Exercise # 1.1

Q:1 Simplify the following

(i) i^{14}

Sol $i^{14} = (i^2)^7 = (-1)^7 = -1$ Ans

(ii) $(-i)^{23}$

Sol $(-i)^{23} = (-)^{23} i^{23}$

$= -(i^2)^{11} i = -(-1)^{11} i$

$= -(-1) i$

$= i$ Ans

(iii) i^{-9}

Sol $i^{-9} = \frac{1}{i^9} = \frac{1}{(i^2)^4 i} = \frac{1}{(-1)^4 i} = \frac{1}{i}$

$= -i$

Note $\frac{1}{i} = -i$

Because $\frac{1}{i} = \frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{-i^2} = \frac{-i}{-(-1)} = -i$

(iv) $(-i)^{-98}$

Sol $(-i)^{-98} = \frac{1}{(-i)^{98}} = \frac{1}{(-)^{98}\, i^{98}} = \frac{1}{i^{98}}$

$= \frac{1}{(i^2)^{49}} = \frac{1}{(-1)^{49}} = \frac{1}{-1} = -1$

Q:2 Add the following complex #s.

(i) $3(1+2i)\,,\ -2(1-3i)$

Sol $3(1+2i) + (-2)(1-3i)$

$= 3 + 6i - 2 + 6i$

$= 3 - 2 + 6i + 6i$

$= 1 + 12i$ Ans

(ii) $\frac{1}{2} - \frac{2}{3}i\,,\ \frac{1}{4} - \frac{1}{3}i$

Sol $\left(\frac{1}{2} - \frac{2}{3}i\right) + \left(\frac{1}{4} - \frac{1}{3}i\right)$

$= \frac{1}{2} + \frac{1}{4} - \frac{2}{3}i - \frac{1}{3}i$

$= \frac{2+1}{4} + \frac{-2i - i}{3}$

$= \frac{3}{4} - \frac{3i}{3} = \frac{3}{4} - i$ Ans

(iii) $(\sqrt{2}, 1)$, $(1, \sqrt{2})$

Sol $(\sqrt{2}+1i) + (1+\sqrt{2}i)$

$= \sqrt{2}+1+1i+\sqrt{2}i$

$= (1+\sqrt{2}) + (1+\sqrt{2})i$ Ans

Q:3 Subtract the following complex #s.

(i) $(3\sqrt{3} - 5\sqrt{7}i)$, $(\sqrt{3} + 2\sqrt{7}i)$

Sol $(3\sqrt{3} - 5\sqrt{7}i) - (\sqrt{3} + 2\sqrt{7}i)$

$= 3\sqrt{3} - 5\sqrt{7}i - \sqrt{3} - 2\sqrt{7}i$

$= 3\sqrt{3} - \sqrt{3} - 5\sqrt{7}i - 2\sqrt{7}i$

$= 2\sqrt{3} - 7\sqrt{7}i$ Ans

(ii) $(-3, \frac{1}{2})$, $(3, \frac{1}{2})$

Sol $(-3, \frac{1}{2}) - (3, \frac{1}{2})$

$= -3 + \frac{1}{2}i - 3 - \frac{1}{2}i$

$= -3 - 3 + \frac{1}{2}i - \frac{1}{2}i$

$= -6 + 0i$ Ans

Quote: The foundation of every state is the education of its youth (By: Diogenes Laertius)

(iii) $(a, 0) - (2, -b)$

$= (a+0i) - (2-bi)$

$= a-2 + 0i + bi$

$= (a-2) + bi$ Ans

Q:4 Multiply the following complex #s

(i) $2i, 3i$

Sol $(2i).3i$

$= 6i^2 = 6(-1) = -6$ Ans

(ii) $3i$, $2(1-i)$

Sol $(3i) \cdot 2(1-i)$

$= (3i)(2-2i)$

$= 6i - 6i^2$

$= 6i - 6(-1)$

$= 6i + 6$

$= 6 + 6i$ Ans

(iii) $\sqrt{2} + \sqrt{3}i$, $2\sqrt{2} - \sqrt{3}i$

Sol $(\sqrt{2} + \sqrt{3}i) \cdot (2\sqrt{2} - \sqrt{3}i)$

$= 2\sqrt{2}\sqrt{2} - \sqrt{2}\sqrt{3}i + 2\sqrt{2}\sqrt{3}i - \sqrt{3}\sqrt{3}i^2$

