3-2x^2-3x}{x^2+7x+6}$

Solution:

$$\frac{9-x^2}{x^4+6x^3} \div \frac{x^3-2x^2-3x}{x^2+7x+6}$$

$$= \frac{-x^2+9}{x^4+6x^3} \times \frac{x^2+7x+6}{x^3-2x^2-3x}$$

$$= \frac{-(x^2-9)}{x^3(x+6)} \times \frac{x^2+1x+6x+6}{x(x^2-2x-3)}$$

$$= \frac{-(x^2-3^2)}{x^3(x+6)} \times \frac{x(x+1)+6(x+1)}{x(x^2-3x+1x-3)}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[x(x-3)+1(x-3)]}$$

$$= \frac{-(x+3)(x-3)}{x^3(x+6)} \times \frac{(x+1)(x+6)}{x[(x-3)(x+1)]}$$

$$= \frac{-(x+3)}{x^3} \times \frac{1}{x}$$

$$= \frac{-(x+3)}{x^4}$$

(x) $\dfrac{ax+ab+cx+bc}{a^2-x^2} \times \dfrac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$

Solution:

$$\frac{ax+ab+cx+bc}{a^2-x^2} \times \frac{x^2-2ax+a^2}{x^2+(b+a)x+ab}$$

$$= \frac{ax+ab+cx+bc}{-x^2+a^2} \times \frac{x^2-2ax+a^2}{x^2+bx+ax+ab}$$

$$= \frac{a(x+b)+c(x+b)}{-(x^2-a^2)} \times \frac{(x-a)^2}{x(x+b)+a(x+b)}$$

$$= -\frac{(x+b)(a+c)}{(x+a)(x-a)} \times \frac{(x-a)(x-a)}{(x+b)(x+a)}$$

$$= -\frac{(a+c)}{(x+a)} \times \frac{(x-a)}{(x+a)}$$

$$= -\frac{(a+c)(x-a)}{(x+a)^2}$$

Ex # 6.3

Square root

Square root of a number is a number that can be multiplied by itself to produce the original

Square root of an algebraic expression can be found out by the following two methods.

(i) Factorization Method

(ii) Division Method

Square root by Factorization

In this method make the expression a perfect square then finds square root.

Example # 20

Find the square root of $\boldsymbol{x^2+ax+\frac{1}{4}a^2}$ ***by factorization***

Solution:

$$x^2+ax+\frac{1}{4}a^2$$

$$x^2+ax+\frac{1}{4}a^2 = (x)^2+2(x)\left(\frac{1}{2}a\right)+\left(\frac{1}{2}a\right)^2$$

$$x^2+ax+\frac{1}{4}a^2 = \left(x+\frac{1}{2}a\right)^2$$

Now take square root on B.S

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \sqrt{\left(x+\frac{1}{2}a\right)^2}$$

$$\sqrt{x^2+ax+\frac{1}{4}a^2} = \pm\left(x+\frac{1}{2}a\right)$$

Example # 21

Find the square root of $\boldsymbol{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}$

Solution:

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+25+2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = x^2+\frac{1}{x^2}+2-10\left(x+\frac{1}{x}\right)+25$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(5)+(5)^2$$

$$x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27 = \left(x+\frac{1}{x}-5\right)^2$$

Chapter # 6

Ex # 6.3

Taking square root on B.S

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left(x+\frac{1}{x}-5\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-10\left(x+\frac{1}{x}\right)+27}=\pm\left(x+\frac{1}{x}-5\right)$$

Square root by Division

طریقہ:

Expression کو Descending ترتیب میں لکھیں۔

پہلے expression کا Square root لینگے پھر Divisor اور Quotient میں لکھیں گے۔

Divisor اور Quotient کو آپس میں Multiply کریں اور پہلے expression کے نیچے لکھیں پھر Subtract کریں تو Remainder حاصل ہو جائے گا

Divisor کو ڈبل کر دے اور Remainder کو اس پر Divide کر دے اور جو Term آئے گا تو Divisor اور Quotient میں اس کو لکھیں۔

