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Class 10th (KPK)

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Chapter # 3

UNIT # 3

VARIATIONS

Ex # 3.1

Ratio

The comparison between two quantities of the same kind (same units) is called ratio.

Example

If a and b are two quantities of the same kind then ration is written as $a : b$ *or in fraction* $\frac{a}{b}$

Example # 1

Write the following ration in simplified form:

(i) $3 : 12$

The simplified form is $1 : 4$

(ii) $6a : 18b$

The simplified form is $a : 3b$

Example # 2

Divide Rs. 5070 among three persons in the ratio $2 : 5 : 6$

Solution:

$$\text{Amount} = Rs. 5070$$
$$Ratio = 2 : 5 : 6$$
$$Sum\ the\ Ratio = 2 + 5 + 6 = 13$$
$$\text{Share of 1st Person} = \frac{2}{13} \times 5070 = 2 \times 390 = \text{Rs. } 780$$
$$\text{Share of 2nd Person} = \frac{5}{13} \times 5070 = 5 \times 390 = \text{Rs. } 1950$$
$$\text{Share of 3rd Person} = \frac{6}{13} \times 5070 = 6 \times 390 = \text{Rs. } 2340$$

Proportion

A proportion is an equation that states that two ratios are equivalent.

Explanation

If a, b, c, d are four quantities then

Ex # 3.1

$$a : b :: c : d$$
$$a : b = c : d$$
$$\frac{a}{b} = \frac{c}{d}$$

Product of mean = Product of extreme

$$a \times d = b \times c$$

Example # 3

$a^3 - b^3, a^2 - b^2, a^2 + ab + b^2$ and x are in a proportion. Find the value of x

Solution:

$$a^3 - b^3, a^2 - b^2, a^2 + ab + b^2 \text{ and } x$$

As these are in proportion

$$a^3 - b^3 : a^2 - b^2 = a^2 + ab + b^2 : x$$

As we have

Product of mean = Product of extreme

$$(a^3 - b^3) \times x = (a^2 - b^2)(a^2 + ab + b^2)$$
$$(a - b)(a^2 + ab + b^2)x = (a + b)(a - b)(a^2 + ab + b^2)$$

Divide B.S $(a - b)(a^2 + ab + b^2)$

$$\frac{(a - b)(a^2 + ab + b^2)x}{(a - b)(a^2 + ab + b^2)} = \frac{(a + b)(a - b)(a^2 + ab + b^2)}{(a - b)(a^2 + ab + b^2)}$$
$$x = a + b$$

Variable quantity

If the value of a quantity changes under different situations, it is called a variable.

Example

Speed of train
Demand of a commodity
Population of a town

Variation

The change of variable parameters is called as variation

Example

If one quantity increase or decrease than what is its effect on other quantity.

Chapter # 3

Ex # 3.1

Direct variation

Direct variation is the relationship between two quantities, whereby if one quantity increases the other also increases or if one quantity decreases the other also decreases.

Explanation

If y varies directly with x

Then

x increases, y also increase

x decreases, y also decreases

Equation:

$$y \propto x$$
$$y = kx$$

Example

If absence fine per day is 5.

Then the fine for One day is 5 and the fine for four days is 20. So, it means if absentee increases fine also increases and when decreases then fine also decreases.

Inverse variation

If one quantity increases, the other decreases or if one quantity decreases the other increases, it is called inverse variation.

Explanation

If y varies inversely with x

Then

x increases, y also decrease

x decreases, y also increases

Equation:

$$y \propto \frac{1}{x}$$
$$y = \frac{k}{x} \quad OR \quad xy = k$$

Example

If workers increase to complete the work, then it will reduce the time

Constant quantities

If the value of a quantity remains unchanged under different situations, it is called a constant.

Example

$$3, \quad 4.45, \quad \frac{22}{7}$$

Ex # 3.1

Example # 4

Given that y varies directly with x and y and $y = 27$ when $x = 3$. Find

An equation connecting x and y

The value of y when $x = 11$

Solution:

As there is direct variation

$y \propto x$

$y = kx$ ……equ(i)

Put $x = 3$ and $y = 27$ in equ(i)

$27 = k(3)$

$\frac{27}{3} = \frac{k(3)}{3}$

$9 = k$

$k = 9$

So equ (i) becomes

$y = 9x$

Thus the equation connecting x and y is $y = 9x$

Now

To Find:

y when $x = 11$

$y = ?, x = 11$

Put $x = 11$ and $k = 9$ in equ(i)

$y = 9(11)$

$y = 99$

Example # 5

If $y \propto x$, then complete the following table.

x	4	5	8		
y	6			18	22.5

Solution:

As there is direct variation

$y \propto x$

$y = kx$ ……equ(i)

Put $x = 4$ and $y = 6$ in equ(i)

$6 = k(4)$

$\frac{6}{4} = \frac{k(4)}{4}$

$\frac{3}{2} = k$

$k = \frac{3}{2}$

Ex # 3.1

Now

Put $x = 5$ and $k = \frac{3}{2}$ in equ(i)

$y = \frac{3}{2}(5)$

$y = \frac{15}{2}$

$y = 7.5$

Now again

Put $x = 8$ and $k = \frac{3}{2}$ in equ(i)

$y = \frac{3}{2}(8)$

$y = \frac{24}{2}$

$y = 12$

Now again

Put $y = 18$ and $k = \frac{3}{2}$ in equ(i)

$18 = \frac{3}{2}(x)$

$\frac{2}{3} \times 18 = x$

$2 \times 6 = x$

$12 = x$

$x = 12$

Now again

Put $y = 22.5$ and $k = \frac{3}{2}$ in equ(i)

$22.5 = \frac{3}{2}(x)$

$\frac{2}{3} \times 22.5 = x$

$\frac{45}{3} = x$

$15 = x$

$x = 15$

x	4	5	8	12	15
y	6	7.5	12	18	22.5

Example # 6

If x varies inversely to y and $x = 3$, when $y = 12$. Find the value of y when $x = 6$

Solution:

As there is Inverse variation

$y \propto \frac{1}{x}$

Ex # 3.1

$y = \frac{k}{x}$ ... ... equ(i)

Put $x = 3$ and $y = 12$ in equ(i)

$12 = \frac{k}{3}$

$12 \times 3 = k$

$36 = k$

$k = 36$

Now

To Find:

y *when* $x = 6$

$y = ?, x = 6$

Put $x = 6$ and $k = 36$ in equ(i)

$y = \frac{36}{6}$

$y = 6$

Example # 7

Given that pressure 'P' on the quantity of gas in a container varies inversely as volume of gas 'V'. When pressure on gas is $10\ \text{N/m}^2$ its volume is 25 m^3. Find pressure when volume is 20 m^3.

Solution:

As there is Inverse variation

$P \propto \frac{1}{V}$

$P = \frac{k}{V}$ ... ... equ(i)

Put $P = 10$ and $V = 25$ in equ(i)

$10 = \frac{k}{25}$

$10 \times 25 = k$

$250 = k$

$k = 250$

Now

To Find:

P *when* $V = 20$

$P = ?, V = 20$

Put $V = 20$ and $k = 250$ in equ(i)

$P = \frac{250}{20}$

$P = 12.5$

Thus Pressure $= 12.5\ \text{N/m}^2$

Ex # 3.1

Page # 56

Q1: Which is the greater ratio, $5:7$ or 151: 208 ?

Solution:

As we have

$5:7$ or 151: 208

Now

$$5:7=\frac{5}{7}=0.714285$$

Also

$$151:208=\frac{151}{208}=0.725961$$

Hence 151: 208 is greater ratio.

Q2: Gold and silver are mixed in the ratio $7:4$. If 36 grams of silver is used. How much gold is used?

Solution:

Let gold used $=x$

$\text{Ratio of Gold and Silver}=7:4$

$Silver\ used=36\ grams$

Now the ratio Gold and Silver

$7:4=x:36$

As we have

$\text{Product of mean}=\text{Product of extreme}$

$4\times x=7\times 36$

$x=\frac{7\times 36}{4}$

$x=63$

Thus 63 grams of Gold is used

Q3: Divide the annual profit of Rs. 40,000 of a factory among 3 partners in the ratio of $5:8:12$

Solution:

$\text{Annual Profit}=Rs.40{,}000$

$Ratio=5:8:12$

$Sum\ the\ Ratio=5+8+12$

$=25$

$\text{Share of 1st Partner}=\frac{5}{25}\times 40000$

$=5\times 1600$

$=\text{Rs.}\,8000$

$\text{Share of 2nd Partner}=\frac{8}{25}\times 40000$

$=8\times 1600$

$=\text{Rs.}\,12800$

Ex # 3.1

$\text{Share of 3rd Partner}=\frac{12}{25}\times 40000$

$=12\times 1600$

$=\text{Rs.}\,19200$

Q4: If $11:x-1=22:27$, find the value of x

Solution:

$$11:x-1=22:27$$

As we have

Product of mean=Product of extreme

$22(x-1)=11\times 27$

$22x-22=297$

Add 22 on B.S

$22x-22+22=297+22$

$22x=319$

Divide B.S by 22

$\frac{22x}{22}=\frac{319}{22}$

$x=14.5$

Q5: There is a direct variation between x^2 and y. When $x=7$, $y=49$. Find:

(i) $y\ when\ x=9$

(ii) $x\ when\ y=100$

Solution:

As there is direct variation

$y\propto x^2$

$y=kx^2$ ... ... equ(i)

Put $x=7$ and $y=49$ in equ(i)

$49=k(7)^2$

$49=k(49)$

$\frac{49}{49}=\frac{k(49)}{49}$

$1=k$

$k=1$

Now

To Find:

$y\ when\ x=9$

$y=?,x=9$

Put $x=9$ and $k=1$ in equ(i)

$y=1(9)^2$

$y=1(81)$

$y=81$

Ex # 3.1

Now again

To Find:

x $when$ $y = 100$

$x = ?, y = 100$

Put $y = 100$ and $k = 1$ in equ(i)

$100 = 1(x^2)$

$100 = x^2$

$x^2 = 100$

Taking Square root on B.S

$\sqrt{x^2} = \sqrt{100}$

$x = 10$

Q6: There is inverse variation between x and y. When $x = 4, y = 6$. Find:

(i) y $when$ $x = 12$

(ii) x $when$ $y = 24$

Solution:

As there is Inverse variation

$y \propto \frac{1}{x}$

$y = \frac{k}{x}$ ……equ(i)

Put $x = 4$ and $y = 6$ in equ(i)

$6 = \frac{k}{4}$

$6 \times 4 = k$

$24 = k$

$k = 24$

Now To Find:

y $when$ $x = 12$

$y = ?, x = 12$

Put $x = 12$ and $k = 24$ in equ(i)

$y = \frac{24}{12}$

$y = 2$

Now again Find:

x $when$ $y = 24$

$x = ?, x = 24$

Put $y = 24$ and $k = 24$ in equ(i)

$24 = \frac{24}{x}$

$x = \frac{24}{24}$

$x = 1$

Ex # 3.1

Q7: $r \propto \frac{1}{p^3}$ and $p = 9$ $when$ $r = 2$. $Find$:

(i) r $when$ $p = 3$

(ii) p $when$ $r = \frac{1}{4}$

Solution:

$r \propto \frac{1}{p^3}$

$r = \frac{k}{p^3}$ ……equ(i)

Put $p = 9$ and $r = 2$ in equ(i)

$2 = \frac{k}{(9)^3}$

$2 = \frac{k}{729}$

$2 \times 729 = k$

$1458 = k$

$k = 1458$

Now

To Find:

r $when$ $p = 3$

$r = ?, p = 3$

Put $p = 3$ and $k = 1458$ in equ(i)

$r = \frac{1458}{(3)^3}$

$r = \frac{1458}{27}$

$r = 54$

Now again

To Find:

p $when$ $r = \frac{1}{4}$

$p = ?, r = \frac{1}{4}$

Put $r = \frac{1}{4}$ and $k = 1458$ in equ(i)

$\frac{1}{4} = \frac{1458}{p^3}$

By Cross Multiplication

$1 \times p^3 = 1458 \times 4$

$p^3 = 18^3$

Taking Cube root on B.S

$\sqrt[3]{p^3} = \sqrt[3]{18^3}$

$p = 18$

Ex # 3.1

Q8: If $y \propto x$, then complete the following table.

x	4	6		15
y	2		3.5	

Solution:

As there is direct variation

$y \propto x$

$y = kx$ ……equ(i)

Put $x = 4$ and $y = 2$ in equ(i)

$2 = k(4)$

$\frac{2}{4} = \frac{k(4)}{4}$

$\frac{1}{2} = k$

$k = \frac{1}{2}$

Now

Put $x = 6$ and $k = \frac{1}{2}$ in equ(i)

$y = \frac{1}{2}(6)$

$y = 3$

Now again

Put $y = 3.5$ and $k = \frac{1}{2}$ in equ(i)

$3.5 = \frac{1}{2}(x)$

$\frac{2}{1} \times 3.5 = x$

$7 = x$

$x = 7$

Now again

Put $x = 15$ and $k = \frac{1}{2}$ in equ(i)

$y = \frac{1}{2}(15)$

$y = \frac{15}{2}$

$y = 7.5$

x	4	6	7	15
y	2	3	3.5	7.5

Ex # 3.2

Third, fourth Mean and Continued Proportion

Continued Proportion

Three quantities are said to be in continued proportion, if the ratio between the first and the second is equal to the ratio between second and third.

Example

If a, b and c are in continued proportion then

$$a : b :: b : c$$

Product of mean=Product of extreme

$$\text{So } b^2 = ac$$

In the above example:

b is called mean proportion or geometric mean

c is called the third proportion

Fourth proportion

If four quantities a, b, c and d are:

$$a : b :: c : d$$

Here d is called fourth proportion

Example # 8

Find the mean proportional of 5 , 15

Solution:

Let the mean proportional $= x$

So 5 , x, 15 are in continued proportional

Now we write it

$5 : x = x : 15$

Product of mean=Product of extreme

$x \times x = 5 \times 15$

$x^2 = 75$

Taking square root on B.S

$\sqrt{x^2} = \sqrt{75}$

$x = \sqrt{25 \times 3}$

$x = \sqrt{25} \times \sqrt{3}$

$x = 5\sqrt{3}$

Example # 9

Find the mean proportional of a^2b^2 and abc

Solution:

Let the third proportional $= x$

So a^2b^2 , abc, x are in continued proportional

Now we write it

$a^2b^2 : abc = abc : x$

Product of mean=Product of extreme

Ex # 3.2

$abc \times abc = a^2b^2 \times x$

$a^2b^2c^2 = a^2b^2 \times x$

Divide B.S a^2b^2

$$\frac{a^2b^2c^2}{a^2b^2} = \frac{a^2b^2 \times x}{a^2b^2}$$

$c^2 = x$

$x = c^2$

Example # 10:

Find fourth proportion of $a^3 - b^3$, $a + b$ and $a^2 + ab + b^2$

Solution:

Let the fourth proportional $= x$

So

$a^3 - b^3, a + b, a^2 + ab + b^2, x$ are in proportional

Now we write it

$a^3 - b^3 : a + b = a^2 + ab + b^2 : x$

Product of mean=Product of extreme

$(a + b)(a^2 + ab + b^2) = (a^3 - b^3)x$

$(a + b)(a^2 + ab + b^2) = (a - b)(a^2 + ab + b^2)x$

Divide B.S $(a - b)(a^2 + ab + b^2)$

$$\frac{(a + b)(a^2 + ab + b^2)}{(a - b)(a^2 + ab + b^2)} = \frac{(a - b)(a^2 + ab + b^2)x}{(a - b)(a^2 + ab + b^2)}$$

$$\frac{(a + b)}{(a - b)} = x$$

$$x = \frac{(a + b)}{(a - b)}$$

Theorems on Proportion

Alternendo Property

If $a : b = c : d$ then $a : c = b : d$

It means that if the second and third term interchange their places, then also the four terms are in proportion.

Example

If $3 : 5 = 6 : 10$ then

$3 : 6 = \mathbf{1} : \mathbf{2} = 5 : 10$

Ex # 3.2

Invertendo Property

If $a : b = c : d$ then $b : a = d : c$

It means that if two ratios are equal, then their inverse are also equal.

Example

$6 : 10 = 9 : 15$ then

$10 : 6 = \mathbf{5} : \mathbf{3} = 15 : 9$

Componendo Property

If $a : b = c : d$ then $(a + b) : b = (c + d) : d$

Or

If $a : b = c : d$ then $\frac{a + b}{b} = \frac{c + d}{d}$

Example:

If $4 : 5 = 8 : 10$ then $(4 + 5) : 5 = (8 + 10) : 10$

Or

If $4 : 5 = 8 : 10$ then $\frac{4 + 5}{5} = \frac{8 + 10}{10}$

Dividendo Property

If $a : b = c : d$ then $(a - b) : b = (c - d) : d$

Or

If $a : b = c : d$ then $\frac{a + b}{b} = \frac{c + d}{d}$

Example:

If $5 : 4 = 10 : 8$ then $(5 - 4) : 4 = (10 - 8) : 8$

Or

If $5 : 4 = 10 : 8$ then $\frac{5 - 4}{4} = \frac{10 - 8}{8}$

Componendo-Dividendo Property

If $a : b :: c : d$ then

$(a + b) : (a - b) = (c + d) : (c - d)$

Or

If $a : b = c : d$ then $\frac{a + b}{a - b} = \frac{c + d}{c - d}$

Example

If $7 : 3 = 14 : 6$ then

$(7 + 3) : (7 - 3) = (14 + 6) : (14 - 6)$

Or

If $7 : 3 = 14 : 6$ then $\frac{7 + 3}{7 - 3} = \frac{14 + 6}{14 - 6}$

Ex # 3.2

Example # 11

If $\frac{a}{b}=\frac{c}{d}$ **then prove that**

$2a+3b : b = 2c+3d : d$

Solution:

As we have

$$\frac{a}{b}=\frac{c}{d}$$

To prove

$$2a+3b : b = 2c+3d : d$$

Now

$$\frac{a}{b}=\frac{c}{d}$$

Multiply on B.S by $\frac{2}{3}$

$$\frac{2}{3}\times\frac{a}{b}=\frac{2}{3}\times\frac{c}{d}$$

$$\frac{2a}{3b}=\frac{2c}{3d}$$

By Componenendo Property

$$\frac{2a+3b}{3b}=\frac{2c+3d}{3d}$$

Multiply on B.S by 3

$$3\times\frac{2a+3b}{3b}=3\times\frac{2c+3d}{3d}$$

$$\frac{2a+3b}{b}=\frac{2c+3d}{d}$$

OR

$$2a+3b : b = 2c+3d : d$$

Hence Proved

Example # 12

If $\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$ **Prove that** $\frac{a}{b}=\frac{c}{d}$

Solution:

As we have

$$\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$$

To prove

$$\frac{a}{b}=\frac{c}{d}$$

Now

$$\frac{3a-4b}{3a+4b}=\frac{3c-4d}{3c+4d}$$

By Componenendo – Dividendo Property

$$\frac{(3a-4b)+(3a+4b)}{(3a-4b)-(3a+4b)}=\frac{(3c-4d)+(3c+4d)}{(3c-4d)-(3c+4d)}$$

Ex # 3.2

$$\frac{3a-4b+3a+4b}{3a-4b-3a-4b}=\frac{3c-4d+3c+4d}{3c-4d-3c-4d}$$

$$\frac{3a+3a-4b+4b}{3a-3a-4b-4b}=\frac{3c+3c-4d+4d}{3c-3c-4d-4d}$$

$$\frac{6a}{-8b}=\frac{6c}{-8d}$$

Multiply on B.S by $\frac{-8}{6}$

$$\frac{-8}{6}\times\frac{6a}{-8b}=\frac{-8}{6}\times\frac{6c}{-8d}$$

$$\frac{a}{b}=\frac{c}{d}$$

Hence Proved

Example # 13

$$\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x+4)^2}=\frac{13}{12}$$

Solution:

$$\frac{(x+3)^2+(x-4)^2}{(x+3)^2-(x+4)^2}=\frac{13}{12}$$

By Componenendo – Dividendo Property

$$\frac{[(x+3)^2+(x-4)^2]+[(x+3)^2-(x+4)^2]}{[(x+3)^2+(x-4)^2]-[(x+3)^2-(x+4)^2]}=\frac{13+12}{13-12}$$

$$\frac{(x+3)^2+(x-4)^2+(x+3)^2-(x-4)^2}{(x+3)^2+(x-4)^2-(x+3)^2+(x-4)^2}=\frac{25}{1}$$

$$\frac{(x+3)^2+(x+3)^2+(x-4)^2-(x-4)^2}{(x+3)^2-(x+3)^2+(x-4)^2+(x-4)^2}=25$$

$$\frac{2(x+3)^2}{2(x-4)^2}=25$$

$$\frac{(x+3)^2}{(x-4)^2}=25$$

$$\left(\frac{x+3}{x-4}\right)^2=25$$

Taking square root on B.S

$$\sqrt{\left(\frac{x+3}{x-4}\right)^2}=\pm\sqrt{25}$$

$$\frac{x+3}{x-4}=\pm5$$

$$\frac{x+3}{x-4}=5 \quad or \quad \frac{x+3}{x-4}=-5$$

Ex # 3.2

Page # 60

Q1: Which of the following quantities are in continued proportion?