$= 2\sqrt{4} - \sqrt{6}i + 2\sqrt{6}i - \sqrt{9}(-1)$

$= 2(2) - \sqrt{6}i + 2\sqrt{6}i + 3$

$= 7 + \sqrt{6}i$ Ans

Q:5 Perform the division and write the answer in the form of $a+bi$.

(i) $\dfrac{1+i}{i}$

Sol $\dfrac{1+i}{i}$ multiply and divide by the conjugate of denominator.

$$= \frac{1+i}{i} \times \frac{-i}{-i}$$

$$= \frac{-i(1+i)}{-i^2}$$

$$= \frac{-i-i^2}{-(-1)}$$

$$= \frac{-i-(-1)}{1}$$

$$= 1-i \quad \text{Ans}$$

(ii) $\dfrac{13}{5-12i}$

$$= \frac{13}{5-12i} \times \frac{5+12i}{5+12i}$$

$$= \frac{13(5+12i)}{(5-12i)(5+12i)}$$

$$= \frac{65+156i}{(5)^2-(12i)^2}$$

$$= \frac{65+156i}{25-144i^2}$$

$$= \frac{65+156i}{25-144(-1)}$$

$$= \frac{65+156i}{25+144} = \frac{65+156i}{169}$$

$$= \frac{65}{169} + \frac{156}{169}i \quad \text{Ans}$$

(iii) $\dfrac{4-3i}{4+3i}$

$$= \frac{4-3i}{4+3i} \times \frac{4-3i}{4-3i} \quad (\text{Rationalizing the denominator})$$

$$= \frac{(4-3i)^2}{(4)^2-(3i)^2}$$

$$= \frac{(4)^2+(3i)^2-2(4)(3i)}{16-9i^2}$$

$$= \frac{16+9i^2-24i}{16+9}$$

$$= \frac{16-9-24i}{25} = \frac{7-24i}{25} = \frac{7}{25} - \frac{24}{25}i \quad \text{Ans}$$

Q:6 Prove that sum and product of a complex number with its conjugate is a real #.

Sol Let $z = a+bi$ is any complex #

Then $\bar{z} = a-bi$ is its conjugate.

Addition: $z+\bar{z} = (a+bi)+(a-bi)$

$= 2a = \text{sum}$

which is a real #

Multiplication: $z.\bar{z} = (a+bi).(a-bi)$

$= (a)^2-(bi)^2 = a^2-b^2i^2 = a^2+b^2 = \text{Product}$

which is real #.

Q:7 If $z_1 = 1+2i$ and $z_2 = 2+3i$. Evaluate

(i) $|z_1 + z_2|$

Sol
$$z_1 + z_2 = (1+2i) + (2+3i)$$
$$z_1 + z_2 = 3+5i$$
Take its modulus
$$\Rightarrow |z_1 + z_2| = |3+5i|$$
$$= \sqrt{3^2+5^2} = \sqrt{34} \text{ Ans}$$

(ii) $|z_1 . z_2|$

Sol
$$z_1 . z_2 = (1+2i).(2+3i)$$
$$= 2+3i+4i+6i^2$$
$$= 2+7i-6$$
$$z_1 . z_2 = -4+7i$$
Taking its magnitude
$$|z_1 . z_2| = |-4+7i|$$
$$= \sqrt{(-4)^2+7^2} = \sqrt{57} \text{ Ans}$$