اب اس Quotient کو پورے Divisor کے ساتھ Multiply کرے پھر Subtract کرے

اب Divisor کے دوسرے Term کو ڈبل کرے اور اوپر کا طریقہ دوبارہ کریں۔

Find the square root of $16x^4-24x^3+25x^2-12x+4$

Solution:

Write the expression in descending order

$16x^4-24x^3+25x^2-12x+4$

Take the square root of first element of expression.

$$\sqrt{16x^4}=4x^2$$

Write $4x^2$ in divisor and quotient

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2 \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4
\end{array}$$

Multiply the divisor and quotient and write it under first expression then subtract from given expression to get the remainder.

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2 \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
 & -24x^3+25x^2-12x+4
\end{array}$$

Now twice the divisor

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2 \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2 & -24x^3+25x^2-12x+4
\end{array}$$

Divide the 2nd expression by this divisor then write that term in quotient and with this divisor.

$$\frac{-24x^3}{8x^2}=-3x$$

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2-3x \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$-3x(8x^2-3x)=-24x^3+9x^2$$

Write $-24x^3+9x^2$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2-3x \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp 24x^3 \pm 9x^2 \\ \cline{2-2}
 & 16x^2-12x+4
\end{array}$$

Now twice the 2nd term of the divisor

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2-3x \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp 24x^3 \pm 9x^2 \\ \cline{2-2}
8x^2-6x & 16x^2-12x+4
\end{array}$$

Repeat the above procedure.

Divide $16x^2$ by divisor $8x^2$ then write that term in quotient and with this divisor.

$$\frac{16x^2}{8x^2}=2$$

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2-3x+2 \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp 24x^3 \pm 9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4
\end{array}$$

Multiply this quotient with entire divisor

$$2(8x^2-6x+2)=16x^2-12x+4$$

Write $16x^2-12x+4$ under given expression then subtract it.

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 4x^2-3x+2 \\ \cline{2-2}
4x^2 & 16x^4-24x^3+25x^2-12x+4 \\
 & \pm 16x^4 \\ \cline{2-2}
8x^2-3x & -24x^3+25x^2-12x+4 \\
 & \mp 24x^3 \pm 9x^2 \\ \cline{2-2}
8x^2-6x+2 & 16x^2-12x+4 \\
 & \pm 16x^2 \mp 12x \pm 4 \\ \cline{2-2}
 & 0
\end{array}$$

Ex # 6.3

Example # 22

Find the square root of $16x^4-24x^3+25x^2-12x+4$

Solution:

Now

$$\begin{array}{r|l} & 4x^2-3x+2 \\ \hline 4x^2 & 16x^4-24x^3+25x^2-12x+4 \\ & \pm16x^4 \\ \hline 8x^2-3x & -24x^3+25x^2-12x+4 \\ & \mp24x^3\pm9x^2 \\ \hline 8x^2-6x+2 & 16x^2-12x+4 \\ & \pm16x^2\mp12x\pm4 \\ \hline & 0 \end{array}$$

So

$$\sqrt{16x^4-24x^3+25x^2-12x+4}=\pm(4x^2-3x+2)$$

Example # 20

Find the square root of $\frac{x^2}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$

Solution:

$$\frac{x^2}{4}+4x^2+\frac{ax^2}{3}+\frac{a^2}{9}-2x^3-\frac{4ax}{3}$$

The descending order of the expression are:

$$\frac{x^2}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}$$

Now

$$\begin{array}{r|l} & \frac{x^2}{2}-2x+\frac{a}{3} \\ \hline \frac{x^2}{2} & \frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\ & \pm\frac{x^4}{4} \\ \hline x^2-2x & -2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\ & \mp2x^3\pm4x^2 \\ \hline x^2-4x+\frac{a}{3} & \frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9} \\ & \pm\frac{ax^2}{3}\mp\frac{4ax}{3}x\pm\frac{a^2}{9} \\ \hline & 0 \end{array}$$