(i) 4 , 12 , 36

Solution:

As 4 , 12 , 36 are in continued proportional
So we can write it
$4 : 12 = 12 : 36$
Product of mean=Product of extreme
$12 \times 12 = 4 \times 36$
$144 = 144$
Thus 4 , 12 , 36 are in continued proportional

(ii) 3 , 12 , 39

Solution:

As 3 , 12 , 39 are in continued proportional
So we can write it
$3 : 12 = 12 : 39$
Product of mean=Product of extreme
$12 \times 12 = 3 \times 39$
$144 = 117$
Thus 4 , 12 , 36 are not in continued proportional

(iii) 72 , 24 , 8

Solution:

As 72 , 24 , 8 are in continued proportional
So we can write it
$72 : 24 = 24 : 8$
Product of mean=Product of extreme
$24 \times 24 = 72 \times 8$
$576 = 576$
Thus 72 , 24 , 8 are in continued proportional

Q2: Find the mean proportional of 12 , 3

Solution:

Let the mean proportional $= x$
So 12 , x, 3 are in continued proportional
Now we write it
$12 : x = x : 3$
Product of mean=Product of extreme
$x \times x = 12 \times 3$
$x^2 = 36$

Ex # 3.2

Taking square root on B. S

$\sqrt{x^2} = \sqrt{36}$
$x = 6$

Q3: If $5 : 15 : x$ are in continued proportional, find the value of x

Solution:

As $5 : 15 : x$ are in continued proportional
So we can write it
$5 : 15 = 15 : x$
Product of mean=Product of extreme
$15 \times 15 = 5 \times x$
$225 = 5x$

Divide B. S by 5

$\frac{225}{5} = \frac{5x}{5}$
$45 = x$
$x = 45$

Q4: If $3x - 1 : 4 : 35$ are in continued proportional, find the value of x

Solution:

As $3x - 1 : 4 : 35$ are in continued proportional
So we can write it
$3x - 1 : 4 = 4 : 35$
As we have
Product of mean=Product of extreme
$4 \times 4 = 35(3x - 1)$
$16 = 105x - 35$

Add 35 on B. S

$16 + 35 = 105x - 35 + 35$
$51 = 105x$

Divide B. S by 105

$\frac{51}{105} = \frac{105x}{105}$
$\frac{19}{51} = x$
$x = \frac{19}{51}$

Ex # 3.2

Q5: **Find the mean proportional of a^2-b^2 and $\frac{a+b}{a-b}$**

Solution:

Let the mean proportional $=x$

So

$a^2-b^2, x, \frac{a+b}{a-b}$ are in continued proportional

Now we write it

$$a^2-b^2 : x = x : \frac{a+b}{a-b}$$

Product of mean=Product of extreme

$$x\times x=(a^2-b^2)\left(\frac{a+b}{a-b}\right)$$

$$x^2=(a+b)(a-b)\left(\frac{a+b}{a-b}\right)$$

$$x^2=(a+b)(a+b)$$

$$x^2=(a+b)^2$$

Taking square root on B.S

$$\sqrt{x^2}=\sqrt{(a+b)^2}$$

$$x=a+b$$

Q6: **If $\frac{a}{b}=\frac{c}{d}$ then prove that $\frac{ac+bd}{ac-bd}=\frac{a^2+b^2}{a^2-b^2}$**

Solution:

As we have

$$\frac{a}{b}=\frac{c}{d}$$

To prove

$$\frac{ac+bd}{ac-bd}=\frac{a^2+b^2}{a^2-b^2}$$

Now

$$\frac{a}{b}=\frac{c}{d}$$

Multiply $\frac{a}{b}$ on B.S

$$\frac{a}{b}\times\frac{a}{b}=\frac{a}{b}\times\frac{c}{d}$$

$$\frac{a^2}{b^2}=\frac{ac}{bd}$$

By Componenendo − Dividendo Property

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac+bd}$$

OR

$$\frac{ac+bd}{ac+bd}=\frac{a^2+b^2}{a^2-b^2}$$

Ex # 3.2

Q7: **Solve the following equations.**

(i) $\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}}=\frac{4}{1}$

Solution:

$$\frac{\sqrt{3x+2}+\sqrt{x}}{\sqrt{3x+2}-\sqrt{x}}=\frac{4}{1}$$

By Componenendo − Dividendo Property

$$\frac{(\sqrt{3x+2}+\sqrt{x})+(\sqrt{3x+2}-\sqrt{x})}{(\sqrt{3x+2}+\sqrt{x})-(\sqrt{3x+2}-\sqrt{x})}=\frac{4+1}{4-1}$$

$$\frac{\sqrt{3x+2}+\sqrt{x}+\sqrt{3x+2}-\sqrt{x}}{\sqrt{3x+2}+\sqrt{x}-\sqrt{3x+2}+\sqrt{x}}=\frac{5}{3}$$

$$\frac{\sqrt{3x+2}+\sqrt{3x+2}+\sqrt{x}-\sqrt{x}}{\sqrt{3x+2}-\sqrt{3x+2}+\sqrt{x}+\sqrt{x}}=\frac{5}{3}$$

$$\frac{2\sqrt{3x+2}}{2\sqrt{x}}=\frac{5}{3}$$

$$\frac{\sqrt{3x+2}}{\sqrt{x}}=\frac{5}{3}$$

$$\sqrt{\frac{3x+2}{x}}=\frac{5}{3}$$

Taking square on B.S

$$\left(\sqrt{\frac{3x+2}{x}}\right)^2=\left(\frac{5}{3}\right)^2$$

$$\frac{3x+2}{x}=\frac{25}{9}$$

By Cross Multiplication

$$9(3x+2)=25\times x$$

$$27x+18=25x$$

$$27x-25x=-18$$

$$2x=-18$$

Divide B.S by 2

$$\frac{2x}{2}=\frac{-18}{2}$$

$$x=-9$$

$$\boldsymbol{S.S=\{-9\}}$$

Ex # 3.2

(ii) $$\frac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=-\frac{17}{8}$$

Solution:

$$\frac{(x-1)^2+(x+2)^2}{(x-1)^2-(x+2)^2}=-\frac{17}{8}$$

By Componenendo – Dividendo Property

$$\frac{[(x-1)^2+(x+2)^2]+[(x-1)^2-(x+2)^2]}{[(x-1)^2+(x+2)^2]-[(x-1)^2-(x+2)^2]}=\frac{-17+8}{-17-8}$$

$$\frac{(x-1)^2+(x+2)^2+(x-1)^2-(x+2)^2}{(x-1)^2+(x+2)^2-(x-1)^2+(x+2)^2}=\frac{9}{25}$$

$$\frac{(x-1)^2+(x-1)^2+(x+2)^2-(x+2)^2}{(x-1)^2-(x-1)^2+(x+2)^2+(x+2)^2}=\frac{9}{25}$$

$$\frac{2(x-1)^2}{2(x+2)^2}=\frac{9}{25}$$

$$\frac{(x-1)^2}{(x+2)^2}=\frac{9}{25}$$

$$\left(\frac{x-1}{x+2}\right)^2=\frac{9}{25}$$

Taking square root on B.S

$$\sqrt{\left(\frac{x-1}{x+2}\right)^2}=\pm\sqrt{\frac{9}{25}}$$

$$\frac{x-1}{x+2}=\pm\frac{3}{5}$$

$$\frac{x-1}{x+2}=\frac{3}{5}\quad or\quad \frac{x-1}{x+2}=-\frac{3}{5}$$

By Cross Multiplication

$$5(x-1)=3(x+2)\quad or\quad 5(x-1)=-3(x+2)$$

$$5x-5=3x+6\quad or\quad 5x-5=-3x-6$$

$$5x-3x=6+5\quad or\quad 5x+3x=-6+5$$

$$2x=11\quad or\quad 8x=-1$$

$$x=\frac{11}{2}\quad or\quad x=\frac{-1}{8}$$

$$\mathbf{S.S}=\left\{\frac{11}{2}\ ,\quad \frac{-1}{8}\right\}$$

(iii) $$\frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\frac{1}{3}$$

Solution:

$$\frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\frac{1}{3}$$

By Componenendo – Dividendo Property

$$\frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)+\left(\sqrt{x^2+a^2}+\sqrt{x^2-a^2}\right)}{\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)-\left(\sqrt{x^2+a^2}+\sqrt{x^2-a^2}\right)}=\frac{1+3}{1-3}$$

$$\frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}+\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}-\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}=\frac{4}{-2}$$

$$\frac{2\sqrt{x^2+a^2}}{-2\sqrt{x^2-a^2}}=-2$$

$$-\frac{2\sqrt{x^2+a^2}}{2\sqrt{x^2-a^2}}=-2$$

$$\frac{\sqrt{x^2+a^2}}{\sqrt{x^2-a^2}}=2$$

$$\sqrt{\frac{x^2+a^2}{x^2-a^2}}=2$$

Taking square on B.S

$$\left(\sqrt{\frac{x^2+a^2}{x^2-a^2}}\right)^2=(2)^2$$

$$\frac{x^2+a^2}{x^2-a^2}=4$$

$$x^2+a^2=4(x^2-a^2)$$

$$x^2+a^2=4x^2-4a^2$$

$$a^2+4a^2=4x^2-x^2$$

$$5a^2=3x^2$$

$$3x^2=5a^2$$

$$x^2=\frac{5a^2}{3}$$

Taking square on B.S

$$\sqrt{x^2}=\pm\sqrt{\frac{5a^2}{3}}$$

$$x=\pm\sqrt{\frac{5}{3}}\,a$$

$$\mathbf{S.S}=\left\{\pm\sqrt{\frac{5}{3}}\,a\right\}$$

Ex # 3.3

Joint variation

A combination of direct and inverse variation of one or more variables forms joint variation.