(iii) $\left|\frac{z_1}{z_2}\right|$

Sol
$$\frac{z_1}{z_2} = \frac{1+2i}{2+3i}$$
$\times$ and $\div$ by $2-3i$, we get
$$\Rightarrow \frac{z_1}{z_2} = \frac{1+2i}{2+3i} \times \frac{2-3i}{2-3i}$$
$$\Rightarrow \frac{z_1}{z_2} = \frac{(1+2i)(2-3i)}{(2+3i)(2-3i)}$$
$$\Rightarrow \frac{z_1}{z_2} = \frac{2-3i+4i-6i^2}{(2)^2-(3i)^2}$$
$$\Rightarrow \frac{z_1}{z_2} = \frac{2+i+6}{4-9i^2} = \frac{8+i}{4+9} = \frac{8+i}{13}$$
Hence $\frac{z_1}{z_2} = \frac{8}{13} + \frac{1}{13} i$

Take magnitude
$$\Rightarrow \left|\frac{z_1}{z_2}\right| = \sqrt{\left(\frac{8}{13}\right)^2 + \left(\frac{1}{13}\right)^2}$$
$$= \sqrt{\frac{64}{169} + \frac{1}{169}} = \sqrt{\frac{65}{169}} \text{ Ans}$$

Q:8 Separate into real and imaginary parts.

(i) $\frac{2+3i}{5-2i}$

Sol $\frac{2+3i}{5-2i}$ Multiplying and dividing by $5+2i$, we get
$$= \frac{2+3i}{5-2i} \times \frac{5+2i}{5+2i}$$
$$= \frac{(2+3i)(5+2i)}{(5-2i)(5+2i)}$$

$$= \frac{10+4i+15i+6i^2}{(5)^2-(2i)^2}$$

$$= \frac{10+19i-6}{25-4i^2} = \frac{4+19i}{25+4} = \frac{4}{29}+\frac{19}{29}i \quad \text{Ans}$$

(ii) $\dfrac{(1+2i)^2}{1-3i}$

Sol $\dfrac{(1+2i)^2}{1-3i}$

$$= \frac{(1)^2+(2i)^2+2(1)(2i)}{1-3i}$$

$$= \frac{1+4i^2+4i}{1-3i}$$

$$= \frac{1-4+4i}{1-3i}$$

$$= \frac{-3+4i}{1-3i}$$

$$= \frac{-3+4i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$= \frac{-3-9i+4i+12i^2}{(1)^2-(3i)^2} = \frac{-3-5i-12}{1-9i^2} = \frac{-15-5i}{1+9}$$

$$= \frac{-15}{10}-\frac{5}{10}i$$

Hence $\dfrac{(1+2i)^2}{1-3i} = \dfrac{-3}{2}-\dfrac{1}{2}i$ Ans

(iii) $\dfrac{1-i}{(1+i)^2}$

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Sol $\dfrac{1-i}{(1+i)^2} = \dfrac{1-i}{1^2+i^2+2i}$

$$= \frac{1-i}{1-1+2i}$$

$$= \frac{1-i}{2i}$$

$$= \frac{1-i}{2i} \times \frac{-2i}{-2i}$$

$$= \frac{-2i(1-i)}{-4i^2}$$

$$= \frac{-2i+2i^2}{-4(-1)}$$

$$= \frac{-2i-2}{4} = \frac{-2-2i}{4}$$

$$= -\frac{2}{4}-\frac{2i}{4}$$

$$= -\frac{1}{2}-\frac{1}{2}i$$

Hence $\dfrac{1-i}{(1+i)^2} = -\dfrac{1}{2}-\dfrac{1}{2}i$ Ans

Exercise # 1.2

Q:1 $z_1 = 1+i$ & $z_2 = 2+i$ verify that

(i) $z_1 + z_2 = z_2 + z_1$

Sol

L.H.S $z_1 + z_2 = (1+i) + (2+i)$

$= 3+2i \longrightarrow (1)$

R.H.S $z_2 + z_1 = (2+i) + (1+i)$

$= 3+2i \longrightarrow (2)$

From eqns (1) and (2) it is proved that

$z_1 + z_2 = z_2 + z_1$ (commutative property w.r.t addition)