So

$$\sqrt{\frac{x^4}{4}-2x^3+4x^2+\frac{ax^2}{3}-\frac{4ax}{3}+\frac{a^2}{9}}=\pm\left(\frac{x^2}{2}-2x+\frac{a}{3}\right)$$

Ex # 6.3

Example # 24

What should be added to

What should be subtracted from

For what value of x

The expression $9x^4-12x^3+10x^2-3x-3$

to make the perfect square

Solution:

$$9x^4-12x^3+10x^2-3x-3$$

$$\begin{array}{r|l} & 3x^2-2x+1 \\ \hline 3x^2 & 9x^4-12x^3+10x^2-3x-3 \\ & \pm9x^4 \\ \hline 6x^2-2x & -12x^3+10x^2-3x-3 \\ & \mp12x^3\pm4x^2 \\ \hline 6x^2-4x+1 & 6x^2-3x-3 \\ & \pm6x^2\mp4x\pm1 \\ \hline & x-4 \end{array}$$

As for perfect square, Remainder $=0$

$-x+4$ should be Added to $9x^4-12x^3+10x^2-3x-3$ will become perfect square.

$-x+4+(x-4)=-x+4+x-4$

$-x+4+(x-4)=0$

$x-4$ should be Subtracted to $9x^4-12x^3+10x^2-3x-3$ will become perfect square.

$x-4-(x-4)=x-4-x+4$

$x-4-(x-4)=0$

For x

$x-4=0$

$x=4$

Chapter # 6

Exercise# 6.3

Page # 169

Q1: Find the square root by factorization method.

$(i)\ \boldsymbol{x^2+4x+4}$

Solution:

x^2+4x+4

$x^2+4x+4 = x^2+2(x)(2)+2^2$

$x^2+4x+4 = (x+2)^2$

Taking Square on B.S

$\sqrt{x^2+4x+4} = \pm\sqrt{(x+2)^2}$

$\sqrt{x^2+4x+4} = \pm(x+2)$

$(ii)\ \boldsymbol{(x-y)^2+6(x-y)+9}$

Solution:

$(x-y)^2+6(x-y)+9$

$(x-y)^2+6(x-y)+9 = (x-y)^2+2(x-y)(3)+3^2$

$(x-y)^2+6(x-y)+9 = (x-y+3)^2$

Taking Square on B.S

$\sqrt{(x-y)^2+6(x-y)+9} = \pm\sqrt{(x-y+3)^2}$

$\sqrt{(x-y)^2+6(x-y)+9} = \pm(x-y+3)$

$(iii)\ \boldsymbol{x^2y^2-8xy+16}$

Solution:

$x^2y^2-8xy+16$

$x^2y^2-8xy+16 = (xy)^2+2(xy)(4)+4^2$

$x^2y^2-8xy+16 = (xy+4)^2$

Taking Square on B.S

$\sqrt{x^2y^2-8xy+16} = \pm\sqrt{(xy+4)^2}$

$\sqrt{x^2y^2-8xy+16} = \pm(xy+4)$

$(iv)\ \boldsymbol{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18}$

Solution:

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18$$

$$= x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+2+16$$

$$= x^2+\frac{1}{x^2}+2-8\left(x+\frac{1}{x}\right)+16$$

$$= \left(x+\frac{1}{x}\right)^2-2\left(x+\frac{1}{x}\right)(4)+(4)^2$$

$$= \left(x-\frac{1}{x}+4\right)^2$$

Now

$$x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18 = \left(x-\frac{1}{x}+4\right)^2$$

Taking Square on B.S

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18} = \pm\sqrt{\left(x-\frac{1}{x}+4\right)^2}$$

$$\sqrt{x^2+\frac{1}{x^2}-8\left(x+\frac{1}{x}\right)+18} = \pm\left(x-\frac{1}{x}+4\right)$$

$(v)\ \boldsymbol{(x+1)(x+2)(x+3)+1}$

Solution:

$x(x+1)(x+2)(x+3)+1$

Rearranging accordingly $0+3=1+2$

$= x(x+3)(x+1)(x+2)+1$

$= (x^2+3x)(x^2+2x+1x+2)+1$

$= (x^2+3x)(x^2+3x+2)+1$

$Let\ x^2+3x = y$

$= y^2+2y+1$

$= (y)^2+2(y)(1)+(1)^2$

$= (y+1)^2$

But y $= x^2+3x$

$= (x^2+3x+1)^2$

Now

$x(x+1)(x+2)(x+3)+1 = (x^2+3x+1)^2$

Taking Square on B.S

$\sqrt{x(x+1)(x+2)(x+3)+1} = \pm\sqrt{(x^2+3x+1)^2}$

$\sqrt{x(x+1)(x+2)(x+3)+1} = \pm(x^2+3x+1)$

Chapter # 6

Ex # 6.3

(vi) $\boldsymbol{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}$

Solution:

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=x^2+\frac{1}{x^2}+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

Subtract and Add 2

$$=x^2+\frac{1}{x^2}-2+2+2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2+4-5\left(x-\frac{1}{x}\right)+\frac{9}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}+4$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9+16}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{25}{4}$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2$$

$$=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Now

$$\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}=\left(x-\frac{1}{x}-\frac{5}{2}\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\sqrt{\left(x-\frac{1}{x}-\frac{5}{2}\right)^2}$$

$$\sqrt{\left(x+\frac{1}{x}\right)^2-5\left(x-\frac{1}{x}\right)+\frac{9}{4}}=\pm\left(x-\frac{1}{x}-\frac{5}{2}\right)$$

(vii) $\boldsymbol{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}+2\right)+12$$

Ex # 6.3

$$=\left(x^2+\frac{1}{x^2}\right)^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2+\frac{1}{x^2}\right)(2)+(4)^2$$

$$=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Now

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12=\left(x^2+\frac{1}{x^2}-2\right)^2$$

Taking square root on B.S

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\sqrt{\left(x^2+\frac{1}{x^2}-2\right)^2}$$

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12}=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

(viii) $\boldsymbol{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}$

Solution:

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$$

$$=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Now

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}=\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2$$

Taking square root on B.S

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\sqrt{\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)^2}$$

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^6}}=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

Chapter # 6

Ex # 6.3

Q2: Find the square root of the following by Division method.

(i) $4x^4 - 4x^3 + 13x^2 - 6x + 9$

Solution:

$$4x^4 - 4x^3 + 13x^2 - 6x + 9$$

$$\begin{array}{c|l} & 2x^2 - x + 3 \\ \hline 2x^2 & 4x^4 - 4x^3 + 13x^2 - 6x + 9 \\ & \pm 4x^4 \\ \hline 4x^2 - x & -4x^3 + 13x^2 - 6x + 9 \\ & \mp 4x^3 \pm x^2 \\ \hline 4x^2 - 2x + 3 & 12x^2 - 6x + 9 \\ & \pm 12x^2 \mp 6x \pm 9 \\ \hline & 0 \end{array}$$

So

$$\sqrt{4x^4 - 4x^3 + 13x^2 - 6x + 9} = \pm(2x^2 - x + 3)$$

(ii) $x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$

Solution:

$$x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16$$

$$\begin{array}{c|l} & x^2 + \frac{x}{2} - 4 \\ \hline x^2 & x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16 \\ & \pm x^4 \\ \hline 2x^2 + \frac{x}{2} & x^3 - \frac{31}{4}x^2 - 4x + 16 \\ & \pm x^3 \pm \frac{x^2}{4} \\ \hline 2x^2 + x - 4 & -8x^2 - 4x + 16 \\ & \mp 8x^2 \mp 4x \pm 16 \\ \hline & 0 \end{array}$$

So

$$\sqrt{x^4 + x^3 - \frac{31}{4}x^2 - 4x + 16} = \pm\left(x^2 + \frac{x}{2} - 4\right)$$

