Exercise # 4.1

Q.1: Classify the following into finite and infinite sequences.

(i) 3, 4, 6, 8, ......... So Finite
(ii) 1, 0, 1, 0, 1, ........ Infinite
(iii) .......-4, 0, 4, 8, ........ Infinite
(iv) 1, -1/2, 1/3, ...... Finite

Q.2: Find the first four terms of the sequence with the general terms

1. Let \( T_n = \frac{n(n+1)}{2} \)

\[
\begin{align*}
\text{n} = 1 & \quad \therefore \quad \text{T}_1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1 \\
\text{n} = 2 & \quad \therefore \quad \text{T}_2 = \frac{2(2+1)}{2} = \frac{6}{2} = 3 \\
\text{n} = 3 & \quad \therefore \quad \text{T}_3 = \frac{3(3+1)}{2} = \frac{12}{2} = 6 \\
\text{n} = 4 & \quad \therefore \quad \text{T}_4 = \frac{4(4+1)}{2} = \frac{20}{2} = 10.
\end{align*}
\]

(ii) \((-1)^{n-1} \frac{n}{n+1}\)

Let \( a_n = (-1)^{n-1} \frac{n}{n+1} \)

\[
\begin{align*}
\text{for } n=1 & \quad a_1 = (-1)^0 \frac{1}{2} = 1/2 = 1 \\
\text{for } n=2 & \quad a_2 = (-1)^1 \frac{2}{3} = -2/3 = -2 \\
\text{for } n=3 & \quad a_3 = (-1)^2 \frac{3}{4} = 3/4 = 3/4 \\
\text{for } n=4 & \quad a_4 = (-1)^3 \frac{4}{5} = -4/5 = -4/5
\end{align*}
\]
3. Write down the nth term of each sequence as suggested by the pattern.

(i) $\frac{1}{2}, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \ldots \ldots$

So, $a_1 = \frac{1}{2}, a_2 = \frac{1}{3}, \ldots$ suggest that $a_n = \frac{n}{n+1}$

(ii) 2, -4, 6, -8, 10, \ldots \ldots

The terms suggest that $a_1 = 2, a_2 = -4, a_3 = 6, a_4 = -8, \ldots$ suggest that $a_n = (-1)^{n+1} \cdot 2n$

(iii) 1, 1, 1, -1, \ldots \ldots

The terms suggest that $a_1 = 1, a_2 = 1, a_3 = 1, a_4 = -1, \ldots$ suggest that $a_n = (-1)^{n+1}$ or $a_n = (-1)^{n-1}$

4.5 Write down each series in expanded form

(i) $\sum_{j=1}^{6} (2j-3)$

Hence $a_j = 2j-3$

For $j=1$ $a_1 = 2(1)-3 = -1$

For $j=2$ $a_2 = 2(2)-3 = 1$

For $j=3$ $a_3 = 2(3)-3 = 3$

For $j=4$ $a_4 = 2(4)-3 = 5$

For $j=5$ $a_5 = 2(5)-3 = 7$

For $j=6$ $a_6 = 2(6)-3 = 9$

Hence $\sum_{j=1}^{6} (2j-3) = (-1) + 1 + 3 + 5 + 7 + 9$
(ii) \[ \sum_{k=1}^{5} (-1)^k a^{k-1} \]

Here \[ a_k = (-1)^k a^{k-1} \]

\[ a_1 = (-1)^1 a^0 = (-1) a^0 = -1 \]
\[ a_2 = (-1)^2 a^1 = (1) a^1 = a \]
\[ a_3 = (-1)^3 a^2 = (1) a^2 = -4 \]
\[ a_4 = (-1)^4 a^3 = (1) a^3 = 10 \]
\[ a_5 = (-1)^5 a^4 = (1) a^4 = -16 \]

Hence \[ \sum_{k=1}^{5} (-1)^k a^{k-1} = (-1) + a + (-4) + (10) + (-16) + \ldots \]

(iii) \[ \sum_{j=1}^{\infty} \frac{1}{a^j} \]

\[ = \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \ldots \]

(iv) \[ \sum_{k=0}^{\infty} \left( \frac{3}{2} \right)^k \]

\[ = \left( \frac{3}{2} \right)^0 + \left( \frac{3}{2} \right)^1 + \left( \frac{3}{2} \right)^2 + \ldots \]
\[ = 1 + \left( \frac{3}{2} \right)^1 + \left( \frac{3}{2} \right)^2 + \ldots \]

**8.6** Write the following in terms of factorial.

(i) \( 8 \times 7 \times 6 \times 5 \)

Set \( X \) and \( V \) by \( n! \)

\[ = \frac{n(n-1)(n-2)(n-3)!}{(n-4)!} = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \]
\[ = \frac{n!}{(n-3)!} \]

(ii) \( n(n-1)(n-2) \)

Set \( X \) and \( V \) by \( (n-3)! \)

\[ = 8 \times 7 \times 6 \times 5 \times 4! \]
\[ = \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \]

\[ = \frac{n!}{(n-3)!} \]

**8.7** Find the Pascal sequence by using its general recursive definition.

(i) \( n=5 \)

Set \( X \).

The recursive definition is

\[ P_{x+1} = \frac{n-x}{y+1} \binom{n}{y} + P_0 = 1 \]

For \( n=5 \)

\[ P_{y+1} = \frac{5-y}{y+1} \binom{5}{y} + P_0 = 1 \]

Put \( y=0 \)

\[ P_{0+1} = \frac{5-0}{0+1} \binom{5}{0} \]

\[ \Rightarrow P_1 = \frac{5}{1} \binom{5}{1} \Rightarrow P_1 = 5 \]

Put \( y=1 \)

\[ P_{i+1} = \frac{5-1}{i+1} \binom{5}{i} \]
\[ P_{1} = \frac{4}{2} = 2 \]
\[ P_{3} = \frac{\binom{4}{2}}{2} = 3 \]
\[ P_{5} = \frac{\binom{6}{2}}{2} = 10 \]
\[ P_{7} = \frac{\binom{8}{2}}{2} = 28 \]
\[ P_{9} = \frac{\binom{10}{2}}{2} = 45 \]
\[ P_{11} = \frac{\binom{12}{2}}{2} = 78 \]
\[ P_{13} = \frac{\binom{14}{2}}{2} = 120 \]
\[ P_{15} = \frac{\binom{16}{2}}{2} = 182 \]
\[ P_{17} = \frac{\binom{18}{2}}{2} = 264 \]

So the Pascal sequence is
\[ P_0, P_1, P_2, P_3, P_4, P_5, \ldots \]
\[ 1, 2, 3, 10, 28, 78, 120, \ldots \]

Available at
www.mathcity.org
Exercise # 4.2

Q.1 Find the common difference, 5th term, 10th term, and nth term of the AP

(i) 2, 7, 12, .......

\[ \text{Common difference } = d = a_2 - a_1 = 7 - 2 \]
\[ \Rightarrow d = 5 \text{ Ans} \]

\[ \text{nth term} \]
\[ a_n = a_1 + (n-1)d \]
\[ a = 2, d = 5 \]
\[ a_n = a_1 + (n-1)d \\
\[ \Rightarrow a_n = 2 + (n-1)5 \text{ Ans} \]

5th term
\[ a_5 = 2 + (5-1)5 = 2 + 4 \times 5 = 22 \text{ Ans} \]

10th term
\[ a_{10} = 2 + (10-1)5 = 2 + 9 \times 5 = 47 \text{ Ans} \]

(ii) -4, -2, 0, 2, .......

\[ \text{Common difference } = d = a_2 - a_1 \\
\[ \Rightarrow d = -2 - (-4) \\
\[ \Rightarrow d = 2 + 4 \\
\[ \Rightarrow (d = 2) \text{ Ans} \]

\[ \text{nth term} \]
\[ a_n = a + (n-1)d \]
\[ \Rightarrow a_n = -4 + (n-1)2 \text{ Ans} \]

5th term
\[ a_5 = -4 + (5-1)2 = -4 + 8 \]
\[ a_5 = 4 \text{ Ans} \]

10th term
\[ a_{10} = -4 + (10-1)2 = -4 + 18 \]
\[ a_{10} = 14 \text{ Ans} \]

Q.2 In AP, \[ a_1 = 43 \text{ and } a_{10} = 7 \] Find \[ a_{15} \].

\[ \text{Ans} \]
\[ a_n = a + (n-1)d \rightarrow 0 \]
\[ \Rightarrow a_{10} = a + (10-1)d \]
\[ \Rightarrow 7 = 43 + 9d \]
\[ \Rightarrow -36 = 9d \Rightarrow d = -4 \]

Now put \[ n = 15 \] in eqn.0
\[ a_{15} = a + (15-1)d = 43 + 14(-4) = 43 - 56 \]
\[ a_{15} = -13 \text{ Ans} \]
\[ a_3 \]

Find \( A \cdot \rho \)

\[
\begin{align*}
\text{If } & \ a_6 + a_4 = 6 \quad \text{and } \ a_6 - a_4 = 2/3 \\
\text{Solve for } & \ a_6 + a_4 = 6 \quad \rightarrow \quad 0 \\
& \ a_6 - a_4 = 2/3 \quad \rightarrow \quad 0 \\
& \text{Eq} \ 0 \ + \ \text{Eq} \ 0, \ \text{we get} \\
& \ 2a_6 = 2a + 2d \ \Rightarrow \ a_6 = 10/3 \\
\text{Eq} \ 0 \Rightarrow & \ \frac{10}{3} + a_4 = 6 \ \Rightarrow \ a_4 = 6 - \frac{10}{3} \\
& \ a_4 = \frac{8}{3} \\
\text{Now } \ & \ a_n = a + (n-1)d \\
\text{Solve for } & \ a_6 + a_4 = 6 \\
& \ a_6 + a_4 = 6 \\
& \ \frac{10}{3} = a + 5d \ \rightarrow \ (1) \ \Rightarrow \ \frac{8}{3} = a + 3d \ \rightarrow \ (2) \\
\text{Eq} \ 0 \ - \ \text{Eq} \ 0 \ \Rightarrow \\
& \ a + 5d = 10/3 \\
& \ a + 3d = 8/3 \\
& \ 2d = 2/3 \\
& \ \Rightarrow \ \left[ d = \frac{1}{3} \right] \\
\end{align*}
\]

Equation 1 \Rightarrow \ a + 5d = 10/3

\[
\Rightarrow \ a + 5\left( \frac{1}{3} \right) = \frac{10}{3}
\]

\[
\Rightarrow \ a = \frac{10}{3} - \frac{5}{3} \\\n\Rightarrow \ a = \frac{5}{3}
\]

Then the A.P will be

\[
\begin{align*}
& a, \ a+d, \ a+2d, \ a+3d, \ldots \ldots \ldots \\
& \Rightarrow \ \frac{5}{3}, \ \frac{5}{3} + \frac{1}{3}, \ \frac{5}{3} + \frac{2}{3}, \ldots \ldots \ldots \\
& \Rightarrow \ \frac{2}{3}, \ \frac{6}{3}, \ \frac{7}{3}, \ldots \ldots \ldots \ \text{Any} \\
\end{align*}
\]

Q.4 How many terms are there in A.P in which the 11st and last terms are \( \frac{33}{4} \) and \( \frac{257}{2} \) respectively and the common difference is \( \frac{1}{3} \)?

Solve: From the statement, it is clear that

\[
\begin{align*}
& a_1 = \frac{33}{4} \\
& a_n = \frac{257}{2} \\
& d = \frac{1}{3} \\
& \text{and } n=? \\
\end{align*}
\]

As

\[
\begin{align*}
& a_n = a + (n-1)d \\
& \Rightarrow \ \frac{25}{2} = \frac{33}{4} + (n-1)\frac{1}{3} \\
& \Rightarrow \ \frac{25}{2} - \frac{33}{4} = \frac{n-1}{3} \\
& \Rightarrow \ \frac{50 - 33}{4} = \frac{n-1}{3} \\
& \Rightarrow \ \frac{17}{4} = \frac{n-1}{3} \\
& \Rightarrow \ \left[ n = 35 \right] \\
\end{align*}
\]

Hence, total terms are 35.
8.5 Which term of the A.P. 4, 1, -2, ..... is -77?