If y varies jointly as x and z

Then

$$y \propto xz$$

If y varies directly as x and inversely as z

Then

$$y \propto \frac{x}{z}$$

Example:

Area of a triangle $= \frac{1}{2}bh$

Here the constant k is $\frac{1}{2}$

Area of a triangle varies jointly with base 'b' and height 'h'

Example # 14 (imp)

If y varies jointly as x and z, and $y = 12$ when $x = 9$ and $z = 3$, find z when $y = 6$ and $x = 15$.

Solution:

As y varies jointly as x and z

So

$y \propto xz$

$y = kxz$ …… equ(i)

Put $y = 12, x = 9$ and $z = 3$ in equ(i)

$12 = k(9)(3)$

$\frac{12}{(9)(3)} = k$

$\frac{4}{(9)(1)} = k$

$\frac{4}{9} = k$

$k = \frac{4}{9}$

Now

To Find:

z when $x = 15$ and $y = 6$

$z = ?, x = 15$ and $y = 6$

Put $x = 15, y = 6$ and $k = \frac{4}{9}$ in equ(i)

$6 = \left(\frac{4}{9}\right)(15)(z)$

$6 = \left(\frac{4}{3}\right)(5)(z)$

Ex # 3.3

$6 = \left(\frac{20}{3}\right)(z)$

$6 \times \frac{3}{20} = z$

$\frac{18}{20} = z$

$\frac{9}{10} = z$

$z = \frac{9}{10}$

Ex # 3.3

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Q1: If y varies jointly as x and z, and $y = 33$ when $x = 9$ and $z = 12$, find y when $x = 16$ and $z = 22$.

Solution:

As y varies jointly as x and z

So

$y \propto xz$

$y = kxz$ …… equ(i)

Put $y = 33, x = 9$ and $z = 12$ in equ(i)

$33 = k(9)(12)$

$\frac{33}{(9)(12)} = k$

$\frac{11}{3(12)} = k$

$\frac{11}{36} = k$

$k = \frac{11}{36}$

Now

To Find:

y when $x = 16$ and $z = 22$

$y = ?, x = 16$ and $z = 22$

Put $x = 16, z = 22$ and $k = 1$ in equ(i)

$y = \left(\frac{11}{36}\right)(16)(22)$

$y = \left(\frac{11}{9}\right)(4)(22)$

$y = \left(\frac{11}{9}\right)(88)$

$y = \frac{968}{9}$

Ex # 3.3

Q2: **If f varies jointly as g and the cube of h, and $f = 200$ when $g = 5$ and $h = 4$, find f when $g = 3$ and $h = 6$**

Solution:

As f varies jointly as g and h^3

So

$f \propto gh^3$

$f = kgh^3$ ... ... equ(i)

Put $f = 200, g = 5$ and $h = 4$ in equ(i)

$200 = k(5)(4)^3$

$200 = k(5)(64)$

$\frac{200}{(5)(64)} = k$

$\frac{40}{(1)(64)} = k$

$\frac{5}{8} = k$

$k = \frac{5}{8}$

Now

To Find:

f when $g = 3$ and $h = 6$

$f = ?, g = 3$ and $h = 6$

Put $g = 3, h = 6$ and $k = \frac{5}{8}$ in equ(i)

$f = \left(\frac{5}{8}\right)(3)(6)^3$

$f = \left(\frac{5}{8}\right)(3)(216)$

$f = (5)(3)(27)$

$f = (15)(27)$

$f = 405$

Q3: **Suppose a is jointly proportional to b and c. If $a = 4$ when $b = 8$ and $c = 9$, then what is a when $b = 2$ and $c = 18$?**

Solution:

As a is jointly proportional to b and c

So

$a \propto bc$

$a = kbc$ ... ... equ(i)

Put $a = 4, b = 8$ and $c = 9$ in equ(i)

$4 = k(8)(9)$

$\frac{4}{(8)(9)} = k$

Ex # 3.3

$\frac{1}{2(9)} = k$

$\frac{1}{18} = k$

$k = \frac{1}{18}$

Now

To Find:

a when $b = 2$ and $c = 18$

$a = ?, b = 2$ and $c = 18$

Put $b = 2, c = 18$ and $k = \frac{1}{18}$ in equ(i)

$a = \left(\frac{1}{18}\right)(2)(18)$

$a = 2$

Q4: **If p varies jointly as q and r squared, and $p = 225$ when $q = 4$ and $r = 3$, find p when $q = 6$ and $r = 8$.**

Solution:

p varies jointly as q and r^2

$p \propto qr^2$

$p = kqr^2$ ... ... equ(i)

Put $p = 225, q = 4$ and $r = 3$ in equ(i)

$225 = k(4)(3)^2$

$225 = k(4)(9)$

$\frac{225}{(4)(9)} = k$

$\frac{25}{(4)(1)} = k$

$\frac{25}{4} = k$

$k = \frac{25}{4}$

To Find:

$p = ?, q = 6$ and $r = 8$

Put $q = 6, r = 8$ and $k = \frac{25}{4}$ in equ(i)

$p = \left(\frac{25}{4}\right)(6)(8)^2$

$p = \left(\frac{25}{4}\right)(6)(64)$

$p = \left(\frac{25}{1}\right)(6)(16)$

$p = (25)(6)(16)$

$p = 2400$

Ex # 3.3

Q5: If a varies jointly as b cubed and c, and $a = 36$ when $b = 4$ and $c = 6$, find a when $b = 2$ and $c = 14$.

Solution:

As a is jointly proportional to b cubed and c

So

$a \propto bc$

$a = kb^3c$ ……equ(i)

Put $a = 36, b = 4$ and $c = 6$ in equ(i)

$36 = k(4)^3(6)$

$36 = k(64)(6)$

$\frac{36}{(64)(6)} = k$

$\frac{6}{64} = k$

$\frac{3}{32} = k$

$k = \frac{3}{32}$

Now

To Find:

a when $b = 2$ and $c = 14$

$a =?, b = 2$ and $c = 14$

Put $b = 2, c = 14$ and $k = \frac{3}{32}$ in equ(i)

$a = \left(\frac{3}{32}\right)(2)^3(14)$

$a = \left(\frac{3}{32}\right)(8)(14)$

$a = \left(\frac{3}{4}\right)(1)(14)$

$a = \left(\frac{3}{2}\right)(1)(7)$

$a = \frac{21}{2}$

Q6: If z varies jointly as x and y, and $z = 12$ when $x = 2$ and $y = 4$, find the constant of variation.

Solution:

As z varies jointly as x and y

So

$z \propto xy$

$z = kxy$ ……equ(i)

Put $z = 12, x = 2$ and y $= 4$ in equ(i)

$12 = k(2)(4)$

Ex # 3.3

$12 = k(8)$

$\frac{12}{8} = k$

$\frac{3}{2} = k$

$k = \frac{3}{2}$

Q7: f y varies jointly as x^2 and z, and $y = 6$ when $x = 4$ and $z = 9$ write y as a function of x and z and determine the value of y when $x = -8$ and $z = 12$.

Solution:

As y varies jointly as x^2 and z

So

$y \propto x^2z$

$y = kx^2z$ ……equ(i)

Put $y = 6, x = 4$ and $z = 9$ in equ(i)

$6 = k(4)^2(9)$

$6 = k(16)(9)$

$\frac{6}{(16)(9)} = k$

$\frac{3}{(8)(9)} = k$

$\frac{1}{(8)(3)} = k$

$\frac{1}{24} = k$

$k = \frac{1}{24}$

Now

To Find:

y when $x = 16$ and $z = 22$

$y =?, x = 16$ and $z = 22$

Put $x = 16, z = 22$ and $k = \frac{1}{24}$ in equ(i)

$y = \left(\frac{1}{24}\right)(16)(22)$

$y = \left(\frac{1}{24}\right)(-8)^2(12)$

$y = \left(\frac{1}{24}\right)(64)(12)$

$y = \left(\frac{1}{2}\right)(64)(1)$

$y = 32$

Ex # 3.3

Q8: If p varies jointly as q and r^2 and inversely as s and t^2, $p = 40$ when $q = 8$ and $r = 5$, $s = 3$ and $t = 2$. Find p in terms of q, r, s and t. Also find the value of when $q = -2$ and $r = 4$, $s = 3$ and $t = -1$.

Solution:

As p varies jointly as q and r^2 and inversely as s and t^2

So

$$p \propto \frac{qr^2}{st^2}$$

$$p = k\frac{qr^2}{st^2} \quad \ldots\ldots \text{equ(i)}$$

Put $p = 40, q = 8, r = 3, s = 3$ and $t = 2$ in equ(i)

$$40 = k\frac{(8)(5)^2}{(3)(2)^2}$$

$$40 = k\frac{(8)(25)}{(3)(4)}$$

$$40 = k\frac{(2)(25)}{(3)(1)}$$

$$40 = k\frac{50}{3}$$

$$40 \times \frac{3}{50} = k$$

$$4 \times \frac{3}{5} = k$$

$$\frac{12}{5} = k$$

$$k = \frac{25}{4}$$

Now

To Find:

$q = -2$ and $r = 4, s = 3$ and $t = -1$

$p = ?, q = -2, r = 4, s = 3$ and $t = -1$

Put $q = -2, r = 4, s = 3, t = -1$ and $k = \frac{12}{5}$ in equ(i)

$$p = \left(\frac{12}{5}\right)\frac{(-2)(4)^2}{(3)(-1)^2}$$

$$p = \left(\frac{12}{5}\right)\frac{(-2)(16)}{(3)(1)}$$

$$p = \left(\frac{4}{5}\right)\frac{(-2)(16)}{(1)(1)}$$

$$p = \frac{-128}{5}$$

Ex # 3.4

K – Method

If $a : b :: c : d$ is a proportion, then putting each ratio equal to k

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k$$

$$a = kb, \qquad c = kd$$

These equations are used to evaluate certain expressions more easily. This method is called K – Method.