(iii) $x^2 - 2x + 1 + 2xy - 2y + y^2$

Solution:

$$x^2 - 2x + 1 + 2xy - 2y + y^2$$

Ex # 6.3

$$\begin{array}{c|l} & x - 1 + y \\ \hline x & x^2 - 2x + 1 + 2xy - 2y + y^2 \\ & \pm x^2 \\ \hline 2x - 1 & -2x + 1 + 2xy - 2y + y^2 \\ & \mp 2x \pm 1 \\ \hline 2x - 2 + y & 2xy - 2y + y^2 \\ & \pm 2xy \mp 2y \pm y^2 \\ \hline & 0 \end{array}$$

So

$$\sqrt{x^2 - 2x + 1 + 2xy - 2y + y^2} = \pm(x - 1 + y)$$

(iv) $\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$

Solution:

$$\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 2(x^2)\left(\frac{1}{x^2}\right) - 12x^2 + \frac{12}{x^2} + 36$$

$$= x^4 + \frac{1}{x^4} - 2 - 12x^2 + \frac{12}{x^2} + 36$$

Arrange it in ascending order

$$= x^4 - 12x^2 - 2 + 36 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$= x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4}$$

$$\begin{array}{c|l} & x^2 - 6 - \frac{1}{x^2} \\ \hline x^2 & x^4 - 12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \pm x^4 \\ \hline 2x^2 - 6 & -12x^2 + 34 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \mp 12x^2 \pm 36 \\ \hline 2x^2 - 12 - \frac{1}{x^2} & -2 + \frac{12}{x^2} + \frac{1}{x^4} \\ & \mp 2 \pm \frac{12}{x^2} \pm \frac{1}{x^4} \\ \hline & 0 \end{array}$$

So

$$\sqrt{\left(x^2 - \frac{1}{x^2}\right)^2 - 12\left(x^2 - \frac{1}{x^2}\right) + 36} = \pm\left(x^2 - 6 - \frac{1}{x^2}\right)$$

Ex # 6.3

Q3 (i): For what value of k the expression

$$4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$$

will become perfect square.

Solution:

$4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$

$2x^2+8+\frac{8}{x^2}$

$2x^2$	$4x^4+32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 4x^4$
$4x^2+8$	$32x^2+96+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 32x^2 \pm 64$
$4x^2+16+\frac{8}{x^2}$	$32+\frac{128}{x^2}+\frac{k}{x^4}$ $\pm 32 \pm \frac{128}{x^2} \pm \frac{64}{x^4}$
	$\frac{k}{x^4}-\frac{64}{x^4}$

As for perfect square, Remainder $=0$

$\frac{k}{x^4}-\frac{64}{x^4}=0$

$\frac{k-64}{x^4}=0$

$k-64=0\times x^4$

$k-64=0$

$k=64$

Q3 (ii):

- **(i) What should be added to**
- **(ii) What should be subtracted to**
- **(iii) For what value of x the expression**

$4x^4-12x^3+17x^2-13x+6$ so that it becomes perfect square

Solution:

$4x^4-12x^3+17x^2-13x+6$

$2x^2-3x+2$

$2x^2$	$4x^4-12x^3+17x^2-13x+6$ $\pm 4x^4$
$4x^2-3x$	$-12x^3+17x^2-13x+6$ $\mp 12x^3 \pm 9x^2$
$4x^2-6x+2$	$8x^2-13x+6$ $\pm 8x^2 \mp 12x \pm 4$
	$-x+2$

As for perfect square, Remainder $=0$

Ex # 6.3

$x-2$ should be Added to $4x^4-12x^3+17x^2-13x+6$ will become perfect square.

$-x+2+(x-2)=-x+2+x-2$

$-x+2+(x-2)=0$

$-x+2$ should be Subtracted to $4x^4-12x^3+17x^2-13x+6$ will become perfect square.

$-x+2-(-x+2)=-x+2+x-2$

$-x+2-(-x+2)=0$

For x

$-x+2=0$

$-x=-2$

$x=2$

Q4: What should be subtracted and added to the expression $x^4-4x^3+10x+7$ so that the expression is made perfect square?