Set

\[ a = 4, \quad d = -3 \]

Now

\[ an = a + (n-1)d \]

\[ = 4 + (n-1)(-3) = -77 \]

\[ -77 = 4 + (n-1)(-3) \]

\[ -81 = -3n + 3 \]

\[ 3n = 84 \]

\[ n = \frac{84}{3} = n = 28 \]

Hence 28th term is -77.

8.6 A ball rolling up an incline covered 24 m in 1st second, 21 m in 2nd second, 18 m in 3rd sec. Find how many meters it covered in the 8th sec.

Set

The distance covered makes A.P. of the pattern

\[ 24, 21, 18, \ldots \]

Hence \( a = 24 \) and \( d = 21 - 24 \)

\[ a = 24, \quad d = -3 \]

Now

\[ an = a + (n-1)d \]

\[ = 24 + (n-1)(-3) = 24 + 7(-3) = 24 - 21 \]

\[ a = 3 \]

Hence in the 8th sec, distance covered is 3 m.

8.7 The population of a town is decreasing by 500 inhabitants each year. If its population at the beginning of 1960 was 20135, what was its population at the beginning of 1970?

\[ a \quad a_2 \quad a_3 \quad a_4 \quad a_5 \]

\[ \text{Years } 1950, \ 1951, \ 1962, \ldots \]

Population \( 20135, 19635, 19135, \ldots \quad d = -500 \)

Put \( n = 11 \) for the beginning of 1970.

\[ a_n = a + (n-1)d \]

\[ a_{11} = 20135 + (11-1)(-500) \]

\[ a_{11} = 20135 + (10)(-500) \]

\[ a_{11} = 20135 - 5000 \]

\[ \boxed{a_{11} = 15135} \]

8.8 Ahmed and Ibrahim can climb 1000 ft in the 1st hour and 100 ft in each succeeding hour. When will they reach the top of a 5400 ft hill?

Set

Hours 1st, 2nd, 3rd, 4th, ...

Distance covered \( 1000, 1100, 1200, \ldots \)

Here \( a = 1000 \)

\[ d = 100 \]
Exercise # 4.3

q1  Find the A.M. b/w

(i) 12 and 18

\[ A = \frac{a+b}{2} \]

\[ \Rightarrow A = \frac{12+18}{2} = \frac{30}{2} = 15 \]

\[ \Rightarrow A = 15 \text{ A.M.} \]

(ii) \[ \frac{1}{3}, \frac{1}{4} \]

\[ A = \frac{\frac{1}{3} + \frac{1}{4}}{2} = \frac{\frac{4+3}{12}}{2} = \frac{7}{24} \]

Hence \[ A = \frac{7}{24} \text{ A.M.} \]

(iii) -6, -216

\[ a = (-6), \ b = (-216) \]

\[ A = \frac{(a+b)^2 + (a-b)^2}{2} \]

\[ \Rightarrow A = \frac{(-6)^2 + (-216)^2}{2} \]

\[ \Rightarrow A = \frac{(-6)^2 + (-216)^2}{2} \]

\[ \Rightarrow A = \frac{6^2 + 216^2}{2} \]

\[ \Rightarrow A = \frac{36 + 46656}{2} \]

\[ \Rightarrow A = \frac{46702}{2} \]

\[ \Rightarrow A = 23351 \text{ A.M.} \]
(iii) Five A.Ms b/w 9 and 33.

Set \( a, a_2, a_3, a_4 \) are five A.Ms b/w 9 and 33.
n
Then \( 9, a_1, a_2, a_3, a_4, a_5, 33 \) is A.P.
n
To find \( d \):
\[
\begin{align*}
A_1 &= a_1 + (n-1)d \\
41 &= 6 + (5-1)d \\
41 - 6 &= 4d \\
35 &= 4d 
\end{align*}
\]
\[ 9 = d \]
n
Now \( A_n = a + nd \)
\[
\begin{align*}
a_1 &= a + 1d = 6 + 1(9) = 15 \\
a_2 &= a + 2d = 6 + 2(9) = 24 \\
a_3 &= a + 3d = 6 + 3(9) = 33 \\
a_n &= a + nd \\
\end{align*}
\]
(i) Four A.Ms b/w 17 and 32.

Set \( a, a_2, a_3, a_4 \) are the four A.Ms b/w 17 and 32.

Then \( 17, a_1, a_2, a_3, a_4, 32 \) is A.P.
\[
\begin{align*}
a_n &= a + (n-1)d \\
32 &= 17 + (6-1)d \\
32 - 17 &= 5d \\
15 &= 5d \\
3 &= d 
\end{align*}
\]
(ii) Insert

1. Three A.Ms b/w 6 and 41.

Set \( a, a_1, a_2, a_3, a_4 \) are the three A.Ms, then
\[
\begin{align*}
a_n &= a + (n-1)d \\
41 &= 6 + (5-1)d \\
41 - 6 &= 4d \\
35 &= 4d 
\end{align*}
\]
\[ 9 = d \]

Now \( A_n = a + nd \)
\[
\begin{align*}
a_1 &= a + 1d = 6 + 1(9) = 15 \\
a_2 &= a + 2d = 6 + 2(9) = 24 \\
a_3 &= a + 3d = 6 + 3(9) = 33 \\
a_n &= a + nd \\
\end{align*}
\]
\[ 2a^{n+1} + 2b^{n+1} = a^n + ab^n + a^n b + b^{n+1} \]

\[ 2a^{n+1} - a^{n+1} + 2b^{n+1} - b^{n+1} = a^n b + a^n b \]

\[ a^{n+1} + b^{n+1} = a^n b + a^n b \]

\[ a^n + b^n = a^n b + a^n b \]

\[ a^n - a^n b = ab^n - b^n \]

\[ (a-b) a^n = (a-b) b^n \]

\[ a^n = b^n \div b.s \text{ by } b^n \]

\[ \frac{a^n}{b^n} = 1 \Rightarrow \left( \frac{a}{b} \right)^n = 1 \]

\[ \Rightarrow \left( \frac{a}{b} \right)^n = \left( \frac{a}{b} \right)^0 \]

\[ \Rightarrow n = 0 \text{ and } a = b \]

Insert five A.M.s b/w 5 and 8. Also show that their sum is equal to 5 times A.M. b/w 5 and 8.

5 \[ \frac{5}{2} \text{ Let } A_1, A_2, A_3, A_4, \text{ and } A_5 \text{ are five A.M.s} \]

\[ \text{b/w } 5 \text{ and } 8. \]

Then 5, 8, 8, 8, 8 and 8 \[ \text{ are A.P.} \]

To find \( d \) \[ a_n = a + (n-1)d \]

\[ 8 = a + (5-1)d \]

\[ \Rightarrow 2 = 6d \]

\[ 3 = 6d \Rightarrow \left( \frac{1}{2} = d \right) \]

Now \[ A_n = a + nd \]

\[ A_1 = a + 1d = 5 + 1 \left( \frac{1}{2} \right) = \frac{13}{2} \]

\[ A_2 = a + 2d = 5 + 2 \left( \frac{1}{2} \right) = 6 \]

\[ A_3 = a + 3d = 5 + 3 \left( \frac{1}{2} \right) = \frac{15}{2} \]

\[ A_4 = a + 4d = 5 + 4 \left( \frac{1}{2} \right) = 7 \]

\[ A_5 = a + 5d = 5 + 5 \left( \frac{1}{2} \right) = \frac{17}{2} \]

Now \[ \text{sum of the five A.M.s} \]

\[ A_1 + A_2 + A_3 + A_4 + A_5 = \frac{5}{2} (A_1 + A_5) \]

\[ = \frac{5}{2} \left( \frac{13}{2} + \frac{17}{2} \right) \]

\[ = \frac{5}{2} \left( \frac{30}{2} \right) \]

\[ = 65/2 \rightarrow (i) \]

Now \[ 5 \text{ times A.M. of } S \text{ and 8} \]

\[ A = \frac{5 + 8}{2} = \frac{13}{2} \]

\[ 5 \text{ A} = 5 \left( \frac{13}{2} \right) = 65/2 \rightarrow (ii) \]

From eqns (i) and (ii) it is proved that \[ A_1 + A_2 + A_3 + A_4 + A_5 = 5 \text{ A.M.} \]
There are $n$ A.M.s between 5 and 32 such that the ratio of 3rd and 7th means is 7:13. Find the value of $n$.

$A_n = a + nd$

$A_3 = a + 3d$  
$A_7 = a + 7d$

Now given that $A_3 : A_7 = 7 : 13$

$\frac{A_3}{A_7} = \frac{7}{13}$

$\frac{a + 3d}{a + 7d} = \frac{7}{13}$

By cross multiplication

$13a + 39d = 7a + 49d$

$13a - 7a = 49d - 39d$

$6a = 10d$

$\frac{a = \frac{5d}{3}}{0}$

Now if $A_1, A_2, A_3, \ldots, A_n$ are $n$ A.M.s between 5 and 32, then

$5, A_1, A_2, A_3, \ldots, A_n, 32$ is A.P.

Then $d = \frac{b - a}{n + 1}$

$\Rightarrow d = \frac{32 - 5}{n + 1}$

$\Rightarrow d = \frac{27}{n + 1}$
Exercise 4.4

Q:1 Find the indicated term and the sum of the number of terms in the following cases.

(1) 9, 7, 5, 3, ....... 20th term; 20 terms.

Let a = 9 and d = 2

a_n = a + (n-1)d

⇒ a_{20} = 9 + (20-1)2

= 9 - 38

Now sum of 1st 20 terms

S_{20} = \frac{n}{2} \{2a + (n-1)d\}

⇒ S_{20} = \frac{20}{2} \{2(9) + 19(-2)\}

= 10 \{18 - 38\}

= 10(-20)

⇒ \boxed{S_{20} = -200}

(ii) 3, \frac{5}{3}, \frac{7}{3}, 2, ....... 11th term; + 11 terms

Let a = 3

d = \frac{2}{3} - 3

⇒ \boxed{d = -\frac{7}{3}}
(ii) \( a_1 = -40 \), \( S_{21} = 210 \)

\[
S_{21} = \frac{n}{2} \left( a_1 + a_n \right)
\]

\[
\Rightarrow S_{21} = \frac{21}{2} \left( a_1 + a_{21} \right)
\]

\[
\Rightarrow 210 = \frac{21}{2} \left( -40 + a_{21} \right)
\]

\[
\Rightarrow \frac{210 \times 2}{21} = -40 + a_{21} \Rightarrow 20 = -40 + a_{21}
\]

\[
\Rightarrow a_{21} = 60 \quad \text{Ans}
\]

And \( a_n = a + (n-1)d \)

\[
\Rightarrow a_{21} = a + (21-1)d
\]

\[
\Rightarrow 60 = -40 + 20d \Rightarrow 100 = 20d \Rightarrow d = 5 \quad \text{Ans}
\]

(iii) \( a_1 = -7 \), \( d = 8 \), \( S_{15} = 225 \)

\[
S_{15} = \frac{15}{2} \left[ 2a + (n-1)d \right]
\]

\[
\Rightarrow 225 = \frac{15}{2} \left[ 2(-7) + (15-1)8 \right]
\]

\[
\Rightarrow 225 = \frac{15}{2} \left[ -14 + 8n - 8 \right]
\]

\[
\Rightarrow 450 = 15 \left[ 8n - 22 \right]
\]

\[
\Rightarrow 450 = 8n^2 - 32n
\]

\[
\Rightarrow 8n^2 - 32n - 450 = 0
\]

By quadratic formula

\[
n = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(8)(-450)}}{2(8)}
\]

\[
\Rightarrow n = \frac{8 \pm \sqrt{484 + 14400}}{16}
\]

\[
\Rightarrow n = \frac{8 \pm \sqrt{14884}}{16}
\]

\[
\Rightarrow n = \frac{22 \pm 122}{16}
\]

\[
\Rightarrow n = \frac{22 + 122}{16} \quad \text{or} \quad n = \frac{22 - 122}{16}
\]

\[
\Rightarrow n = \frac{144}{16} \quad \text{or} \quad n = \frac{-100}{16}
\]

\[
\Rightarrow n = 9 \quad \text{Ans}
\]

\( \Rightarrow \) which is not possible.