Example # 15:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ then prove that each of the ratios is equal to $\frac{la + mc + ne}{lb + md + nf}$

Solution:

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \ldots\ldots \text{equ(i)}$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k, \qquad \frac{e}{f} = k$$

$$a = kb, \qquad c = kd, \qquad e = kf$$

$$\frac{la + mc + ne}{lb + md + nf} = \frac{lkb + mkd + nkf}{lb + md + nf}$$

$$\frac{la + mc + ne}{lb + md + nf} = \frac{k(lb + md + nf)}{lb + md + nf}$$

$$\frac{la + mc + ne}{lb + md + nf} = k \quad \ldots\ldots \text{equ(ii)}$$

Thus from equ (i) and equ (ii)

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{la + mc + ne}{lb + md + nf}$$

Example # 16:

Prove that $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a + c + e}{b + d + f}$

Solution:

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \ldots\ldots \text{equ(i)}$$

$$\frac{a}{b} = k, \qquad \frac{c}{d} = k, \qquad \frac{e}{f} = k$$

$$a = kb, \qquad c = kd, \qquad e = kf$$

$$\frac{a + c + e}{b + d + f} = \frac{kb + kd + kf}{b + d + f}$$

$$\frac{a + c + e}{b + d + f} = \frac{k(b + d + f)}{b + d + f}$$

Ex # 3.4

$$\frac{a+c+e}{b+d+f} = k \quad \dots\dots \text{equ(ii)}$$

Thus from equ (i) and equ (ii)

$$\frac{a+c+e}{b+d+f} = \frac{a+c+e}{b+d+f}$$

Example # 17:

Prove that $\boldsymbol{\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}}$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \quad \dots\dots \text{equ(i)}$

$\frac{a}{b} = k, \quad \frac{c}{d} = k, \quad \frac{e}{f} = k$

$a = kb, \quad c = kd, \quad e = kf$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{(kb)^2+(kd)^2+(kf)^2}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{k^2b^2+k^2d^2+k^2f^2}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{k^2}$$

$$\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = k \quad \dots\dots \text{equ(i)}$$

Thus from equ (i) and equ (ii)

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

Example # 18:

If $\boldsymbol{\frac{a}{x} = \frac{b}{y} = \frac{c}{z}}$ **then prove that**

$$\boldsymbol{\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}}$$

Solution:

Let $\frac{a}{x} = \frac{b}{y} = \frac{c}{z} = k$

As each ratio equal to k

So

Ex # 3.4

$\frac{a}{x} = k, \quad \frac{b}{y} = k, \quad \frac{c}{z} = k$

Now take the reciprocal

$\frac{x}{a} = \frac{1}{k}, \quad \frac{y}{b} = \frac{1}{k}, \quad \frac{z}{c} = \frac{1}{k} \quad \dots\dots \text{equ(i)}$

To Prove:

$$\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Now

Take cube on equ(i)

$$\left(\frac{x}{a}\right)^3 = \left(\frac{1}{k}\right)^3, \left(\frac{y}{b}\right)^3 = \left(\frac{1}{k}\right)^3, \left(\frac{z}{c}\right)^3 = \left(\frac{1}{k}\right)^3$$

$\frac{x^3}{a^3} = \frac{1}{k^3}, \quad \frac{y^3}{b^3} = \frac{1}{k^3}, \quad \frac{z^3}{c^3} = \frac{1}{k^3}$

Now add them

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{1}{k^3} + \frac{1}{k^3} + \frac{1}{k^3}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{1+1+1}{k^3}$$

$$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3}{k^3} \quad \dots\dots \text{equ(ii)}$$

Now take the product of equ (i)

$$\frac{x}{a} \cdot \frac{y}{b} \cdot \frac{z}{c} = \frac{1}{k} \cdot \frac{1}{k} \cdot \frac{1}{k}$$

$$\frac{xyz}{abc} = \frac{1}{k^3}$$

Multiply B.S by 3

$$3 \times \frac{xyz}{abc} = 3 \times \frac{1}{k^3}$$

$$\frac{3xyz}{abc} = \frac{3}{k^3} \quad \dots\dots \text{equ(iii)}$$

From equ (ii) and equ (iii)

$$\frac{x^3}{a^3} = \frac{y^3}{b^3} = \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Ex # 3.4

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Q1: (i) **If $\frac{a}{b}=\frac{c}{d}$ then prove that**

$$\frac{2a+3b}{2a-3b}=\frac{2c+3d}{2c-3d}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k,\quad \frac{c}{d}=k$

$a=kb,\quad c=kd$

As we have

$$\frac{2a+3b}{2a-3b}=\frac{2c+3d}{2c-3d}$$

L.H.S:

$$\frac{2a+3b}{2a-3b}=\frac{2kb+3b}{2kb-3b}$$

$$\frac{2a+3b}{2a-3b}=\frac{b(2k+3)}{b(2k-3)}$$

$$\frac{2a+3b}{2a-3b}=\frac{2k+3}{2k-3}$$

R.H.S:

$$\frac{2c+3d}{2c-3d}=\frac{2kd+3d}{2kd-3d}$$

$$\frac{2c+3d}{2c-3d}=\frac{d(2k+3)}{d(2k-3)}$$

$$\frac{2c+3d}{2c-3d}=\frac{2k+3}{2k-3}$$

L.H.S=R.H.S

Q1: (ii) **If $\frac{a}{b}=\frac{c}{d}$ then prove that $\frac{pa+qb}{ma-nb}=\frac{pc+qd}{mc-nd}$**

Solution:

Let $\frac{a}{b}=\frac{c}{d}=k$

$\frac{a}{b}=k,\quad \frac{c}{d}=k$

$a=kb,\quad c=kd$

As we have

$$\frac{pa+qb}{ma-nb}=\frac{pc+qd}{mc-nd}$$

L.H.S:

$$\frac{pa+qb}{ma-nb}=\frac{pkb+qb}{mkb-nb}$$

$$=\frac{b(pk+q)}{b(mk-n)}$$

$$=\frac{pk+q}{mk-n}$$

Ex # 3.4

R.H.S:

$$\frac{pc+qd}{mc-nd}=\frac{pkd+qd}{mkd-nd}$$

$$=\frac{d(pk+q)}{d(mk-n)}$$

$$=\frac{pk+q}{mk-n}$$

L.H.S=R.H.S

Q2: **Prove that $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$**

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ …… equ(i)

$\frac{a}{b}=k,\quad \frac{c}{d}=k,\quad \frac{e}{f}=k$

$a=kb,\quad c=kd,\quad e=kf$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{p(kb)^2+q(kd)^2+(kf)^2}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{pk^2b^2+qk^2d^2+k^2f^2}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{\frac{k^2(pb^2+qd^2+f^2)}{pb^2+qd^2+f^2}}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=\sqrt{k^2}$$

$$\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}=k \text{ …… equ(ii)}$$

From equ(i) and equ(ii), we get

$$\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\sqrt{\frac{pa^2+qc^2+e^2}{pb^2+qd^2+f^2}}$$

Ex # 3.4

Q3: **If** $\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}$ **then prove that** $x=y=z$ *where* x, y *and* z *are non − zero numbers and* $z+y+z\neq 0$

Solution:

As we have

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}$$

Now we know that

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=\frac{x-y+y-z+z-x}{z+x+y}$$

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=\frac{x-x-y+y-z+z}{x+y+z}$$

$$\frac{x-y}{z}=\frac{y-z}{x}=\frac{z-x}{y}=0$$

$$\frac{x-y}{z}=0,\quad \frac{y-z}{x}=0,\quad \frac{z-x}{y}=0$$

$$x-y=0\times z\ ,y-z=0\times x\ ,z-x=0\times y$$

$$x-y=0,\quad y-z=0,\quad z-x=0$$

$$x=y,\quad y=z,\quad z=x$$

Now by Transitive Property

$$x=y=z$$

Hence Proved

Q4: $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}$ **then prove that** $\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$

Solution:

Let $\frac{2y+2z-x}{a}=\frac{2z+2x-y}{b}=\frac{2x+2y-z}{c}=k$

So

$$\frac{2y+2z-x}{a}=k$$

$$2y+2z-x=ak\quad \ldots\ldots \text{equ(i)}$$

$$\frac{2z+2x-y}{b}=k$$

$$2z+2x-y=bk\quad \ldots\ldots \text{equ(ii)}$$

$$\frac{2x+2y-z}{c}=k$$

$$2x+2y-z=ck\quad \ldots\ldots \text{equ(iii)}$$

Arrange them

$$-x+2y+2z=ak\quad \ldots\ldots \text{equ(iv)}$$

$$2x-y+2z=bk\quad \ldots\ldots \text{equ(v)}$$

$$2x+2y-z=ck\quad \ldots\ldots \text{equ(vi)}$$

Multiplied equ (iv) by − 1 $\quad x-2y-2z=-ak$
Multiplied equ (v) by 2 $\quad 4x-2y+4z=2bk$
Multiplied equ (vi) by 2 $\quad 4x+4y-2z=2ck$
Now Add them $\quad 9x=-ak+2bk+2ck$

$$9x=k(-a+2b+2c)$$

$$9x=k(2b+2c-a)$$

$$\frac{x}{2b+2c-a}=\frac{k}{9}\quad \ldots\ldots \text{equ(vii)}$$

Multiplied equ (iv) by 2 $\quad -2x+4y+4z=2ak$
Multiplied equ (v) by − 1 $\quad -2x+y-2z=-bk$
Multiplied equ (vi) by 2 $\quad 4x+4y-2z=2ck$
Now Add them $\quad 9y=2ak-bk+2ck$

$$9y=k(2a-b+2c)$$

$$9y=k(2c+2a-b)$$

$$\frac{x}{2c+2a-b}=\frac{k}{9}\quad \ldots\ldots \text{equ(viii)}$$

Multiplied equ (iv) by 2 $\quad -2x+4y+4z=2ak$
Multiplied equ (v) by 2 $\quad 4x-2y+4z=2bk$
Multiplied equ (vi) by − 1 $\quad -2x-2y+z=-ck$
Now Add them $\quad 9z=2ak+2bk-ck$

$$9z=k(2a+2b-c)$$

$$9y=k(2a+2b-c)$$

$$\frac{x}{2a+2b-c}=\frac{k}{9}\quad \ldots\ldots \text{equ(ix)}$$

From equ (vii), (viii) and (ix), we get

$$\frac{x}{2b+2c-a}=\frac{y}{2c+2a-b}=\frac{z}{2a+2b-c}$$

Q5: **Prove that each of its fraction in** $\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}$ **is equal to** $\frac{x+y+z}{a+b+c}$