Solution:

$x^4-4x^3+10x+7$

x^2-2x-2

x^2	$x^4-4x^3+10x+7$ $\pm x^4$
$2x^2-2x$	$-4x^3+10x+7$ $\mp 4x^3 \quad \pm 4x^2$
$2x^2-4x-2$	$-4x^2+10x+7$ $\mp 4x^2 \pm 8x \pm 4$
	$2x+3$

As for perfect square, Remainder $=0$

$-2x-3$ should be Added to $x^4-4x^3+10x+7$ will become perfect square.

$-2x-3+(2x+3)=2x+3-2x-3$

$-2x-3+(2x+3)=0$

$2x+3$ should be Subtracted to $x^4-4x^3+10x+7$ will become perfect square.

$2x+3-(2x+3)=2x+3-2x-3$

$2x+3-(2x+3)=0$

Ex # 6.3

Q5 (i): Find the value of l and m for which expression will become perfect square

$$\boldsymbol{x^4+4x^3+16x^2+lx+m}$$

Solution:

$$x^4+4x^3+16x^2+lx+m$$

$$\begin{array}{r|l} & x^2+2x+6 \\ \hline x^2 & x^4+4x^3+16x^2+lx+m \\ & \pm x^4 \\ \hline 2x^2+2x & 4x^3+16x^2+lx+m \\ & \pm 4x^3 \pm 4x^2 \\ \hline 2x^2+4x+6 & 12x^2+lx+m \\ & \pm 12x^2 \pm 24x \pm 36 \\ \hline & lx-24x+m-36 \end{array}$$

$$\text{As for perfect square, Remainder } = 0$$

$lx-24x+m-36=0$

$(l-24)x+(m-36)=0$

This $(l-24)x+(m-36)=0$ when

$(l-24)x+(m-36)=0x+0$

By compare the co-efficient of x and constant

$$l-24=0$$
$$l=24$$

And $m-36=0$

$$m=36$$

Hence

$$l=24 \text{ and } m=36$$

Q5 (ii): Find the value of l and m for which expression will become perfect square

$$\boldsymbol{49x^4-70x^3+109x^2+lx-m}$$

Solution:

$$49x^4-70x^3+109x^2+lx-m$$

$$\begin{array}{r|l} & 7x^2-5x+6 \\ \hline 7x^2 & 49x^4-70x^3+109x^2+lx-m \\ & \pm 49x^4 \\ \hline 14x^2-5x & -70x^3+109x^2+lx-m \\ & \mp 70x^3 \pm 25x^2 \\ \hline 14x^2-10x+6 & 84x^2+lx-m \\ & \pm 84x^2 \mp 60x \pm 36 \\ \hline & lx+60x-m-36 \end{array}$$

$$\text{As for perfect square, Remainder } = 0$$

$lx+60x-m-36=0$

$(l+60)x+(-m-36)=0$

This $(l+60)x+(-m-36)=0$ when

$(l+60)x+(-m-36)=0x+0$

By compare the co-efficient of x and constant

$$l+60=0$$
$$l=-60$$

And $-m-36=0$

$$-m=36$$
$$m=-36$$

Hence

$$l=-60 \text{ and } m=-36$$

Review Exercise # 6

Page # 171

Q2: Simplify the following.

$$\textbf{(i)}: \frac{\boldsymbol{5}}{\boldsymbol{2s+4}}-\frac{\boldsymbol{3}}{\boldsymbol{s^2+3s+2}}+\frac{\boldsymbol{s}}{\boldsymbol{s^2-s-2}}$$