Now \( a_n = a + (n-1)d \)

\[
\Rightarrow a_9 = a + (9-1)d
\]

\[
\Rightarrow a_9 = -7 + 8 (8)
\]

\[
\Rightarrow a_9 = 57 \quad \text{Ans}
\]

(iv) \( a_{15} = 4 \), \( S_{15} = 30 \)

\[
S_{15} = \frac{15}{2} \left[ 2a + (n-1)d \right]
\]

\[
\Rightarrow 30 = \frac{15}{2} \left[ 2a + 14d \right]
\]

\[
\Rightarrow a_{15} = a + 14d \quad \Rightarrow \quad \text{(3)}
\]

\[
\Rightarrow 4 = a + 14d
\]

\[
\Rightarrow 4 = 2a + 14d \Rightarrow \frac{4}{2} = a
\]

\[
\Rightarrow 4 \neq 8 \quad \Rightarrow \quad a = 0
\]

\[
\Rightarrow 4 = 2(0) + 14d
\]

\[
\Rightarrow 4 = 14d \Rightarrow \frac{4}{14} = d
\]

\[
\Rightarrow d = \frac{2}{7}
\]

\[\text{Hence, } n = 15, a = 0 \quad \text{and} \quad d = 2/7\]
6. Find the sum of all the nos. divisible by 5 from 25 to 350.

The numbers divisible by 5 from 25 to 350 are 25, 30, 35, 40, ..., 345, 350.

To find the number of terms, we have

\[ a_n = a + (n-1)d \]

Here \( a = 25 \) and \( d = 5 \)

\[ 350 = 25 + (n-1)5 \]

\[ 325 = (n-1)5 \]

\[ n-1 = \frac{325}{5} \Rightarrow n-1 = 65 \Rightarrow n = 66 \]

Now, the sum of the terms is 25 + 30 + 35 + ... + 350

is given by the formula

\[ S_n = \frac{n}{2} \left[ a_1 + a_n \right] \]

\[ S_{66} = \frac{66}{2} \left[ 25 + 350 \right] \]

\[ S_{66} = 33 \times 375 \]

\[ S_{66} = 12,375 \text{ Ans} \]

64. The sum of the three numbers in an A.P is 36 and the sum of their cubes is 6336. Find the nos.

Let the numbers be

\[ a-d, a, a+d \]

Sum = 36

\[ (a-d) + a + (a+d) = 36 \]

\[ 3a = 36 \Rightarrow a = 12 \]

Also, the sum of the cubes is 6336.

\[ (a-d)^3 + a^3 + (a+d)^3 = 6336 \]

\[ (a^3 - 3a^2d + 3ad^2) + a^3 + (a^3 + 3a^2d + 3ad^2) = 6336 \]

\[ 3a^3 + 6ad^2 = 6336 \]

\[ \div \text{by} \ 3 \]

\[ a^3 + 2ad^2 = 2112 \]

\[ (a)^3 + 2ad^2 = 2112 \]

\[ 1728 + 2ad^2 = 2112 \Rightarrow 2ad^2 = 2112 - 1728 \]

\[ 2ad^2 = 384 \Rightarrow ad^2 = 192 \]

\[ \frac{ad^2}{a} = 192 \Rightarrow d^2 = \frac{192}{a} \]

\[ d^2 = \frac{192}{12} \]

\[ d^2 = 16 \]

\[ d = \pm 4 \]
Now the terms are
\[\begin{align*}
a &= 12 + 4 = 16 \\
16 - d &= 12 - 4 = 8 \\
a + d &= 12 + 4 = 16 \\
16 - d &= 12 - (4) = 8.
\end{align*}\]

So the required terms are
\[8, 12, 16 \text{ of } 16, 12, 8\]

\[\text{By formula } S_n = \frac{n}{2} \{2a + (n-1)d\} \]
\[\Rightarrow S_n = \frac{n}{2} \{2(-1) + (n-1)6\} \]
\[= \frac{n}{2} \{2 - 6n + 6\} \]
\[= \frac{n}{2} \{8 - 6n\} \]
\[= \frac{n}{2} \{2(3n-4)\} \]
\[= \frac{n}{2} \{3n-4\} \]
\[= 3n^2 - 4n \text{ Ans.}\]

Method #02:
For any three terms we can represent the given series as sum of three series, as
\[1 + 3 + 5 + 7 + 9 - 11 + 13 + 15 - 17 + \ldots \text{ upto } 3n \text{ terms}\]
\[= \frac{n}{3} \left\{2a + (n-1)d\right\} + \frac{n}{3} \left\{2a + (n-1)c\right\} - \frac{n}{3} \left\{2a + (n-1)e\right\}\]
\[= \frac{n}{3} \{2 + (n-1)3\} \quad \frac{n}{3} \{2 + (n-1)0\} \quad \frac{n}{3} \{2 + (n-1)(-1)\}\]
\[= \frac{n}{3} \{2 + 6n - 6\} + \frac{n}{3} \{6 + 6n - 6\} - \frac{n}{3} \{2 + 6n - 6\}\]
\[= \frac{n}{3} [6n - 4] + \frac{n}{3} [6n] - \frac{n}{3} [6n + 1]
\]
\[\text{take } n/2 \text{ as common}\]
\[= \frac{n}{3} \left\{(6n - 4) + 6n - (6n + 4)\right\}\]
\[= \frac{n}{3} \{6n - 4 + 6n - 6n + 4\}\]
\[= \frac{n}{3} \{6n - 8\} \]
\[= \frac{n}{3} \{2(3n-4)\} \]
\[= n(3n-4) \]
\[= 3n^2 - 4n \text{ Ans.}\]

\[\text{Quote:}\]
\[\text{Education is the ability to listen to almost anything without losing your temper or your self-confidence.}\]

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(Robert Frost)
show that the sum of 1st n positive odd integers is n^2.

The sequence of 1st n positive odd numbers is
1, 3, 5, 7, .........

And the series becomes
1 + 3 + 5 + 7 + .........

Here a = 1, d = 2 and terms = n (given)

Then the series will be

\[ s_n = \frac{n}{2} \left[ 2a + (n-1)d \right] \]

\[ s_n = \frac{n}{2} \left[ 2(1) + (n-1)2 \right] \]

\[ s_n = \frac{n}{2} \left[ 2n - 2 \right] \]

\[ s_n = \frac{n}{2} \left[ 2n^2 \right] = n^2 \]

So the series is

1 + 3 + 5 + ......... n-term = n^2

6.7 Find four numbers in A.P., whose sum is 20 and the sum of whose squares is 120.

Let the #s are

a-3d, a-d, a+d, a+3d

Sum = 20 (Given)

\[ (a-3d) + (a-d) + (a+d) + (a+3d) = 20 \]

\[ \Rightarrow 4a = 20 \]

\[ \Rightarrow a = 5 \]

Now sum of their squares = 120 (Given)

\[ (a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120 \]

\[ a^2 + 9d^2 - 6ad + a^2 + d^2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 120 \]

\[ 4a^2 + 20d^2 = 120 \]

\[ \div by 4 \]

\[ a^2 + 5d^2 = 30 \]

\[ s^2 + 5d^2 = 30 \]

\[ \Rightarrow 25 + 5d^2 = 30 \]

\[ \Rightarrow 5d^2 = 5 \]

\[ \Rightarrow d^2 = 1 \]

\[ \Rightarrow d = +1 \]

Then the required #s are

\[ a = 5 \]

If d = 1

a-3d = 5-3(1) = 2

a-d = 5-1 = 4

a+d = 5+1 = 6

a+3d = 5+3(1) = 8

Hence the #s are 2, 4, 6, 8

8, 6, 4, 2

A. 2

The sum of Rs. 1000 is to be divided among four people so that each person after the 1st gets Rs. 20 less than the preceding one. How much does each person receive?

Total amount = Rs. 1000

5.2
Let 1st person get = Rs \(x\)

2nd gets = \(x - 20\)

and 3rd = \(x - 40\)

and 4th = \(x - 60\)

So the series becomes \(x + (x - 20) + (x - 40) + (x - 60)\).

Now sum = Rs 1000

\[ a + (x - 20) + (x - 40) + (x - 60) = Rs \ 1000 \]

\[ 4x - 120 = 1000 \]

\[ 4x = 1120 \]

\[ x = 280 \]

So 1st person gets = Rs 280

2nd = Rs 260 (\(x - 20\))

3rd = Rs 240

4th = Rs 220

Ag To dig a well a company costs \$10 for 1st ft,

\$12.5 for 2nd ft, \$15 for 3rd ft and so on.

What is the dept of a well that costs \$2925 to dig?

So 1st ft, 2nd ft, 3rd ft,...

Dollars 10, 12.5, 15, ...

Hence \(a = 10\), \(d = 12.5 - 10 = 2.5\)

\[ S_n = \frac{n}{2} [2a + (n-1)d] \]

\[ S_n = \text{sum of all dollars} \]

\[ 2925 = \frac{n}{2} [2(10) + (n-1)2.5] \]

\[ 5850 = n [25n + 17.5] \]

\[ 5850 = 2.5n^2 + 17.5n \]

\[ 2.5n^2 - 17.5n - 5850 = 0 \]

By quadratic formula

\[ n = \frac{-(-17.5) \pm \sqrt{(-17.5)^2 - 4(2.5)(-5850)}}{2(2.5)} \]

\[ n = 17.5 \pm \sqrt{56800} \]

\[ n = 17.5 \pm 242.5 \]

\[ n = \frac{17.5 + 242.5}{5} \]

\[ n = \frac{260}{5} \]

\[ n = 52 \] (not possible)

Since each term represents one year also

Hence total ft over 5 yrs = 52 ft.
The distance which an object dropped from a cliff will fall 16 ft. the 1st second, 48 ft. the next second, 80 ft. the third second, and so on. What is the total distance the object will fall in six seconds?

The distance travelled is:

1st second, 2nd sec, 3rd sec, ---
16, 48, 80, ---

which is an AP with \(d = 32, \ a = 16\)

Now, total distance of the 6 seconds is:

\[
S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]
\]

\[
S_6 = \frac{6}{2} \left[ 2(16) + (6-1)32 \right]
\]

\[
= 3 \left[ 32 + 160 \right]
\]

\[
= 3 \times 192
\]

\[
= 576
\]

\[
S_6 = 576 \text{ ft}
\]

is the total distance of the 6 sec

Bill Affan saves \(Rs.1\) the 1st day, \(Rs.2\) the 2nd day, \(Rs.3\) the third day and so on for thirty days. What is his total saving for the 30 days?