Solution:

As we have

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}$$

Now we know that

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{x+y+y+z+z+x}{a+b+b+c+c+a}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{2x+2y+2z}{2a+2b+2c}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{2(x+y+z)}{2(a+b+c)}$$

$$\frac{x+y}{a+b}=\frac{y+z}{b+c}=\frac{z+x}{c+a}=\frac{x+y+z}{a+b+c}$$

Hence Proved

Ex # 3.4

Q6: If $\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}$ then $(a+b+c)(x+y+z)=ax+by+cz$

Solution:

Let

$$\frac{bz+cy}{b-c}=\frac{cx+az}{c-a}=\frac{ay+bx}{a-b}=k$$

$$\frac{bz+cy}{b-c}=k,\qquad \frac{cx+az}{c-a}=k,\qquad \frac{ay+bx}{a-b}=k$$

$$bz+cy=k(b-c)$$

$$bz+cy=kb-kc$$

$$cx+az=k(c-a)$$

$$cx+az=kc-ka$$

$$ay+bx=k(a-b)$$

$$ay+bx=ka-kb$$

Add equ (i), (ii) and (iii)

$$bz+cy+cx+az+ay+bx=kb-kc+kc-ka+ka-kb$$

$$bz+cy+cx+az+ay+bx=0$$

Add ax, by, cz on B.S

$$bz+cy+cx+az+ay+bx+ax+by+cz=ax+by+cz$$

Re-arrange it

$$ax+ay+az+bx+by+bz+cx+cy+cz=ax+by+cz$$

Now

$$a(x+y+z)+b(x+y+z)+c(x+y+z)=ax+by+cz$$

$$(x+y+z)(a+b+c)=ax+by+cz$$

$$(a+b+c)(x+y+z)=ax+by+cz$$

Q7: If $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$ then $(b-c)x+(c-a)y+(a-b)z=0$

Solution:

Let $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}=k$

$$\frac{x}{b+c-a}=k,\qquad \frac{y}{c+a-b}=k,\qquad \frac{z}{a+b-c}=k$$

$$\frac{x}{b+c-a}=k$$

$$x=k(b+c-a)$$

$$x=kb+kc-ka$$

$$\frac{y}{c+a-b}=k$$

$$y=k(c+a-b)$$

$$y=kc+ka-kb$$

$$\frac{z}{a+b-c}=k$$

$$z=k(a+b-c)$$

$$z=ka+kb-kc$$

Ex # 3.4

L.H.S:

$(b-c)x+(c-a)y+(a-b)z$

Put the values of x, y and z

$(b-c)(kb+kc-ka)+(c-a)(kc+ka-kb)+(a-b)(ka+kb-kc)$

$=kb^2+kbc-kab-kbc-kc^2+kac+kc^2+kac-kbc-kac-ka^2+kab+ka^2+kab-kac-kab-kb^2+kbc$

$=0$

R.H.S

Q8: **If $2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$ then $7x+6y+8z=0$**

Solution:

As we know that $a:b:c=x:y:z$ Then $a+b+c=x+y+c$

So

$2x+3y:3y+4z:4z+5x=4a-5b:3b-a:2b-3a$

$2x+3y+3y+4z+4z+5x=4a-5b+3b-a+2b-3a$

$2x+5x+3y+3y+4z+4z=4a-a-3a-5b+3b+2b$

$7x+6y+8z=3a-3a-5b+5b$

$7x+6y+8z=0$

Q9: **If $\frac{a-b}{d-e}=\frac{b-c}{e-f}$ then each of them is equal to $\frac{b\{(f-d)+(cd-af)\}}{e(f-d)}$**

Solution:

Let $\frac{a-b}{d-e}=\frac{b-c}{e-f}=k$

$\frac{a-b}{d-e}=k,\qquad \frac{b-c}{e-f}=k$

$a-b=k(d-e)$

$a-b=dk-ek$

Multiply B.S by f

$f(a-b)=f(dk-ek)$

$af-bf=dfk-efk$ ... ... equ(i)

Also $b-c=k(e-f)$

$b-c=ek-fk$

Multiply B.S by d

$d(b-c)=d(ek-fk)$

$bd-cd=dek-dfk$ ... ... equ(ii)

Add equ(i) and equ(ii)

$af-bf+bd-cd=dfk-efk+dek-dfk$

$af-bf+bd-cd=-efk+dek$

Multiply B.S by -1

$-1(af-bf+bd-cd)=-1(-efk+dek)$

$-af+bf-bd+cd=efk-dek$

$bf-bd+cd-af=k(ef-de)$

$\frac{b(f-d)+cd-af}{ef-de}=k$

$\frac{b(f-d)+(cd-af)}{e(f-d)}=k$

Ex # 3.5

Example 19:

A stone is dropped from the top of a hill. The distance it falls is proportional to the square of the time of fall. The stone falls 19.6 m after 2 seconds, how far does it fall after 3 seconds?

Solution:

As there is direct variation. Thus

$d \propto t^2$

$d \propto t^2$ ... ... equ(i)

Put $d = 19.6$ and $t = 2$ in equ(i)

$19.6 = k(2)^2$

$19.6 = k(4)$

$\frac{19.6}{4} = k$

$4.9 = k$

$k = 4.9$

Now

To Find:

$d\ when\ t = 3$

$d = ?$ and $t = 3$

Put $t = 3$ and $k = 4.9$ in equ(i)

$d = (4.9)(3)^2$

$d = (4.9)(9)$

$d = 44.1$

Thus it has fallen 44.1 m after 3 seconds

Example 20:

Height of an image y on a screen varies directly as distance x of the projector from the screen. Height of the image is 20 cm when distance of the projector from the screen is 100 cm. At what distance should the projector kept from the screen so that the height of an image on the screen be 15 cm.

Solution:

As Height of an image $= y$

And Distance of projector $= x$

As there is direct variation. Thus

$y \propto x$

$y = kx$ ... ... equ(i)

Put $x = 100$ and $y = 20$ in equ(i)

$20 = k(100)$

$\frac{20}{100} = \frac{k(100)}{100}$

Ex # 3.5

$\frac{1}{5} = k$

$k = \frac{1}{5}$

Now

Put $y = 15$ and $k = \frac{1}{5}$ in equ(i)

$15 = \frac{1}{5}(x)$

$5 \times 15 = x$

$75 = x$

$x = 75$

Thus Distance of projectro from screen = 75 cm

Example 21:

The ratio of the mass of sand to cement in a particular type of concrete is $4.8 : 2$. If 6 kg of sand are used, how much cement is needed?

Solution:

Let the cement required $= x\ kg$

Now the ratio between sand and cement is given by:

$sand : cement$

$4.8 : 2$ ↓

$6 : x$ ↓

As there is direct variation

So

$\frac{4.8}{6} = \frac{2}{x}$

By cross multiplication

$4.8 \times x = 2 \times 6$

$4.8x = 12$

Divide B.S by 4.8

$\frac{4.8x}{4.8} = \frac{12}{4.8}$

$x = 2.5$

Thus the cement required $= 2.5\ kg$

Ex # 3.5

Example 22:

4 people can paint a fence in 3 hours.
How long will it take 6 people to paint it?
How many people are needed to complete the job in half an hour?

Solution:

$\text{As Number of people} = \text{P}$

$\text{And Time to complete work} = \text{T}$

As there is Inverse variation

$T \propto \frac{1}{P}$

$T = \frac{k}{P} \quad \dots\dots \text{equ(i)}$

$\text{Put } T = 3 \text{ and P} = 4 \text{ in equ(i)}$

$3 = \frac{k}{4}$

$3 \times 4 = k$

$12 = k$

$k = 12$

Now

To Find:

$T \; when \; P = 10$

$T = ?, P = 6$

$\text{Put } P = 6 \text{ and } k = 12 \text{ in equ(i)}$

$T = \frac{12}{6}$

$T = 2$

$\text{Thus Time to complete work} = 2 \text{ hrs}$

Now again

To Find:

$P \; when \; T = \frac{1}{2}$

$P = ?, T = 6$

$\text{Put } T = 6 \text{ and } k = 12 \text{ in equ(i)}$

$\frac{1}{2} = \frac{12}{P}$

By Cross Multiplication

$1 \times P = 12 \times 2$

$P = 24$

$\text{Thus Number of people required} = 24$

Ex # 3.5

Page # 68

Q1: **A hedge is made of wooden planks. The thickness (T) of the hedge varies directly as number of planks (N). 4 planks make 12 cm thick edge. Find**

(i) **Thickness of the hedge when number of planks is 6.**

(ii) **Number of planks when thickness of the hedge is 9cm**

Solution:

$\text{As thickness of the hedge} = \text{T}$

$\text{And number of planks} = \text{N}$

$\text{As there is direct variation. Thus}$

$T \propto N$

$T = kN \quad \dots\dots \text{equ(i)}$

$\text{Put T} = 12 \text{ and N} = 12 \text{ in equ(i)}$

$12 = k(4)$

Divide B.S by 4

$\frac{12}{4} = \frac{k(4)}{4}$

$3 = k$

$k = 3$

Now

To Find:

$T \; when \; N = 6$

$T = ?, N = 6$

$\text{Put } N = 6 \text{ and } k = 3 \text{ in equ(i)}$

$T = 3(6)$

$T = 18$

$\text{Thus thickness of the hedge} = 18 \text{ cm}$

Now again

To Find:

$N \; when \; T = 9$

$N = ?, T = 9$

$\text{Put } T = 9 \text{ and } k = 3 \text{ in equ(i)}$

$9 = (3)\text{N}$

Divide B.S by 3

$\frac{9}{3} = \frac{(3)\text{N}}{3}$

$3 = N$

$N = 3$

$\text{Also number of planks} = 3$

Ex # 3.5

Q2: In a fountain, the pressure "P" of water at any internal point varies directly as depth 'd' from the surface. Pressure is $51\ \text{Newton/cm}^2$ when depth is 3cm. find pressure when depth is 7cm.