Solution:

$$\frac{5}{2s+4}-\frac{3}{s^2+3s+2}+\frac{s}{s^2-s-2}$$

$$=\frac{5}{2(s+2)}-\frac{3}{s^2+2s+1s+2}+\frac{s}{s^2-2s+1s-2}$$

$$=\frac{5}{2(s+2)}-\frac{3}{s(s+2)+1(s+2)}+\frac{s}{s(s-2)+1(s-2)}$$

$$=\frac{5}{2(s+2)}-\frac{3}{(s+2)(s+1)}+\frac{s}{(s-2)(s+1)}$$

$$=\frac{5(s+1)(s-2)-3\times 2(s-2)+s\times 2(s+2)}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5(s^2-2s+1s-2)-6(s-2)+2s(s+2)}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5(s^2-1s-2)-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5s^2-5s-10-6s+12+2s^2+4s}{2(s+2)(s+1)(s-2)}$$

$$=\frac{5s^2+2s^2-5s-6s+4s-10+12}{2(s+2)(s+1)(s-2)}$$

$$=\frac{7s^2-11s+4s-2}{2(s+2)(s+1)(s-2)}$$

$$=\frac{7s^2-7s-2}{2(s+2)(s+1)(s-2)}$$

Chapter # 6

Review Ex # 6

$$(ii).\frac{\boldsymbol{a}}{(\boldsymbol{c}-\boldsymbol{a})(\boldsymbol{a}-\boldsymbol{b})}+\frac{\boldsymbol{b}}{(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{b}-\boldsymbol{c})}+\frac{\boldsymbol{c}}{(\boldsymbol{b}-\boldsymbol{c})(\boldsymbol{c}-\boldsymbol{a})}$$

Solution:

$$\frac{a}{(c-a)(a-b)}+\frac{b}{(a-b)(b-c)}+\frac{c}{(b-c)(c-a)}$$

$$=\frac{a(b-c)+b(c-a)+c(a-b)}{(c-a)(a-b)(b-c)}$$

$$=\frac{ab-ac+bc-ab+ac-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{ab-ab-ac+ac+bc-bc}{(a-b)(b-c)(c-a)}$$

$$=\frac{0}{(a-b)(b-c)(c-a)}$$

$$=0$$

$$(\mathbf{iii}):\ \frac{\boldsymbol{x^2-4}}{\boldsymbol{xy^2}}\cdot\frac{\boldsymbol{2xy}}{\boldsymbol{x^2-4x+4}}$$

Solution:

$$\frac{x^2-4}{xy^2}\cdot\frac{2xy}{x^2-4x+4}$$

$$=\frac{x^2-2^2}{xyy}\cdot\frac{2xy}{x^2-2(x)(2)+2^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)^2}$$

$$=\frac{(x+2)(x-2)}{xyy}\cdot\frac{2xy}{(x+2)(x+2)}$$

$$=\frac{(x-2)}{y}\cdot\frac{2}{(x+2)}$$

$$=\frac{2(x-2)}{y(x+2)}$$

$$(\mathbf{iv}):\ \frac{\boldsymbol{a^3-b^3}}{\boldsymbol{a^4-b^4}}\div\frac{\boldsymbol{a^2+ab+b^2}}{\boldsymbol{a^2+b^2}}$$

Solution:

$$\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}$$

$$=\frac{a^3-b^3}{a^4-b^4}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

Review Ex # 6

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}\times\frac{a^2+b^2}{a^2+ab+b^2}$$

$$=\frac{1}{a+b}\times\frac{1}{1}$$

$$=\frac{1}{a+b}$$

Chapter # 6

Review Ex # 6

Q3: **Find L.C.M of $x^3-6x^2+11x-6$ and x^3-4x+3**

$x^3-6x^2+11x-6$ and x^3-4x+3

Solution:

$Let\ A = x^3-6x^2+11x-6$

$and\ B = x^3-4x+3$

As we have:

$$L.C.M = \frac{A \times B}{H.C.F} --equ\ (i)$$

First we find H.C.F

$$x^3-4x+3 \;\big)\; x^3-6x^2+11x-6 \;\big(\; 1$$
$$\pm x^3 \qquad \mp 4x \;\pm 3$$
$$-3 \;\big)\; -6x^2+15x-9$$
$$2x^2-5x+3 \;\big)\; x^3-4x+3 \;\big(\; x+5$$
$$\times 2$$
$$2x^3-8x+6$$
$$\pm 2x^3 \pm 3x \qquad \mp 5x^2$$
$$5x^2-11x+6$$
$$\times 2$$
$$10x^2-22x+12$$
$$\pm 10x^2 \mp 25x \pm 15$$
$$3 \;\big)\; 3x-3$$
$$x-1 \;\big)\; 2x^2-5x+3 \;\big(\; 2x-3$$
$$\pm 2x^2 \mp 2x$$
$$-3x+3$$
$$\mp 3x \pm 3$$
$$\times$$