1st day, 2nd day, 3rd day, --- 30th day

\[
S_n = \frac{n}{2} \left[ 2a + (n-1)d \right]
\]

\[
S_{30} = \frac{30}{2} \left[ 2(1) + (30-1)1 \right]
\]

\[
= 15 \left[ 2 + 29 \right]
\]

\[
= 15 \times 31
\]

\[
= 465
\]

A contest will have five cash prizes totaling Rs.5000 and there will be a Rs.100 difference between successive prizes. And the 1st prize.

\[
S_5 \quad \text{let 1st prize} = x
\]

Then 2nd prize = \(x+100\)

\[
d = \text{2nd} \rightarrow = x + 200
\]

5th prize
Here \( a = x, d = 100, n = 5 \) and \( \text{sum} = 5000 \\
\text{Now by formula} \\
S_n = \frac{n}{2} [2a + (n-1)d] \\
\Rightarrow S_5 = \frac{5}{2} [2x + (5-1)100] \\
9000 = \frac{5}{2} [2x + 400] \\
\Rightarrow \frac{2 \times 9000}{5} = 2x + 400 \\
2000 = 2x + 400 \\
\Rightarrow 2x = 1600 \Rightarrow [x = 800] \\
\text{Hence sum of 5th term in \( R_3 \), 800.} \ \ \ \ \text{Ans}

8.13
A theater has 40 rows with 20 seats in the 1st row, 25 in the second row, 26 in the third row and so on. How many seats are in the theater? \\

\text{The seats in the rows are:} \\
1st row, 2nd row, 3rd row, \ldots \ldots \ 40 \text{ terms } \\
20, 25, \ldots \ldots 26, \ldots \ldots \ \text{A}_{40} \\
\text{a=20, d=3, n=40} \\
S_n = \frac{n}{2} \{2a + (n-1)d\} \\
\Rightarrow S_{40} = \frac{40}{2} \{2(20) + (40-1)3\} \\
\quad = 20 \times 40 + 117\} = 20 \{417\} = 3140 \\
\Rightarrow S_{40} = 3140. \\
\text{So the theater has 3140 seats.}
\[ a_1 = \frac{-2}{\sqrt{3}}, \quad r = -\sqrt{3} \]

\[ a_n = ar^{n-1} \]

\[ a_2 = ar^2 = \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{-\sqrt{3}}{2}\right) = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} = \frac{2}{2} = 1 \]

\[ a_3 = ar^3 = \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = -\frac{3}{4} \]

\[ a_4 = ar^4 = \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{-\sqrt{3}}{2}\right)^3 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{3}{8} \]

\[ a_5 = ar^5 = \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{-\sqrt{3}}{2}\right)^4 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{3}{16} \]

Hence the first five terms are:

\[ -\frac{2}{\sqrt{3}}, \quad \frac{1}{4}, \quad \frac{1}{8}, \quad -\frac{1}{2} \]

(iv) \[ a_1 = \frac{2}{\sqrt{3}}, \quad r = -\sqrt{3} \]

\[ a_n = ar^{n-1} \]

\[ a_2 = ar^2 = a_n \cdot \left(\frac{x}{y}\right) \left(\frac{-\sqrt{3}}{2}\right) = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} = \frac{2}{2} = 1 \]

\[ a_3 = ar^3 = a_n \cdot \left(\frac{x}{y}\right)^2 \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = -\frac{3}{4} \]

\[ a_4 = ar^4 = a_n \cdot \left(\frac{x}{y}\right)^3 \left(\frac{-\sqrt{3}}{2}\right)^3 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{3}{8} \]

\[ a_5 = ar^5 = a_n \cdot \left(\frac{x}{y}\right)^4 \left(\frac{-\sqrt{3}}{2}\right)^4 = \frac{2}{\sqrt{3}} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} = \frac{3}{16} \]

Hence the first five terms are:

\[ -\frac{2}{\sqrt{3}}, \quad \frac{1}{4}, \quad \frac{1}{8}, \quad -\frac{1}{2} \]

8.2

Suppose that the third term of a G.P is 27 and the 5th term is 243. Find the first term and the common ratio of the sequence.

\[ a_3 = 27 \quad \Rightarrow \quad ar^2 = 27 \quad \Rightarrow \quad a = 27 \]

\[ a_5 = 243 \quad \Rightarrow \quad ar^4 = 243 \quad \Rightarrow \quad a = 243 \]

From Eqn (1) \[ a = 27 \]

P.V in Eqn (5)

\[ a = 27 \quad \Rightarrow \quad a \neq 243 \]

\[ \frac{27}{243} \quad \Rightarrow \quad 27 \cdot 9 = 243 \]

\[ \frac{9}{243} \quad \Rightarrow \quad 9 \quad \Rightarrow \quad (y \neq 3) \]

Now Eqn (1) \[ a = 27 \]

\[ a (27) = (27)^2 \quad \Rightarrow \quad (a = 3) \]

Hence 1st term \[ a = 3 \quad \Rightarrow \quad \frac{a}{x} = \frac{3}{x} = \frac{3}{1} \]

Ratio \[ r = \frac{3}{x} \]

8.3

Find the seventh term of a G.P whose second and third terms are \( 2 + \sqrt{3} \).

\[ a_1 = \frac{a_n}{r^{n-1}} \]

\[ a_2 = ar^{n-1} \quad \text{and} \quad a_3 = ar^{3-1} \]

\[ a_4 = ar^4 \quad \Rightarrow \quad \sqrt{3} = a_4 \]

\[ a_5 = ar^5 \quad \Rightarrow \quad \sqrt{3} = a_5 \]

\[ a_6 = ar^6 \quad \Rightarrow \quad \sqrt{3} = a_6 \]

\[ a_7 = ar^7 \quad \Rightarrow \quad \sqrt{3} = a_7 \]

\[ a_2 = ar^2 \quad \Rightarrow \quad \sqrt{3} = a_2 \]

\[ a_3 = ar^3 \quad \Rightarrow \quad \sqrt{3} = a_3 \]

\[ a_4 = ar^4 \quad \Rightarrow \quad \sqrt{3} = a_4 \]

\[ a_5 = ar^5 \quad \Rightarrow \quad \sqrt{3} = a_5 \]

\[ a_6 = ar^6 \quad \Rightarrow \quad \sqrt{3} = a_6 \]

\[ a_7 = ar^7 \quad \Rightarrow \quad \sqrt{3} = a_7 \]
Q. 4
How many terms are there in a G.P. if its first term is 16, last term is $\frac{1}{64}$, and common ratio is $\frac{1}{2}$.

\[a = 16, \quad \text{l.t.} = \frac{1}{64}, \quad r = \frac{1}{2}\]

\[\therefore a_n = a_1 \times r^{n-1}\]
\[\Rightarrow \frac{1}{64} = 16 \left(\frac{1}{2}\right)^{n-1}\]
\[\Rightarrow \frac{1}{128} = \left(\frac{1}{2}\right)^{n-1}\]
\[\Rightarrow \left(\frac{1}{2}\right)^{10} = \left(\frac{1}{2}\right)^{n-1}\]
\[\Rightarrow 10 = n-1 \Rightarrow n = 11\]

Q. 5
Find \(n\) if \(n + 7, \quad n - 3, \quad 2 - 8\) form G.P in the given order. Also, give the sequence.

\[\therefore \text{G.P.} \quad n + 7, \quad n - 3, \quad 2 - 8\]
\[\Rightarrow \frac{a_3}{a_1} = \frac{a_3}{a_2} \quad \text{for G.P.}\]
\[\Rightarrow \frac{n-3}{n+7} = \frac{n-8}{n-3}\]
\[\Rightarrow (n-3)^2 = (n+7)(n-8)\]
\[\Rightarrow n^2 + 9 - 6n = n^2 + 3n - 56\]
\[\Rightarrow 9n + 3 = -56\]
\[\Rightarrow n = 7\]

Hence 7th term is \(a_7 = -\frac{3}{2}\).
Show that the reciprocals of the terms of a G.P.
also form a G.P.

General representation of G.P is
\[ a, ar, ar^2, ar^3, \ldots, ar^{n-1} \]

Then its reciprocal will be
\[ \frac{1}{a}, \frac{1}{ar}, \frac{1}{ar^2}, \frac{1}{ar^3}, \ldots, \frac{1}{ar^{n-1}} \]

End the ratio \( \frac{1}{a} \)w consecutive term
\[ \frac{A_2}{A_1} = \frac{\frac{1}{a}r}{a} = \frac{1}{a} \times \frac{a}{1} = \frac{1}{a} \]
\[ \frac{A_3}{A_2} = \frac{\frac{1}{a}r^2}{\frac{1}{a}r} = \frac{1}{a} \times \frac{a}{1} = \frac{1}{a} \]
\[ \frac{A_4}{A_3} = \frac{\frac{1}{a}r^3}{\frac{1}{a}r^2} = \frac{1}{a} \times \frac{a}{1} = \frac{1}{a} \]

End the reciprocal of the terms
also form G.P because the ratio is same.

The yearly depreciation of a certain machine is 20% of its value at the beginning of the
year. If the original cost of the machine is Rs 5000, find its value after 5 years.

Initial value = 0, = Rs 5000
After one year depreciation is 20% of 500
= \frac{10}{100} (5000)
= 1000
So at 2nd year, its value is $= 5000 - 1000$
\[ \Rightarrow A_2 = 1000 \]

Then the sequence of value is:
5000, 4000, .......

Total 5 yrs \[ A_1, A_2, A_3, A_4, A_5, A_6 \]
\[ A_1 = 1000, \quad r = \frac{4000}{5000} = 0.8 \]

Now \[ A_2 = A_1 \times 0.8 \]
\[ \Rightarrow A_3 = A_2 \times 0.8 \]
\[ \quad = (5000)(0.8)^2 = 5000(0.32768) = 1638.4 \]

Hence after 5 yrs, value will be $A_5 = 1538.4$

Q: A tennis ball is dropped from a height of 10 ft and it bounces to 75% of the previous bounce. How high does the ball bounce on the 6th bounce?

\[ A = 10 \text{ ft} \]
\[ A_2 = 75\% \times A \]
\[ = 75 \left( \frac{1}{100} \right) (10) \]
\[ = 7.5 \text{ ft} \]

Then the sequence becomes
10, 7.5, .......
\[ \Rightarrow r = 7.5 / 10 \Rightarrow r = 0.75 \]

On 6th bounce the height will be 7th
\[ A_7 = A_3 \times 0.8^4 \]
\[ = 10 \times (0.75)^6 \]
\[ = 10 \times (0.1779) \]
\[ \Rightarrow [A_7 = 1.78 \text{ ft}] \]

Ex.45
P.12

Q.10
Find three numbers such that their sum is 3.
The numbers form A.P. but their squares form G.P.

Let the three numbers are \(a-d, a, a+d\)
sum = 3
\[ \Rightarrow (a-d) + a + (a+d) = 3 \]
\[ \Rightarrow 3a = 3 \Rightarrow [a = 1] \]

Squares of the 3s form G.P.
\[ \Rightarrow (a-d)^2, a^2, (a+d)^2 \text{ form G.P.} \]
\[ \Rightarrow \frac{a^2}{(a-d)^2} = \frac{(a+d)^2}{a^2} \]
\[ a^2, a^2 = (a-d)^2 (a+d)^2 \]
\[ a^4 = (a-d)^4 (a+d)^4 \]
\[ p = (l-d)^2 (l+d)^2 \Rightarrow 1 = \left[ (l-d) (l+d) \right]^2 \]
\[ l = (l-d)^2 \Rightarrow 1 = (a+d)^2 - 2d \cdot \frac{d^2}{2} \]
\[ a^2 + 2a \cdot d = 0 \]
\[ a^2 + d^2 \]
\[ d = \frac{1}{2} \Rightarrow d = \pm \sqrt{2} \]

New the required numbers are:
\[ a = 1 \]
\[ d = \sqrt{2} \]
\[ a-d = 1-\sqrt{2} \]
\[ a-d = 1-(-\sqrt{2}) = 1+\sqrt{2} \]
\[ a = 1 \]
\[ a = 1 \]
\[ a+d = 1+\sqrt{2} \]
\[ a+d = 1+(-\sqrt{2}) = 1-\sqrt{2} \]

\[ \text{Annual interest} = 11 \% \]
\[ = \frac{11}{100} \]
\[ = 0.11 \]

The common ratio is \[ 1 + 0.11 = 1.11 \]
At the end of four years we have \[ a_5 \]
\[ a_4 = a_3 \]
\[ a_2 \]
\[ a_1 = 1\text{st yr} \]
\[ a_2 = 2000 \times 1.11 \]
\[ a_3 = 2000 \times (1.11)^2 \]
\[ a_4 = 2000 \times (1.11)^3 \]
\[ a_5 = 2000 \times (1.11)^4 \]
\[ a_5 = \text{Rs} \ 3036.14 \text{ Rs} \]

\[ \text{Initial amount} = a_1 = \text{Rs} \ 2000 \]

\[ \text{Quote} \]
our prayers should be for our blessings in general, for God knows best what is good for us. (Socrates)

Available at www.mathcity.org
Exercise # 4.6

2
2
1

3. Insert two G.M.s b/w \( \sqrt{3} \) and 3.

Example 6

Let \( G_1 \) and \( G_2 \) are the two G.M.s, then \( \sqrt{3}, G_1, G_2, 3 \) is G.P.