Solution:

As Pressure = P

And depth = d

As there is direct variation. Thus

$P \propto d$

$P = kd \quad \ldots\ldots \text{equ(i)}$

Put $P = 51$ and $d = 3$ in equ(i)

$51 = k(3)$

Divide B.S by 3

$\frac{51}{3} = \frac{k(3)}{3}$

$17 = k$

$k = 17$

Now

To Find:

P *when* $d = 7$

$P = ?, d = 7$

Put $d = 7$ and $k = 17$ in equ(i)

$P = 17(7)$

$P = 119$

Thus Pressure = $119\ \text{Newton/cm}^2$

Q3: Pressure P of gas in a container varies directly as temperature T. When pressure is $50\ \text{Newton/m}^2$, temperature is 75 ^{0}C. Find the pressure when temperature is 150 ^{0}C.

Solution:

As Pressure = P

And Temperature = T

As there is direct variation. Thus

$P \propto T$

$P = kT \quad \ldots\ldots \text{equ(i)}$

Put $P = 50$ and $T = 75$ in equ(i)

$50 = k(75)$

Divide B.S by 75

$\frac{50}{75} = \frac{k(75)}{75}$

$\frac{2}{3} = k$

$k = \frac{2}{3}$

Ex # 3.5

Now

To Find:

P *when* $T = 150$

$P = ?, T = 150$

Put $T = 150$ and $k = \frac{2}{3}$ in equ(i)

$P = \frac{2}{3}(150)$

$P = 2(50)$

$P = 100$

Thus Pressure = $100\ \text{Newton/m}^2$

Q4: If 8 persons complete a work in 10 days then how many days would 10 persons take to complete same work?

Solution:

As Number of persons = P

And number of Days = N

As there is Inverse variation

$N \propto \frac{1}{P}$

$N = \frac{k}{P} \quad \ldots\ldots \text{equ(i)}$

Put $N = 10$ and $P = 8$ in equ(i)

$8 = \frac{k}{10}$

$8 \times 10 = k$

$80 = k$

$k = 80$

Now

To Find:

N *when* $P = 10$

$N = ?, P = 10$

Put $P = 10$ and $k = 80$ in equ(i)

$N = \frac{80}{10}$

$N = 8$

Thus Number of days = 8

Ex # 3.5

Q5: Volume of gas 'V' varies inversely as pressure 'P'. $P = 300\ \mathrm{N/m^2}$ when $V = 4m^3$. Find pressure when $V = 3m^3$.

Solution:

As Volume = V

And Pressure = P

As there is Inverse variation

$P \propto \frac{1}{V}$

$P = \frac{k}{V}$ ……equ(i)

Put P = 300 and V = 4 in equ(i)

$300 = \frac{k}{4}$

$300 \times 4 = k$

$1200 = k$

$k = 1200$

Now

To Find:

P when $V = 3$

$P = ?, V = 3$

Put $V = 3$ and $k = 1200$ in equ(i)

$P = \frac{1200}{3}$

$P = 400$

Thus Pressure = $400\ \mathrm{N/m^2}$

Q6: Attraction force 'F' between two magnets vary inversely as square of the distance 'd' between them. F is 18 Newton when d is 2cm. find the distance when attraction force is 2 Newton.

Solution:

As Force = F

And Distance = d

As there is Inverse variation

$F \propto \frac{1}{d^2}$

$F = \frac{k}{d^2}$ ……equ(i)

Put F = 18 and d = 2 in equ(i)

$18 = \frac{k}{(2)^2}$

$18 = \frac{k}{4}$

$18 \times 4 = k$

Ex # 3.5

$72 = k$

$k = 72$

Now

To Find:

d when $F = 2$

$d = ?, F = 2$

Put $F = 2$ and $k = 72$ in equ(i)

$2 = \frac{72}{d^2}$

$d^2 = \frac{72}{2}$

$d^2 = 36$

Taking square on B.S

$\sqrt{d^2} = \sqrt{36}$

$d = 6$

Thus Distance between magnets = 6cm

Q7: The volume of a right circular cylinder varies jointly as the height and the square of the radius. The volume of a right circular cylinder, with radius 4centimeters and height 7centimeters, is 352 cm^3. Find the volume of another cylinder with radius 8 centimeters and height 14centimeters.

Solution:

Let Volume of right circular cylinder = V

And Height of right circular cylinder = h

Radius of right circular cylinder = r

According to condition

V varies jointly as h and r^2

So

$V \propto hr^2$

$V = khr^2$ ……equ(i)

Put $V = 352, h = 7$ and $r = 4$ in equ(i)

$352 = k(7)(4)^2$

$352 = k(7)(16)$

$\frac{352}{(7)(16)} = k$

$\frac{22}{7} = k$

$k = \frac{22}{7}$

Ex # 3.5

Now

To Find:

$V\ when\ h = 14 \text{ and } r = 8$

$V = ?, h = 14 \text{ and } r = 8$

Put $h = 14, r = 8 \text{ and } k = \frac{22}{7}$ in equ(i)

$V = \left(\frac{22}{7}\right)(14)(8)^2$

$V = \left(\frac{22}{7}\right)(14)(64)$

$V = (22)(2)(64)$

$V = 2816$

Thus volume of a right circular cylinder = 2816cm^3

Review Ex # 3

Page # 69-70

Q2: Find the constant of variation when $s \propto t^2$ and $t = 10$ when $s = 5$

Solution:

As there is direct variation

$s \propto t^2$

$s = kt^2 \quad \ldots\ldots \text{equ(i)}$

Put $s = 5$ and $t = 10$ in equ(i)

$5 = k(10)^2$

$5 = k(100)$

$\frac{5}{100} = \frac{k(100)}{100}$

$\frac{1}{20} = k$

$k = \frac{1}{20}$

Q3: $y \propto \frac{1}{x^2}$ and $y = 4$ When $x = 3$. Find x when $y = 9$

Solution:

As there is Inverse variation

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2} \quad \ldots\ldots \text{equ(i)}$

Put $x = 3$ and $y = 4$ in equ(i)

$4 = \frac{k}{(3)^2}$

Review Ex # 3

$4 = \frac{k}{9}$

$4 \times 9 = k$

$36 = k$

$k = 36$

Now

To Find:

$x\ when\ y = 9$

$x = ?, y = 9$

Put $y = 9$ and $k = 36$ in equ(i)

$9 = \frac{36}{x^2}$

$x^2 = \frac{36}{9}$

$x^2 = 4$

Taking square root on B.S

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Q4: Pressure of gas in the closed vessel varies directly with the temperature. If pressure is 150 unit the temperature is 70 units. What will be the pressure if temperature is 140 units?

Solution:

As Pressure = P

And Temperature = T

As there is direct variation. Thus

$P \propto T$

$P = kT \quad \ldots\ldots \text{equ(i)}$

Put P = 150 and T = 70 in equ(i)

$150 = k(70)$

Divide B.S by 70

$\frac{150}{70} = \frac{k(70)}{70}$

$\frac{15}{7} = k$

$k = \frac{15}{7}$

Now To Find:

$P = ?, T = 140$

Put T = 140 and $k = \frac{15}{7}$ in equ(i)

$P = \frac{15}{7}(140)$

$P = 15(20)$

$P = 300$

Thus Pressure = 300 units

Review Ex # 3

5: In an electric circuit, current varies inversely as resistance. When current is 44 amp, the resistance is 30 ohm. How much current will flow if resistance becomes 22 ohm.

Solution:

As Electric current $= \mathrm{I}$
And Resistance $= \mathrm{R}$

As there is Inverse variation

$I \propto \frac{1}{R}$

$I = \frac{k}{R}$ ... ... equ(i)

Put $I = 44$ and $R = 30$ in equ(i)

$44 = \frac{k}{30}$

$44 \times 30 = k$

$1320 = k$

$k = 1320$

Now

To Find:

$I \; when \; R = 22$

$I = ?, R = 22$

Put $R = 22$ and $k = 1320$ in equ(i)

$\boldsymbol{I = \frac{1320}{22}}$

$I = 60$

Thus Electric current $= 60$ amp

6: If a varies jointly as b and square root of c. If $a = 21$ when $b = 5$ and $c = 36$, Find a when $b = 9$ and $c = 225$

Solution:

As a varies jointly as b and square root of c

So

$$a \propto b\sqrt{c}$$

$$a = kb\sqrt{c} \quad \text{... ... equ(i)}$$

Put $a = 21, b = 5$ and $c = 36$ in equ(i)

$$21 = k(5)\sqrt{36}$$

$$21 = k(5)(6)$$

$$\frac{21}{(5)(6)} = k$$

$$\frac{7}{(5)(2)} = k$$

$$\frac{7}{10} = k$$

Review Ex # 3

$k = \frac{7}{10}$

Now

To Find:

$a \; when \; b = 9$ and $c = 225$

$a = ?, b = 9$ and $c = 225$

Put $b = 9, c = 225$ and $k = \frac{7}{10}$ in equ(i)

$a = \left(\frac{7}{10}\right)(9)\sqrt{225}$

$a = \left(\frac{7}{10}\right)(9)(15)$

$a = \frac{945}{10}$

$a = 94.5$

Q7: What number should be added to each of number 3, 8, 11 and 20 to make them in proportion?