$H.C.F = x-1$

Now put the values in equ (i)

$$L.C.M = \frac{(x^3-6x^2+11x-6\,)(x^3-4x+3)}{x-1}$$

Now by Simple Division

$$x^2-5x+6$$
$$x-1 \;\big)\; x^3-6x^2+11x-6$$
$$\pm x^3 \;\mp x^2$$
$$-5x^2+11x-6$$
$$\mp 5x^2 \pm 5x$$
$$6x-6$$
$$\pm 6x \mp 6$$
$$\times$$

$So\ L.C.M = (x^2-5x+6)(x^3-4x+3)$

Chapter # 6

Review Ex # 6

Q4: Find the square root of :

(i): $\mathbf{4x^2 - 12x + 9}$

Solution:

$4x^2 - 12x + 9$

$4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + (3)^2$

$4x^2 - 12x + 9 = (2x-3)^2$

Taking Square on B.S

$$\sqrt{4x^2 - 12x + 9} = \pm\sqrt{(2x-3)^2}$$

$$\sqrt{4x^2 - 12x + 9} = \pm(2x-3)$$

Review Ex # 6

(ii): $\mathbf{x^4 + 4x^3 + 6x^2 + 4x + 1}$

Solution:

$x^4 + 4x^3 + 6x^2 + 4x + 1$

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ \hline x^2 & x^4 + 4x^3 + 6x^2 + 4x + 1 \\ & \pm x^4 \\ \hline 2x^2 + 2x & 4x^3 + 6x^2 + 4x + 1 \\ & \pm 4x^3 \pm 4x^2 \\ \hline 2x^2 + 4x + 1 & 2x^2 + 4x + 1 \\ & \pm 2x^2 \pm 4x \pm 1 \\ \hline & 0 \end{array}$$

So

$$\sqrt{x^4 + 4x^3 + 6x^2 + 4x + 1} = \pm(x^2 + 2x + 1)$$

Think

$\mathbf{Q5}$**: Simplify** $\dfrac{\mathbf{x^3 - y^3}}{\mathbf{x^3 - z^3}} \times \dfrac{\mathbf{x^2 + xy + xz + yz}}{\mathbf{x^4 + x^2y^2 + y^4}} \times \dfrac{\mathbf{x^3 + y^3}}{\mathbf{x^2 - y^2}}$

Solution:

$$\frac{x^3 - y^3}{x^3 + z^3} \times \frac{x^2 + xy + xz + yz}{x^4 + x^2y^2 + y^4} \times \frac{x^3 + y^3}{x^2 - y^2}$$

$$= \frac{(x-y)(x^2 + xy + y^2)}{(x+z)(x^2 - xz + z^2)} \times \frac{x(x+y) + z(x+y)}{x^4 + y^4 + x^2y^2} \times \frac{(x+y)(x^2 - xy + y^2)}{(x+y)(x-y)}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)}{(x^2)^2 + (y^2)^2 + 2x^2y^2 - 2x^2y^2 + x^2y^2} \times \frac{(x^2 - xy + y^2)}{1}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - x^2y^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2)^2 - (xy)^2}$$

$$= \frac{(x^2 + xy + y^2)}{(x^2 - xz + z^2)} \times \frac{(x+y)(x^2 - xy + y^2)}{(x^2 + y^2 + xy)(x^2 + y^2 - xy)}$$

$$= \frac{1}{(x^2 - xz + z^2)} \times \frac{(x+y)}{1}$$

$$= \frac{(x+y)}{(x-z)(x^2 + xz + z^2)}$$