To find \( G_1 \), we have

\[
G_1 = a \gamma^{n-1}
\]

\[
3 = \sqrt{3} \cdot 3^{\frac{1}{2}}
\]

\[
\Rightarrow 3^2 = 3^3
\]

\[
\Rightarrow x = 3^n
\]

\[
\Rightarrow (3^n)\frac{1}{2} = (3^3)\frac{1}{2}
\]

\[
\Rightarrow 3^2 = 3^4
\]

Now

\[
G_2 = a \gamma^n
\]

\[
G_1 = a^3 = \sqrt{3} \cdot (3^{\frac{3}{2}}) = 3^{\frac{3}{2}} \cdot 3^{\frac{3}{2}} = 3^{\frac{3}{2} + \frac{3}{2}} = 3^3
\]

\[
G_2 = a^7 = \sqrt{3} \cdot (3^{\frac{3}{2}}) = 3^{\frac{3}{2}} \cdot 3^{\frac{3}{2}} = 3^{\frac{3}{2} + \frac{3}{2}} = 3^3
\]


3

Let \( G_1, G_2, \) and \( G_3 \) are the three G.M.s b/w \( a^4 \) and \( b^4 \).

Then

\[
a^4, G_1, G_2, G_3, b^4 \text{ is G.P.}
\]

To find \( \gamma \), we have

\[
G_1 = a \gamma^{n-1}
\]

\[
b^4 = a^4 \gamma^{5-1}
\]

\[
\Rightarrow \frac{b^4}{a^4} = \gamma^4 \Rightarrow (\frac{b}{a})^4 = \gamma^4 \Rightarrow \frac{b}{a} = \gamma
\]
Now \( G_m = a_m^n \)

\( \Rightarrow G_1 = a_1^{*1} = a_1 \left( \frac{b}{a} \right) = a_1 \cdot \frac{b}{a} = a_1 \cdot \frac{b_m}{a_m} \)

\( G_2 = a_2^{*2} = a_2 \left( \frac{b}{a} \right)^2 = a_2 \left( \frac{b_m}{a_m} \right)^2 \)

\( G_3 = a_3^{*3} = a_3 \left( \frac{b}{a} \right)^3 = a_3 \left( \frac{b_m}{a_m} \right)^3 \)

\( G_4 = a_4^{*4} = a_4 \left( \frac{b}{a} \right)^4 = a_4 \left( \frac{b_m}{a_m} \right)^4 \)

Q5: Find two numbers, if the difference b/w them is 48 and their A.M exceeds their G.M by 18 (i.e., A.M is 18 more than G.M).

\( \Rightarrow a-b=48 \) (given, that their difference is 43)

and \( G.M + 18 = A.M \) ( \( \because \) A.M is 18 more than G.M).

\( \Rightarrow \sqrt{a} + 18 = \frac{a+b}{2} \)

(\(G.M\) + 18 = AM)

\( \Rightarrow \sqrt{a} + 36 = a + b \)

\( \Rightarrow \sqrt{a} + 36 = a + b \quad \text{(1)} \)

From eqn (1) \( b = a - 48 \), put in eqn (1)

\( \Rightarrow 2\sqrt{a(a - 48)} + 36 = a + (a - 48) \)

\( \Rightarrow 2\sqrt{a^2 - 48a} + 36 = 2a - 48 \)

\( \Rightarrow 2\sqrt{a^2 - 48a} = 2a - 48 - 36 \)

\( \Rightarrow 2\sqrt{a^2 - 48a} = 2a - 84 \Rightarrow 2\sqrt{a^2 - 48a} = 2(a - 42) \)

\( \Rightarrow \sqrt{a^2 - 48a} = a - 42 \quad \text{squaring both sides} \)

\( \Rightarrow (\sqrt{a^2 - 48a})^2 = (a - 42)^2 \)

\( \Rightarrow a^2 - 48a = a^2 + (42)^2 - 2(a)(42) \)

\( \Rightarrow a^2 - 48a = a^2 + 1764 - 84a \)

\( \Rightarrow -48a = 1764 - 84a \)

\( \Rightarrow 84a - 48a = 1764 \)
\[ 36a = 1764 \div 6.5 \text{ by 36} \]
\[ a = 49 \]

Then \[ a - b = 48 \Rightarrow b = a - 48 \]
\[ b = 49 - 48 \]
\[ b = 1 \]

Hence the required numbers are 49 and 1.

**Q.6:** Prove that the product of \( n \) G.M.s b/w \( a \) & \( b \) is equal to the \( n \)th power of a single G.M b/w them.

**Sol.:** To prove that the product of \( n \) G.M.s b/w \( a \) and \( b \) is equal to \( n \)th power of a single G.M b/w them

\[ G_1G_2G_3 \ldots \ldots G_n = \sqrt[n]{ab} \]

Let \( G_1, G_2, G_3, \ldots \; G_n \) be \( n \) G.M.s b/w \( a \) and \( b \) then \( a, G_1, G_2, G_3, \ldots \; G_n, b \) is G.P

To find \( \frac{a - b}{a^2} \)
\[ a = ar^{n-1} \]
\[ b = ar^n \]
\[ \frac{b}{a} = r^{n+1} \Rightarrow \left( \frac{b}{a} \right)^{\frac{1}{n+1}} = r \]

**L.H.S.**
\[ G_1G_2G_3 \ldots \ldots G_n = a r^1 \cdot ar^2 \cdot ar^3 \ldots \; ar^n \]
\[ = (a, a^2 \ldots n \text{ times}) \cdot (r, r^2 \ldots r^3 \ldots r^n) \]
\[ = a^{n+1} \ldots \; n \text{ times} \; r^{1+2+3+\ldots+n} \]

**R.H.S.**
\[ \\]

\[ \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab} \]
\[ \frac{(a^n + b^n)^{\frac{n+1}{n}}}{a^n + b^n} = (ab)^{\frac{1}{2}} \]

\[ \text{From eqn (i) and (ii), we have} \]
\[ G^n = (ab)^{\frac{1}{2}} \]

**Q.7:** For what value of \( n \), \( \frac{a^{n+1} + b^{n+1}}{a^n + b^n} \) is G.M.

b/w \( a \) and \( b \).

**Sol.:**
\[ \frac{a^n + b^n}{a^n + b^n} = G.M \]
\[ \frac{a^n + b^n}{a^n + b^n} = \sqrt{ab} \]
\[ \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = (ab)^{\frac{1}{2}} \]
\[
\begin{align*}
\text{Exercise } \#4.7 \\
\text{Q1: Compute the sum} \\
\text{a) } 3 + 6 + 12 + \cdots + 3 \cdot 2^9 \\
\text{So } \qquad \frac{a}{2} = \frac{3}{2} \Rightarrow q > 1 \\
\text{To find } n \\
\text{An} = a_0 \cdot q^{n-1} \\
q = 3 \cdot 2^9 = 3 \cdot (2)^{n-1} \\
q = a_0 \cdot q^{n-1} \\
9 = n-1 \Rightarrow 10 = n \\
\text{Now } S_n = a \left\{ q^n - 1 \right\} \\
\text{r-1} \\
\Rightarrow S_{10} = 3 \left\{ 2^{10} - 1 \right\} \\
\text{r-1} \\
\Rightarrow S_{10} = 3 \left\{ 2^{10} - 1 \right\} \\
\end{align*}
\]

\[8 + 4 + 2 + 1 + \cdots + \frac{1}{15} \quad q = \frac{2}{3} = \frac{1}{2} \]

\[
\begin{align*}
\text{So } \qquad \text{An} = a_0 \cdot q^{n-1} \\
\Rightarrow \frac{1}{15} = 8 \left( \frac{1}{2} \right)^{n-1} \\
\Rightarrow \frac{1}{15 \times 2} = \left( \frac{1}{2} \right)^{n-1} \\
\end{align*}
\]
\[ \frac{1}{2^n} = \left(\frac{1}{2}\right)^{n-1} \]

\[ \frac{1}{2^7} = \left(\frac{1}{2}\right)^{7-1} \]

\[ \Rightarrow B = n \]

\[ S_n = \frac{\alpha \left\{ \gamma^n - 1 \right\}}{\gamma - 1} \]

\[ S_3 = \frac{8 \left\{ \left(\frac{1}{2}\right)^3 - 1 \right\}}{\frac{1}{2} - 1} = \frac{8 \left\{ -\frac{1}{2} \right\}}{-\frac{1}{2}} = 8 \left\{ \frac{2}{2} \right\} = 16 \left(\frac{255}{256}\right) = \frac{255}{16} \text{ Mw} \]

\[ a^4 + a^5 + a^6 + \ldots + a^{10} \]

Here \( a = 2^4 \) and \( \gamma = 2 \) and \( n = 7 \), \( \therefore a_n = a_7 \gamma^{n-1} \)

\[ S_n = \frac{a \left\{ \gamma^n - 1 \right\}}{\gamma - 1} \]

\[ S_7 = \frac{a^4 \left\{ \gamma^7 - 1 \right\}}{2 - 1} = 4 \left\{ 128 - 1 \right\} = 16(127) = 3072 \text{ Mw} \]

(i) \( a_1 = 1 \), \( \gamma = -2 \), \( a_n = 64 \)

\[ a_1 = \alpha \gamma^{n-1} \]

\[ \Rightarrow 64 = 1 (-2)^{n-1} \]

\[ \Rightarrow \frac{64}{(-2)^{n-1}} = (-2)^{n-1} \]

\[ \Rightarrow G = n - 1 \Rightarrow [2^n - 1] \text{ Mw} \]

\[ \text{And } S_7 = \frac{a \left\{ \gamma^7 - 1 \right\}}{(\gamma - 1)} \]

\[ S_n = \frac{a \left\{ \gamma^n - 1 \right\}}{\gamma - 1} \]
\[ S_{3} = \frac{1}{2} \left( \frac{-129}{-2} \right) = \frac{-129}{-4} = 43 \text{ } \text{ Eq.} \]

\[ H_{n} = \text{given} \quad n = 7 \]

\[ S_{3} = 43 \]

(iii) \[ \gamma = \frac{1}{2} \quad \text{and} \quad a_{9} = 1 \]

\[ S_{9} = a \left\{ \gamma^{9-1} \right\} \]

\[ a_{9} = a \gamma^{9-1} \]

\[ 1 = a \left( \frac{1}{2} \right)^{9} \Rightarrow 1 = a \left( \frac{1}{512} \right) \Rightarrow 512 = a \] \text{ Eq.}

Now

\[ S_{9} = \frac{a \left\{ \gamma^{9} - 1 \right\}}{\gamma - 1} \]

\[ S_{9} = 256 \left\{ \frac{\gamma^{9} - 1}{\gamma - 1} \right\} = 256 \left\{ \frac{\gamma^{9} - 1}{512} \right\} \]

\[ = \frac{256 \gamma^{9} - 256}{512} \times \frac{511}{511} = 511 \text{ } \text{ Eq.} \]

(iii) \[ \gamma = -2 \quad S_{n} = 63 \quad a_{6} = -96 \]

\[ a_{6} = a \gamma^{6-1} \]

\[ a_{6} = a \gamma^{5} \]

\[ -96 = a (-2)^{5} \]

\[ \Rightarrow -96 = a (32) \quad \Rightarrow \quad a = -96 \frac{3}{2} \]

\[ \Rightarrow a = 3 \quad \text{Eq.} \]

Now

\[ S_{n} = \frac{a \left\{ \gamma^{n} - 1 \right\}}{\gamma - 1} \]

\[ \Rightarrow -63 = 3 \left\{ \left( \gamma^{7} - 1 \right) \right\} \]

\[ \Rightarrow -63 = 3 \left\{ \left( -2 \right)^{7} - 1 \right\} \]

\[ \Rightarrow 63 = \left( -2 \right)^{7} - 1 \Rightarrow 64 = \left(-2\right)^{7} \Rightarrow (-2)^{7} = (-2)^{7} \]

\[ \Rightarrow a = 3 \quad \text{Eq.} \]

\[ n = 6 \]

Q3: And \( \nu \), such that

\[ S_{0} = 244 S_{5} \]

\[ S_{8} \quad S_{n} = a \left\{ \gamma^{n} - 1 \right\} \quad S_{10} = a \left\{ \gamma^{10} - 1 \right\} \]