Solution:

Suppose the number $= x$

As x is added to each of number

So according to condition

$3 + x : 8 + x = 11 + x : 20 + x$

Product of mean $=$ Product of extreme

$(8 + x)(11 + x) = (3 + x)(20 + x)$

$88 + 8x + 11x + x^2 = 60 + 3x + 20x + x^2$

$88 + 19x + x^2 = 60 + 23x + x^2$

$x^2 + 19x + 88 = x^2 + 23x + 60$

$x^2 - x^2 + 19x - 23x + 88 - 60 = 0$

$-4x + 28 = 0$

$-4x = -28$

Divide B.S by -4

$\frac{-4x}{-4} = \frac{-28}{-4}$

$x = 7$

Thus 7 should be added to each number

So, number becomes

$3 + 7 : 8 + 7 = 11 + 7 : 20 + 7$

$10 : 15 = 18 : 27$

Review Ex # 3

8: **What number should be subtracted to each f the number 6, 8, 7 and 11 so that the remaining numbers are in proportion?**

Solution:

Suppose the number $= x$

As x is subtracted to each of number

So according to condition

$6 - x : 8 - x = 7 - x : 11 - x$

Product of mean $=$ Product of extreme

$(8 - x)(7 - x) = (6 - x)(11 - x)$

$56 - 8x - 7x + x^2 = 66 - 6x - 11x + x^2$

$56 - 15x + x^2 = 66 - 17x + x^2$

$x^2 - 15x + 56 = x^2 - 17x + 66$

$x^2 - x^2 - 15x + 17x + 56 - 66 = 0$

$2x - 10 = 0$

$2x = 10$

Divide B.S by 2

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Thus 5 should be subtracted to each number

So, number becomes

$6 - 5 : 8 - 5 = 7 - 5 : 11 - 5$

$1 : 3 = 2 : 6$

9: **The ratio between two numbers is 8 : 3 and their difference is 20. Find the numbers.**

Solution:

Let the two numbers are x and y

According to first condition

$x : y = 8 : 3$

$\frac{x}{y} = \frac{8}{3}$

By cross multiplication

$3x = 8y$ ....equ(i)

Now according to second condition

$x - y = 20$

$x = 20 + y$ ....equ(ii)

Put the value of x in equ(i)

$3(20 + y) = 8y$

$60 + 3y = 8y$

$60 = 8y - 3y$

$60 = 5y$

Review Ex # 3

$\frac{60}{5} = y$

$12 = y$

$y = 12$

Put the value of y in equ(ii)

$x = 20 + 12$

$x = 32$

Thus the required numbers are 32 and 12 is 8 : 3 and their difference is 20.

OR

$Let\ first\ number = 8x$

$Second\ number = 3x$

$According\ to\ condition$

$8x - 3x = 20$

$5x = 20$

$x = \frac{20}{5}$

$x = 4$

$Now\ first\ number = 8x = 8(4) = 32$

$Second\ number = 3x = 3(4) = 12$

Thus, the required numbers are 32 and 12 is 8 : 3 and their difference is 20.

10: **Find the number in continued proportion such that their sum is 14 and sum of their squared is 84.**

Solution:

Let x, y and z be the three numbers

As they are in continued proportion

$x : y = y : z$

$y^2 = xz$ ...... equ (i)

According to conditions

$x + y + z = 14$ ...... equ (ii)

$x^2 + y^2 + z^2 = 84$ ...... equ (iii)

Put $y^2 = xz$ in equ (iii)

$x^2 + xz + z^2 = 84$ ...... equ (iv)

From equ(ii)

$x + z = 14 - y$ ...... equ (v)

Taking square on B.S

$(x + z)^2 = (14 - y)^2$

$z^2 + 2xz + z^2 = 196 - 28y + y^2$

Put $y^2 = xz$

$z^2 + 2xz + z^2 = 196 - 28y + xz$

$z^2 + 2xz - xz + z^2 = 196 - 28y$

Review Ex # 3

$z^2 + xz + z^2 = 196 - 28y$ ... ... equ (vi)

Compare equ(iv) and equ(vi)

$84 = 196 - 28y$

$84 - 196 = -28y$

$-112 = -28y$

$4 = y$

$y = 4$

Put $y = 4$ in equ(i) and equ(ii)

$(4)^2 = xz$

$16 = xz$

$xz = 16$ ... ... equ (vii)

Now

$x + 4 + z = 14$

$x + z = 14 - 4$

$x + z = 10$

$z = 10 - x$ ... ... equ (viii)

Put equ(viii) in equ(vii)

$x(10 - x) = 16$

$10x - x^2 = 16$

$0 = x^2 - 10x + 16$

$x^2 - 10x + 16 = 0$

$x^2 - 2x - 8x + 16 = 0$

$x(x - 2) - 8(x - 2) = 0$

$(x - 2)(x - 8) = 0$

$x - 2 = 0 \quad or \quad x - 8 = 0$

$x = 2 \quad or \quad x = 8$

Now Put $x = 2$ in equ(viii)

$z = 10 - 2$

$z = 8$

Also Put $x = 8$ in equ(viii)

$z = 10 - 8$

$z = 2$

Thus the required numbers are:

2, 4, 8

OR

8, 4, 2

11: The mean proportion of two numbers is 6 and their sum is 13. Find the number.

Solution:

Let x and y be the numbers

As the mean proportion=6

$x : 6 = 6 : y$

$36 = xy$

Review Ex # 3

$xy = 36$ ... ... equ (i)

According to condition

$x + y = 13$

$y = 13 - x$ ... ... equ (ii)

Put the value of x in equ(i)

$x(13 - x) = 36$

$13x - x^2 = 36$

$0 = x^2 - 13x + 36$

$x^2 - 13x + 36 = 0$

$x^2 - 4x - 9x + 36 = 0$

$x(x - 4) - 9(x - 4) = 0$

$(x - 4)(x - 9) = 0$

$x - 4 = 0 \quad or \quad x - 9 = 0$

$x = 4 \quad or \quad x = 9$

Now Put $x = 4$ in equ(iii)

$y = 13 - 4$

$y = 9$

Also Put $x = 9$ in equ(iii)

$y = 13 - 9$

$y = 4$

Thus the required numbers are:

4 and 9

OR

9 and 4

12: Find angle of a triangle which are in ratio 3 : 4 : 5

Solution:

As triangle has three angles

Also we know that

Sum of angles=180

As ratio of given triangle=3 : 4 : 5

Sum the Ratio $= 3 + 4 + 5$

$= 12$

First Angle $= \frac{3}{12} \times 180^0$

$= 3 \times 15^0$

$= 45^0$

Second Angle $= \frac{4}{12} \times 180^0$

$= 4 \times 15^0$

$= 60^0$

Third Angle $= \frac{5}{12} \times 180^0$

$= 5 \times 15^0$

$= 75^0$

Review Ex # 3

13: **If $\frac{a}{b}=\frac{c}{d}$ then prove that**

$$\frac{ac(a+c)}{bd(b+d)}=\frac{(a+c)^3}{(b+d)^3}$$

Solution:

$As\ \frac{a}{b}=\frac{c}{d}$

Now $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$

So $\frac{a}{b}=\frac{a+c}{b+d}$ ... ... equ(i)

And $\frac{c}{d}=\frac{a+c}{b+d}$ ... ... equ(ii)

As $\frac{a}{b}=\frac{a+c}{b+d}$

Multiply B.S by $\frac{c}{d}$

$\frac{a}{b}\times\frac{c}{d}=\left(\frac{c}{d}\right)\left(\frac{a+c}{b+d}\right)$

As $\frac{c}{d}=\frac{a+c}{b+d}$

So $\frac{ac}{bd}=\left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)$

Now multiply B.S by $\left(\frac{a+c}{b+d}\right)$

$\frac{ac}{bd}\left(\frac{a+c}{b+d}\right)=\left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)\left(\frac{a+c}{b+d}\right)$

$\frac{ac(a+c)}{bd(b+d)}=\left(\frac{a+c}{b+d}\right)^3$

14: **If a, b, c are in continued proportion then prove that $\frac{a}{c}=\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$**

Solution:

As a, b and c are in continued proportion

So

$a:b=b:c$

$\frac{a}{b}=\frac{b}{c}$

Let $\frac{a}{b}=\frac{b}{c}=k$

$\frac{a}{b}=k,\quad \frac{b}{c}=k$

$a=kb$ ... equ (i), $\quad b=kc$ ... equ (ii)

Put $b=kc$ in equ(i)

$a=k(kc)$

$a=k^2c$ ... ... equ (iii)

Review Ex # 3

$\frac{a}{c}=k^2$ ... ... equ (iv)

Now

$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{(k^2c)^2+(k^2c)(kc)+(kc)^2}{(kc)^2+(kc)c+c^2}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{k^4c^2+k^3c^2+k^2c^2}{k^2c^2+kc^2+c^2}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{k^2c^2(k^2+k+1)}{c^2(k^2+k+1)}$

$\frac{a^2+ab+b^2}{b^2+bc+c^2}=k^2$ ... ... equ (v)

Now again

$\frac{a^2-b^2}{b^2-c^2}=\frac{(k^2c)^2-(kc)^2}{(kc)^2-c^2}$

$\frac{a^2-b^2}{b^2-c^2}=\frac{k^4c^2-k^2c^2}{k^2c^2-c^2}$

$\frac{a^2-b^2}{b^2-c^2}=\frac{k^2c^2(k^2-1)}{c^2(k^2-1)}$

$\frac{a^2-b^2}{b^2-c^2}=k^2$ ... ... equ (vi)

From equ(iv), equ(v) and equ(vi)

$\frac{a}{c}=\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$

Review Ex # 3

Q15: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then prove that

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\frac{a}{b}=k,\quad \frac{c}{d}=k,\quad \frac{e}{f}=k$

$a=kb,\quad c=kd,\quad e=kf$

To prove:

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Now L.H.S:

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{(kb)^3+(kd)^3+(kf)^3}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{k^3b^3+k^3d^3+k^3f^3}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{k^3(b^3+d^3+f^3)}{b^3+d^3+f^3}$$

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=k^3$$

Also R.H.S:

$$\frac{ace}{bdf}=\frac{(kb)(kd)(kf)}{bdf}$$

$$\frac{ace}{bdf}=\frac{k^3bdf}{bdf}$$

$$\frac{ace}{bdf}=k^3$$

Thus L.H.S=R.H.S

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$$

Hence Proved:

Chapter # 3