R.T.V in

\[ S_{10} = 244 S_{5} \]

\[ S_{5} = a \left\{ \gamma^{5} - 1 \right\} \]

\[ \Rightarrow a \left\{ \gamma^{10} - 1 \right\} = 244 a \left\{ \gamma^{5} - 1 \right\} \]

\[ \Rightarrow \gamma^{10} - 1 = 244 \gamma^{5} - 244 \]

\[ \Rightarrow \gamma^{10} - 244 \gamma^{5} + 243 = 0 \]

\[ \Rightarrow \gamma^{10} - 243 \gamma^{5} + 243 = 0 \]

\[ \Rightarrow \gamma^{5} \left( \gamma^{5} - 243 \right) - 1 \left( \gamma^{5} - 243 \right) = 0 \]

\[ \Rightarrow \left( \gamma^{5} - 243 \right) \left( \gamma^{5} - 1 \right) = 0 \]
\[ a^2 \left( y_{n-1} \right) \left\{ y^{3n} - y^{2n} \right\} \]
As

Find the sum $S_n$ of the first $n$ terms of the sequence $\left(\frac{1}{2}\right)^n$.

$$a_n = \left(\frac{1}{2}\right)^n$$

$$a_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

$$a_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{The series will be}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

which is a Geometric Progression (G.P)

Here $a = \frac{1}{2}$ and $r = \frac{1}{2}$

$$S_n = a \cdot \frac{r^n - 1}{r - 1}$$

$$a_n = \left(\frac{1}{2}\right) \cdot \frac{(\frac{1}{2})^{n-1} - 1}{\frac{1}{2} - 1}$$

$$S_n = \left(\frac{1}{2}\right) \cdot \frac{(\frac{1}{2})^{n-1} - 1}{\frac{1}{2} - 1} = \left(\frac{1}{2}\right) \cdot \frac{-\left(\frac{1}{2}\right)^n + 1}{-1}$$

$$= -\left(\frac{1}{2}\right)^n + 1$$

$$= 1 - \left(\frac{1}{2}\right)^n$$

$$\Rightarrow S_n = 1 - \frac{1}{2^n}$$

A ball re-bounces to half the height from which it is dropped. If it is dropped from 10 ft, how far does it travel from the moment it is dropped until the moment of its eigth bounce?

5 ft up
\[\uparrow\downarrow\]
6 ft down
So the sequence becomes

$$10 + \left\{5 + 5 + 5 + \cdots\right\}$$

$$= 10 + 2 \left\{5 + \frac{5}{2} + \cdots\right\}a_2$$

$$= 10 + 2 \left\{5 \cdot \left(\frac{1}{2}\right)^2 - 1\right\}$$

$$= 10 + 2 \left\{5 \cdot \frac{1}{4} - 1\right\}$$

$$= 10 + 2 \left\{\frac{5}{4} - 1\right\}$$

$$= 2.875$$

of 29 $\frac{29}{28}$ ft
A man wishes to save money by setting aside Rs. 1 the 1st day, Rs. 2 the 2nd day, Rs. 4 the third day and so on, doubling the amount each day. If this continued, how much must be set aside on the 15th day? What is the total amount saved at the end of 30 days?

Day 1 Day 2 Day 3 Day 4 ……… Day 30
Saving → Rs. 1, Rs. 2, Rs. 4, ………. Rs. 30

Amount of 15th day

\[ a = a^n - 1 \]
\[ a_1 = a^2 \]
\[ a_2 = (a^2)^2 \]
\[ a_3 = 16384 \text{ Ans.} \]

Total Amount of the 30 days:

\[ S_n = \frac{a^n}{a-1} \]
\[ S_{30} = \frac{1(1073741824-1)}{2-1} = 1073741823 \text{ Rs.} \]

The population of an insect is found to triple each week in the summer months. If there are twenty insects in the colony at the beginning of the summer, how many are present at the end of 11 weeks assuming no deaths are there?

Weeks 1st 2nd 3rd ………. 11th
Insects 20, 60, 180, ………. 120 because starting insects = 20 and each week they triple.
Total insects at 11 weeks = 120

\[ a = 20 \quad r = 3 \]
\[ S_n = \frac{a(r^n-1)}{r-1} \]
\[ S_{11} = \frac{20(3^{12}-1)}{2-1} \]
\[ S_{11} = 10(3^{12}-1) = 531440 \text{ Ans.} \]
Exercise # 4.8

(i) Find the sum of each of the given infinite G.P.

(i) \(16 + 12 + 9 + \ldots\)

The geometric series will be

\[a = 16, \quad r = \frac{12}{16} = \frac{3}{4} \quad (1 < 1)\]

\[ S_{\infty} = \frac{a}{1-r} \]

\[ \Rightarrow S_{\infty} = \frac{16}{1 - \frac{3}{4}} = \frac{16}{\frac{1}{4}} = 64 \]

(ii) \(\frac{1}{25}, \frac{1}{5}, 1, 5, \ldots\)

\[ S_{\infty} = \frac{a}{1-r} \]

\[ \Rightarrow S_{\infty} = \frac{\frac{1}{25}}{1 - \frac{1}{5}} = \frac{\frac{1}{25}}{\frac{4}{5}} = \frac{5}{4} \]

\[ \Rightarrow S_{\infty} \text{ is not possible} \]

(iii) \(\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \ldots\)

\[ S_{\infty} = \frac{a}{1-r} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{\sqrt{2}}} = \frac{\frac{2}{\sqrt{3}}}{\frac{\sqrt{2} - 1}{\sqrt{2}} \cdot \sqrt{2}} = \frac{2}{\sqrt{3} - 1} \]

(iv) Find the 1st five terms of the following infinite geometric sequence.

(i) \(a_1 = 25, \quad S_0 = 125\)

\[ a_1 = \frac{a}{1-r} \]

\[ \Rightarrow 125 = \frac{25}{1-\frac{1}{5}} \]

\[ \Rightarrow 1 - \frac{1}{5} = \frac{25}{125} \]

\[ \Rightarrow \frac{4}{5} = \frac{1}{5} \]

\[ \Rightarrow \frac{4}{5} - 1 = \frac{4}{5} \]

\[ \Rightarrow a = 25 \]

\[ a_1 = a = 25 \]

\[ a_2 = ar = 25 \left( \frac{4}{5} \right) = 20 \]

\[ a_3 = ar^2 = 25 \left( \frac{4}{5} \right)^2 = 25 \left( \frac{16}{25} \right) = 16 \]

\[ a_4 = ar^3 = 25 \left( \frac{4}{5} \right)^3 = 25 \left( \frac{64}{125} \right) = \frac{64}{5} \]

\[ a_5 = ar^4 = 25 \left( \frac{4}{5} \right)^4 = 25 \left( \frac{256}{625} \right) = \frac{256}{25} \]
(ii) $a_1 = 4$ and $S = 7$

$S = a_1 \frac{S_r}{1 - r}$

\[ \Rightarrow -7 = \frac{4}{1 - r} \Rightarrow 1 - r = -\frac{4}{7} \Rightarrow 1 + \frac{4}{7} = r \]

Since $|r| > 1$

Therefore, such an infinite geometric series does not exist.

(iii) Find the first five terms and the sum of an infinite geometric sequence having $a_2 = 2$ and $a_3 = 1$.

$S_3 = a + ar + ar^2 = 2 + a + a^2$

\[ \Rightarrow 2 = a^2 \quad \text{and} \quad 1 = a^3 \Rightarrow a = 2 \]

New

Now $a = \frac{2}{3}$

\[ \Rightarrow \frac{2}{3} \quad \Rightarrow \quad a = 1 \]

Then the terms are...

\[ a_1 = a = 2 \]

\[ a_2 = a^2 = 2(\frac{2}{3})^2 \]

\[ a_3 = a^3 = 2(\frac{2}{3})^3 \]

\[ a_4 = a^4 = 2(\frac{2}{3})^4 \]

\[ a_5 = a^5 = 2(\frac{2}{3})^5 \]

Hence $S_5 = \frac{2}{1 - \frac{2}{3}} = \frac{2}{1 - \frac{1}{3}} = \frac{2}{\frac{2}{3}} = \frac{3}{2}$

\[ \Rightarrow 1.63 = \frac{3}{2} \]

\[ \Rightarrow 1.63 = 1.63636363... \]

\[ = 1 + 0.63636363... \]

\[ = 1 + \left\{ \frac{0.63 + 0.6363 + 0.636363...}{0.63} \right\} \]

\[ = \frac{1 + \frac{0.63}{1 - 0.63}}{0.99} \]

\[ = 1 + \frac{0.63}{0.99} \quad \text{where } a = 0.63 \]

\[ = 1 + \frac{0.63}{0.99} \quad \text{and } r = \frac{a_2}{a_1} = \frac{0.6363}{0.63} \]

\[ = 1 + \frac{0.63}{0.99} = \frac{99 + 63}{99} = \frac{162}{99} \]

Hence $1.63 = \frac{162}{99}$
(i) \[ 2.15 \]

Let \[ 2.15 = 2.151515 \ldots \]

\[ = 2 + \frac{0.15}{1 - 0.01} \]

\[ = 2 + \frac{0.15}{0.99} \]

\[ = 2 + \frac{199 + 15}{99} \]

\[ = 2 + \frac{214}{99} \]

\[ = 2.15 \]

Self question

(ii) \[ 2.15 \]

\[ = 2 + \frac{0.15}{0.99} \]

\[ = 2 + \frac{a}{1 - y} \]

where \[ a = 0.15 \]

\[ y = \frac{a}{a_1} = \frac{0.0015}{0.15} = 0.01 \]

\[ = 2 + \frac{0.15}{1 - 0.01} \]

\[ = \frac{199 + 15}{99} \]

\[ = \frac{214}{99} \]

\[ = 2.15 \]

\[ \text{The sum of infinite geometric series is } 15 \]

\[ \text{and the sum of their squares is } 4.5. \text{ Find the series.} \]

\[ \Rightarrow \frac{a}{1 - y} = 15 \Rightarrow a = 15(1 - y) \rightarrow 1 \]

\[ \text{And given that the sum of their squares is } 4.5 \]

\[ \Rightarrow a^2 + (ay)^2 + (a^2y^2)^2 + \ldots = 4.5 \]

\[ \Rightarrow a^2 + a^2y^2 + a^4y^4 + \ldots = 4.5 \]

\[ \Rightarrow a^2 \left( 1 + y^2 + y^4 + \ldots \right) = 4.5 \]

\[ \Rightarrow \frac{a^2}{1 - y} = 4.5 \Rightarrow \frac{a^2}{1 - y^2} = 4.5 \]
According to the condition

Ex 4.8

Sum of 1st 6 terms is 9 times the sum of 1st 3 terms

\[ S_6 = 9S_3 \]

\[ a^6 = 9a^3 \]

\[ \frac{a^6 - 1}{a - 1} = 9 \cdot \frac{a^3 - 1}{a - 1} \]

\[ a^6 - 1 = 9a^3 - 9 \]

\[ a^6 - 9a^3 + 8 = 0 \]

Let \( y^3 = a \)

\[ y^3 - 9y + 8 = 0 \]

\[ y^2 - 8y - x + 8 = 0 \]

\[ x(x - 8) - 1(x - 8) = 0 \]

\[ (x - 8)(x - 1) = 0 \]

\[ x - 8 = 0 \quad \text{or} \quad x - 1 = 0 \]

\[ x = 8 \quad \text{or} \quad x = 1 \]

\[ y^3 = 8 \quad \text{or} \quad y^2 = 1 \]

\[ x^2 = a^2 \quad \text{or} \quad y^2 = 1^2 \]

\[ a = 2 \quad \text{or} \quad y = 1 \]

\[ a = 1 \text{ is not possible.} \]

Available at
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8.3. How many terms of the series \(1 + \sqrt{3} + 3 + \ldots\) be added to get the sum \(40 + 13\sqrt{3}\). 

**Sol.** Geometric series is 
\[ 1 + \sqrt{3} + 3 + \ldots \]
Here \(a = 1\) and \(r = \sqrt{3}\), \(S_n = 40 + 13\sqrt{3}\), \(n = ?\)

\[ S_n = \frac{a \left(r^n - 1\right)}{r - 1} \]

\[ 40 + 13\sqrt{3} = \frac{1 \left((\sqrt{3})^n - 1\right)}{\sqrt{3} - 1} \]

\[ (40 + 13\sqrt{3})(\sqrt{3} - 1) = (\sqrt{3})^n - 1 \]

\[ 40\sqrt{3} - 40 + 13(\sqrt{3})^2 - 13\sqrt{3} = (\sqrt{3})^n - 1 \]

\[ 40\sqrt{3} - 40 + 39 - 13\sqrt{3} = (\sqrt{3})^n - 1 \]

\[ 40\sqrt{3} - 1 - 13\sqrt{3} = (\sqrt{3})^n - 1 \]

\[ 40\sqrt{3} - 13\sqrt{3} = (\sqrt{3})^n - 1 \]

\[ 27\sqrt{3} = (\sqrt{3})^n \]

\[ 3^{3.5} = (\sqrt{3})^n \]

\[ 3^{3+\frac{1}{2}} = 3^{\frac{7}{2}} \]

\[ 3^{\frac{7}{2}} = 3^{\frac{7}{2}} \]

**compare the powers** 
\[ \frac{7}{2} = \frac{n}{2} \]

\[ \boxed{n = 7} \] 

\[ a + ar + ar^2 + \cdots \]

\[ = \frac{2\frac{1}{5}}{5} + \frac{2\frac{4}{5}}{5} \left(\frac{1}{5}\right) + \cdots \]

\[ = \frac{2\frac{1}{5}}{5} + \frac{2\frac{4}{5}}{5} \cdot \frac{1}{5} + \cdots \]

\[ = 2\frac{1}{5} + 2\frac{4}{25} + 2\frac{4}{125} + \cdots \]
\[ S_0 = \text{Total distance travelled in Ex 4.8} \]
\[ 24 + \{8 + \frac{8}{3} \times \frac{8}{3} + \cdots \} \]
\[ = 24 + \frac{8}{3} \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + \frac{8}{3} \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]
\[ = 24 + 2 \times \left\{ \frac{8}{1 - \frac{8}{3}} \right\} \]

\[ 9.1 \text{ The number of bacteria in a culture increased geometrically from 64000 to 729000 in 6 days.} \]
\[ \text{Find the daily rate of increase if the rate is assumed to be constant.} \]

\[ a_1, a_2, a_3, a_4, a_5, a_6, a_7 \]
1 day 2nd day 3rd day 4th day 5th day 6th day
\[ \text{start} = a_1 = 64000 \]
\[ \text{Total days} = n = 7 \]
\[ \text{End} = a_n = 729000 \]
\[ \text{To find } a_n: a_n = a_1 \times r^{n-1} \]
\[ = 729000 = 64000 \times r^{7-1} \]

\[ \text{Diagram} \]
\[ y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \cdots \]
where \( 0 < x < 3 \)

\[ \text{Show that } x = \frac{2y}{1+y} \]

\[ y = \frac{x}{3} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \cdots \]

\[ \text{is an infinite geometric series with } a = \frac{x}{3}, r = \frac{x}{3} \]

\[ y = \frac{a}{1-r} \]

\[ y = \frac{x/3}{1-x/3} \Rightarrow y = \frac{x/3}{3} \Rightarrow y = \frac{x}{3-x} \Rightarrow y = \frac{x}{3} \]

Then \( (3-x) y = x \Rightarrow 3y - xy = x \)
\[ \Rightarrow 3y = x + xy \]
\[ \Rightarrow 3y = x + xy \]
\[ \Rightarrow 3y = x + xy \]
\[ \Rightarrow \left[ x = \frac{3y}{1+y} \right] \text{ proved.} \]

\[ 6.10 \text{ Find how far a ball travels before coming to rest, if it is dropped from a height of 84 meters and each time it hits the ground, it rebounds one third of the distance from which it fell?} \]

\[ \text{Diagram} \]
\[ 24 \]
\[ \frac{729000}{64000} = \gamma^6 \]
\[ \frac{729}{64} = \gamma^6 \]
\[ \gamma^6 = 11.390625 \]
\[ \gamma^6 = 11.390625 \]
\[ \frac{3}{2} = \gamma^6 \]
\[ \left( \frac{3}{2} \right)^6 = \gamma^6 \]
\[ \Rightarrow \gamma = \frac{3}{2} \]

Now, the common ratio of 1.5 implies that quantity 1 becomes 1.5 after the increase. So the rate of increase = 50%. Increase = 0.5.

\[ = 0.5 \times \frac{100}{100} \]
\[ = 0.5 \times \frac{1}{100} \]
\[ = 50 \text{%} \]

\[ \text{Rate} = 5\% \]
\[ = 5 \left( \frac{1}{100} \right) = 0.05 \]

Then, the common ratio = 1 + 0.05 = 1.05. After 6 years term will be \( a_7 \).

\[ a_7 = a_0 \cdot 1.05^6 \]
\[ = (300,000) \cdot (1.05)^6 \]
\[ = (300,000) \cdot (1.34005) \]
\[ = 402,028.69 \]

\[ a_7 = 402,028.69 \text{ AU} \]

\[ \text{A machine which contains 16,000 liters of water. Each day one half of the water in the tank is removed with out replacement. How much water remains in the tank at the end of the 8th day?} \]

\[ \text{Initial water} = a_1 = 16,000 \text{ liters} \]

\[ \text{Removed per day} = \frac{1}{2} \text{ or half} \]

\[ \text{So water at end day} = \frac{1}{2} (16000) = 8000 \]
\[ \Rightarrow a_2 = 8000 \]

\[ \text{So the sequence becomes} \]
\[ 16,000 \; 8000 \; 4000, \ldots \Rightarrow \gamma = \frac{1}{2} \]

\[ a_1 \; a_2 \; a_3 \; a_4 \; \ldots \]

1st day \; 2nd day \; 3rd day
End of 8th day implies $a_9$.

\[ a_n = a_8n - 1 \]

\[ a_9 = a_8^9 - 1 = 16000 \left( \frac{1}{2} \right)^9 = 16000 \left( \frac{1}{512} \right) = 62.5 \]

Hence $a_9 = 62.5 \text{ letters}$.

### Exercise 4.9

1. Find the indicated term of the H.P

(i) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \ldots \text{ 9th term}$

(ii) H.P is $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \ldots$ 

\[ a = 2, \quad d = 3, \quad \text{then} \]

\[ a_n = a + (n-1)d \]

\[ a_9 = 2 + (9-1) \times 3 \]

In H.P: $a_{26}$

\[ \Rightarrow \text{In H.P} \quad a_{26} = \frac{1}{50} \text{ Ans.}$

(iii) $6, 2, \frac{6}{5}, \ldots \ldots \text{ 20th term}$

H.P is $6, 2, \frac{6}{5}, \ldots \ldots$

A.P will be $\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \ldots \ldots$

\[ \Rightarrow a = \frac{1}{6} \quad d = \frac{1}{3} - \frac{1}{6} = \frac{1}{6} \]

\[ d = \frac{3}{6} \quad \Rightarrow d = \frac{1}{2} \]

\[ a_n = a + (n-1)d \]

\[ a_9 = \frac{3}{17} + (8-1) \times \frac{1}{17} = \frac{3}{17} + \frac{14}{17} - \frac{3}{17} = \frac{14}{17} = 1 \]

\[ a_8 = 1 \text{ in A.P} \]

\[ \text{In H.P} \quad a_8 = \frac{1}{4} \]

\[ \Rightarrow a_8 = 1 \text{ A.N.} \]

Ex 4.9

CH-04

P-21
8.3 And five more terms of the H.P
\[ \frac{1}{3}, 1, -1, \ldots \]

8.4 H.P is \( \frac{1}{3}, 1, -1, \ldots \)
H.P will be \( \frac{2}{3}, \frac{1}{3}, \frac{1}{4}, \ldots \)
\[ 3, 1, -1, \ldots \]
\[ \Rightarrow a = 3 \quad d = -2 \quad (\because d = a_2 - a_1 = 1 - 3) \]

8.5 \( a_n = a + (n-1)d \) in H.P
\[ a_4 = a + (4-1)d = 3 + 3(-2) = -3 \]
\[ a_5 = a + (5-1)d = 3 + 4(-2) = -5 \]
\[ a_6 = a + (6-1)d = 3 + 5(-2) = -7 \]
\[ a_7 = a + (7-1)d = 3 + 6(-2) = -9 \]
\[ a_8 = a + (8-1)d = 3 + 7(-2) = -11 \]

In H.P \( a_4 = \frac{-1}{3}, a_5 = \frac{-1}{5}, a_6 = \frac{-1}{7}, a_7 = \frac{-1}{9}, a_8 = \frac{-1}{11} \)

8.6 The second term of H.P is \( \frac{1}{2} \) and the first term is \( -\frac{1}{4} \). Find the 12th term.

8.7 \( a_2 = \frac{1}{2} \) and \( a_5 = -\frac{1}{4} \) in H.P

In H.P \( a_2 = 2 \) and \( a_5 = -4 \)
\[ a + 1d = 2 \]
\[ a + 4d = -4 \quad \Rightarrow \quad a_n = a + (n-1)d \]

\[ \text{Eqn (7) - Eqn (1)} \]
\[ a + 4d = -4 \]
\[ \frac{a + 1d = -2}{3d = -6 \Rightarrow d = -2} \]

\[ \text{Eqn (6) \Rightarrow a + d = 2} \]
\[ a - 2 = 2 \quad \Rightarrow a = 4 \]

Now \( a_n = a + (n-1)d \)
\[ a_{12} = a + (12-1)d \]
\[ = 4 + 11(-2) = -18 \]

So in H.P \( a_{12} = -18 \)
\[ \Rightarrow \text{In H.P} \quad a_{12} = -\frac{1}{12} \text{nd} \]

8.8 And the A.M, H.M and G.M of each of the following. Also verify that
\( AH = G^2 \)

8.9
\[ \begin{align*}
&3.14, 2.71 \\
\text{So} & \quad a = 3.14, b = 2.71
\end{align*} \]
\[ A = \frac{a + b}{2} = \frac{3.14 + 2.71}{2} = 2.925 \Rightarrow \text{[A = 2.925]} \]
\[ G = \sqrt{ab} = \sqrt{3.14 \times 2.71} = \sqrt{8.5094} = 2.927 \Rightarrow \text{[G = 2.92]} \]
\[ H = \frac{2ab}{a + b} = \frac{2 	imes 3.14 \times 2.71}{3.14 + 2.71} = \frac{17.0188}{5.85} = 2.909 \Rightarrow \text{[H = 2.91]} \]
To verify \(A \cdot M \times H \cdot M = (a \cdot M)^2\):

\[
A \cdot H = 9^2
\]

\[
\text{L.H.S.}
\]

\[
A \cdot H = (8.915)(2.9) = 8.517
\]

\[
\Rightarrow A \cdot H = 8.52
\]

\[
\text{R.H.S.}
\]

\[
G = \pm 2.92
\]

\[
\Rightarrow (a)^2 = (\pm 2.92)^2
\]

\[
\Rightarrow G^2 = 8.52
\]

From eqns (i) and (ii), it is verified that

\[
A \cdot H = G^2
\]

(ii) \(-6\) and \(-216\)

\[
\Rightarrow a = -6 \quad \text{and} \quad b = -216
\]

\[
A \cdot M = \frac{a + b}{2} = \frac{-6 + (-216)}{2} = \frac{-222}{-2} = -111
\]

\[
H \cdot M = \frac{2ab}{a + b} = \frac{2(-6)(-216)}{(-6) + (-216)} = \frac{2592}{-222} = -11.68
\]

\[
G \cdot M = \sqrt{ab} = \sqrt{-6 \times -216} = \sqrt{1368} = \pm 36
\]

\[
\text{Now}
\]

\[
A \cdot H = (-111)(-11.68) = 1296
\]

\[
G^2 = (\pm 36)^2 = 1296
\]

Hence, \(A \cdot H = G^2\)

(iii) \(\lambda + y, \lambda - y\)

\[
\text{Let}
\]

\[
a = \lambda + y, \quad b = \lambda - y
\]

\[
A = \frac{a + b}{2} = \frac{\lambda + y + \lambda - y}{2} = \frac{2\lambda}{2} = \lambda
\]

\[
H = \frac{2ab}{a + b} = \frac{2(\lambda + y)(\lambda - y)}{(\lambda + y) + (\lambda - y)} = \frac{2(\lambda^2 - y^2)}{2\lambda} = \frac{\lambda^2 - y^2}{\lambda}
\]

\[
G = \sqrt{ab} = \sqrt{(\lambda + y)(\lambda - y)} = \sqrt{\lambda^2 - y^2}
\]

New \(\text{L.H.S.}\)

\[
A \cdot H = \lambda \sqrt{\frac{\lambda^2 - y^2}{\lambda}} = \lambda \sqrt{\lambda^2 - y^2}
\]

\[
\text{R.H.S.}
\]

\[
G = \pm \sqrt{\lambda^2 - y^2}
\]

\[
\Rightarrow G^2 = \lambda^2 - y^2
\]

\[
\text{Hence}
\]

\[
A \cdot H = G^2
\]

(b) \(\sqrt{2} + 3, \sqrt{2} - 3\)

\[
\text{Let}
\]

\[
a = \sqrt{2} + 3, \quad b = \sqrt{2} - 3
\]

\[
A = \frac{a + b}{2} = \frac{(\sqrt{2} + 3) + (\sqrt{2} - 3)}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}
\]

\[
H = \frac{2ab}{a + b} = \frac{2(\sqrt{2} + 3)(\sqrt{2} - 3)}{(\sqrt{2} + 3) + (\sqrt{2} - 3)} = \frac{(\sqrt{2})^2 - 3^2}{2\sqrt{2}} = \sqrt{2} - 3
\]

\[
G = \frac{\sqrt{ab}}{2} = \frac{\sqrt{(\sqrt{2} + 3)(\sqrt{2} - 3)}}{2} = \sqrt{(\sqrt{2})^2 - 3^2} = \sqrt{2} - 3 = \sqrt{2} - \sqrt{3}
\]

\[
G = \text{Undefined}
\]
Now, \( H = \frac{2a+8}{a+b} \) and \( H = \frac{2b+8}{a+b} \).

Since \( G = \text{Undefined} \), so \( a^2 = b^2 \) is also not valid.

For what value of \( a \) and \( b \), \( a^2 + b^2 = a^2 + b^2 \) in \( H = a + b \).

\[
\frac{a^2 + b^2}{a^2 + b^2} = \frac{2ab}{a+b}
\]

For two H.M.'s \( H_1, H_2 \), \( H_1 = b \) and \( H_2 = a \). Hence, \( \frac{a}{b} = \frac{b}{a} \).

\[
\frac{a}{b} = \frac{b}{a} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2}
\]

\[
\frac{a+b}{a+b} = \frac{2ab}{a+b}
\]

\[
H = \frac{2a+8}{a+b} = \frac{2b+8}{a+b}
\]

\[
H = \frac{2a+8}{a+b} = \frac{2b+8}{a+b}
\]

\[
\frac{a^2 + b^2}{a^2 + b^2} = \frac{2ab}{a+b}
\]

\[
\frac{a}{b} = \frac{b}{a} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2}
\]

\[
\frac{a+b}{a+b} = \frac{2ab}{a+b}
\]

\[
H = \frac{2a+8}{a+b} = \frac{2b+8}{a+b}
\]

\[
H = \frac{2a+8}{a+b} = \frac{2b+8}{a+b}
\]

\[
\frac{a^2 + b^2}{a^2 + b^2} = \frac{2ab}{a+b}
\]

\[
\frac{a}{b} = \frac{b}{a} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{a^2}{b^2} = \frac{b^2}{a^2}
\]

\[
\frac{a+b}{a+b} = \frac{2ab}{a+b}
\]
Now
\[ H = \frac{2ab}{a+b} = 2 \frac{(\sqrt{2}+3)(\sqrt{2}-3)}{(\sqrt{2}+3) + (\sqrt{2}-3)} \]
\[ H = 2 \frac{(\sqrt{2})^2 - 3^2}{2\sqrt{2}} \]
\[ = \frac{2-9}{\sqrt{2}} = \frac{7}{\sqrt{2}} \text{ Ans} \]

Since \( G = \text{undefined} \), so \( AH = G^2 \) is also not verified.

Q:5 For what value of \( n \), \( a^{n+1} + b^{n+1} \) is H.M.\( b/w \ a \) and \( b \).
\[ \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \text{H.M} \]
\[ \Rightarrow \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{2ab}{a+b} \]
\[ \Rightarrow (a+b)(a^{n+1} + b^{n+1}) = 2ab(a^n + b^n) \]
\[ \Rightarrow a^{n+1} + a^{n+1} + b^{n+1} + b^{n+1} = 2ab a^n + 2ab b^n \]
\[ \Rightarrow a^{n+1} + a^{n+1} + b a^{n+1} + b a^{n+1} = 2ab a^n + 2ab b^n \]
\[ \Rightarrow a a^{n+1} + b b^{n+1} = 2ba^{n+1} + 2ab b^n \]
\[ \Rightarrow a a^{n+1} + b b^{n+1} = b a^{n+1} + ab b^n \]

Q:6 Insert two H.Ms \( b/w \) 12 and 48.

Sol: Let \( H_1 \) and \( H_2 \) are two H.Ms \( b/w \) 12 and 48.
Then \( 12, H_1, H_2, 48 \) is H.P.
\[ \Rightarrow \frac{1}{12}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{48} \text{ is A.P.} \]

To find \( d \)
\[ aH = a + (n-1)d \]
\[ \frac{1}{48} = \frac{1}{12} + (4-1)d \]
\[ \frac{1}{48} - \frac{1}{12} = 3d \]
\[ \Rightarrow \frac{1-4}{48} = 3d \Rightarrow 3d = \frac{3}{48} \Rightarrow d = \frac{1}{16} \]
Now \( H_n = \frac{1}{a + nd} \)

\[ H_1 = \frac{1}{a + 1d} = \frac{1}{\frac{1}{12} + \frac{1}{78}} = \frac{1}{\frac{47}{48}} = \frac{48}{47} = \frac{1}{\frac{47}{48}} = 1 \]

\[ H_2 = \frac{1}{a + 2d} = \frac{1}{\frac{1}{12} + \frac{2}{(\frac{1}{78})}} = \frac{1}{\frac{1}{12} - \frac{1}{24}} = \frac{1}{\frac{12}{24}} = \frac{1}{2} \]

\[ \text{Hence, } \frac{H_1}{H_2} = 2 \]

\[ \text{Insert: } 3 \text{ H.M.s?} \]

5 \[ \text{Let } H_1, H_2, H_3, H_4 \text{ are four H.M.s by } \frac{5}{2} \text{ and } \frac{2}{11} \]

Then \( \frac{5}{2}, H_1, H_2, H_3, H_4, \frac{2}{11} \) is \( \text{H.P.} \) \( \Rightarrow \frac{\frac{5}{2}}{H_1, \frac{1}{H_2}, H_3, H_4, \frac{2}{11}} \) is \( \text{H.P.} \)

\[ H_n = \frac{1}{a + nd} \]

\[ H_1 = \frac{1}{a + 1d} = \frac{1}{\frac{5}{2} + \frac{5}{33}} = \frac{35}{33} = \frac{\frac{1}{2}}{\frac{5}{33}} = \frac{1}{\frac{5}{2} - \frac{5}{33}} = \frac{5}{3} \]

\[ H_2 = \frac{1}{a + 2d} = \frac{1}{\frac{5}{2} + \frac{2}{33}} = \frac{35}{33} = \frac{\frac{1}{2}}{\frac{5}{33}} = \frac{1}{\frac{5}{2} - \frac{2}{33}} = \frac{35}{31} \]

\[ H_3 = \frac{1}{a + 3d} = \frac{1}{\frac{5}{2} + \frac{3}{33}} = \frac{35}{33} = \frac{\frac{1}{2}}{\frac{5}{33}} = \frac{1}{\frac{5}{2} - \frac{3}{33}} = \frac{35}{29} \]

\[ H_4 = \frac{1}{a + 4d} = \frac{1}{\frac{5}{2} + \frac{4}{33}} = \frac{35}{33} = \frac{\frac{1}{2}}{\frac{5}{33}} = \frac{1}{\frac{5}{2} - \frac{4}{33}} = \frac{35}{47} \]

\[ \text{G.H. Prove that the square of the G.M. of two numbers, equal the product of A.M. and H.M. of the two numbers.} \]

5 \[ \text{Let } a \text{ and } b \text{ are the two numbers. To prove } AH = G^2 \]

Then \( \frac{a + \frac{b}{2}}{2}, \frac{a + \frac{b}{2}}{2}, H = \frac{2ab}{a + b} \)

\[ \text{L.H.S. } \]

\[ AH = \left( \frac{a + b}{2} \right) \left( \frac{2ab}{a + b} \right) = ab \]

\[ \text{R.H.S. } \]

\[ G = \pm \sqrt{ab} \]

\[ G^2 = ab \rightarrow \text{(i)} \]

From (i) and (ii) it is proved \( G^2 = \text{AH} \)

5 \[ \text{The A.M. of two numbers is } 8 \text{ and the H.M. is } 6. \text{ Find the numbers.} \]

5 \[ \text{Let } a \text{ and } b \text{ are the numbers.} \]

Then \( \frac{a + b}{2}, H = \frac{2ab}{a + b} \)

\[ \Rightarrow \frac{a}{2} = \frac{a + b}{2} \]

\[ \Rightarrow 2a = a + b \]

\[ \Rightarrow a + b = 16 \]

\[ \Rightarrow 6a + 6b = 2ab \rightarrow \text{(i)} \]

\[ \Rightarrow a = 16 - b \]

\[ \text{P.T. of } a \]

\[ \Rightarrow 6(16 - b) + 6b = 2(16 - b) \]

\[ \Rightarrow 96 - 6b + 6b = 32 - 2b \]

\[ \Rightarrow 96 = 32 - 2b \]
\[ 2b^2 - 32b + 96 = 0 \]
\[ \div \text{ by 2} \]
\[ b^2 - 16b + 48 = 0 \]
\[ \text{by factorization} \]
\[ b^2 - 12b - 4b + 48 = 0 \]
\[ b(b-12) - 4(b-12) = 0 \]
\[ (b-12)(b-4) = 0 \]
\[ b = 12 \quad \text{or} \quad b = 4 \]

\[ \text{Now} \]
\[ a = 16 - b \]
\[ b=12 \Rightarrow a = 16 - 12 = 4 \]
\[ b=12 \Rightarrow a = 16 - 12 = 4 \]

Hence the two \#s are 12, 4 \quad \text{or} \quad 4, 12

**Q:10**

The H.M b/w two numbers is \(\frac{4}{5}\) and their G.M is 6. What are the numbers?

Let \(a\) and \(b\) be the numbers.

Then, \[ \text{H.M} = \frac{2ab}{a+b} \quad \text{and} \quad \text{G.M} = \sqrt{ab} \]

\[ 4 \cdot \frac{4}{5} = \frac{2ab}{a+b} \quad \text{and} \quad 6 = \sqrt{ab} \]

\[ 2 \cdot \frac{4}{5} = \frac{2ab}{a+b} \quad \Rightarrow \quad \frac{b^2}{a} = ab \]

\[ 36 = ab \]

\[ 24a + 24b = 10ab \]

\[ \text{put} \quad a = \frac{36}{b} \]

\[ 24\left(\frac{36}{b}\right) + 24b = 10\left(\frac{36}{b}\right)b \]

\[ \frac{864}{b} + 24b = 360 \]

\[ \times \text{ by } b \]

\[ 864 + 24b^2 = 360b \]

\[ 24b^2 - 360b + 864 = 0 \]

Divide by 24

\[ b^2 - 15b + 36 = 0 \]

\[ b = 12 - b = 3 \]

\[ b(12 - 3)(b-12) = 0 \]

\[ (b-12)(b-3) = 0 \]

\[ b = 12 \quad \text{or} \quad b = 3 \]

\[ \text{Now} \quad a = \frac{36}{b} \]

for \(b=3\) \[ a = \frac{36}{3} = 12 \]

for \(b=12\) \[ a = \frac{36}{12} = 3 \]

Hence the numbers are \(3, 12\) or \(12, 3\).

\[ \text{Hurrah! That's the end of chapter #04} \]

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