$z_1+z_2=z_2+z_1$

(ii) $z_1 z_2 = z_2 z_1$

L.H.S $= z_1 z_2 = (1+i)(2+i)$

$= 2+i+2i+i^2$

$= 2+3i-1$

$= 1+3i \longrightarrow (1)$

commutative property w.r.t multiplication

$z_1 z_2 = z_2 z_1$

R.H.S $z_2 z_1 = (2+i)(1+i)$

$= 2+2i+i+i^2$

$= 2+3i-1 = 1+3i \longrightarrow (2)$

From eqns (1) & (2), it is proved $z_1 z_2 = z_2 z_1$

Q:2: $z_1 = -1-i$, $z_2 = 3+2i$, $z_3 = -2+3i$

verify that

(i) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

L.H.S

$z_2 + z_3 = (3+2i) + (-2+3i)$

$= 1+5i$

Now $z_1 + (z_2+z_3) = (-1-i) + (1+5i)$

$= 0+4i \longrightarrow (1)$

R.H.S $z_1 + z_2 = (-1-i) + (3+2i)$

$= 2+i$

Then $(z_1 + z_2) + z_3 = (2+i) + (-2+3i)$

$= 0+4i \longrightarrow (2)$

From eqns (1) and (2), it is proved that

$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

This property is called associative property of addition.

(ii) $z_1(z_2 z_3) = (z_1 z_2) z_3$

L.H.S $z_2 z_3 = (3+2i)(-2+3i)$

$= -6+9i-4i+6i^2$

$= -6+5i-6$

$= -12+5i$

Now $z_1(z_2z_3) = (-1-i)(-12+5i)$

$= 12-5i+12i-5i^2$

$= 12+7i+5$

$= 17+7i \longrightarrow (a)$

R.H.S

$z_1z_2 = (-1-i)(3+2i)$

$= -3-2i-3i-2i^2$

$= -3-5i+2$

$= -1-5i$

Now $(z_1z_2)z_3 = (-1-5i)\cdot(-2+3i)$

$= 2-3i+10i-15i^2$

$= 2+7i+15$

$= 17+7i \longrightarrow (b)$

From eqns (a) and (b) it is verified that

$z_1(z_2z_3) = (z_1z_2)z_3$

This property is called associative property of multiplication.

Q:3 $z_1 = \sqrt{3}+\sqrt{2}\,i$, $z_2 = \sqrt{3}-\sqrt{2}\,i$, $z_3 = 2-2i$

verify that $z_1(z_2+z_3) = z_1z_2 + z_1z_3$

Sol L.H.S $z_2+z_3 = (\sqrt{3}-\sqrt{2}\,i)+(2-2i)$

$= 2+\sqrt{3}-\sqrt{2}\,i-2i$

Now $z_1\cdot(z_2+z_3) = (\sqrt{3}+\sqrt{2}\,i)\cdot(2+\sqrt{3}-\sqrt{2}\,i-2i)$

$= 2\sqrt{3}+\sqrt{3}\sqrt{3}-\sqrt{3}\sqrt{2}\,i-2\sqrt{3}\,i+2\sqrt{2}\,i+\sqrt{3}\sqrt{2}\,i$
$-\sqrt{2}\sqrt{2}\,i^2-2\sqrt{2}\,i^2$

$\Rightarrow z_1\cdot(z_2+z_3) = 2\sqrt{3}+(\sqrt{3})^2-\sqrt{3\times2}\,i-2\sqrt{3}\,i+2\sqrt{2}\,i+\sqrt{3\times2}\,i$
$-(\sqrt{2})^2(-1)-2\sqrt{2}(-1)$

$\Rightarrow z_1\cdot(z_2+z_3) = 2\sqrt{3}+3-\sqrt{6}\,i-2\sqrt{3}\,i+2\sqrt{2}\,i+\sqrt{6}\,i+2+2\sqrt{2}$

$\Rightarrow z_1\cdot(z_2+z_3) = 5+2\sqrt{3}+2\sqrt{2}-2\sqrt{3}\,i+2\sqrt{2}\,i \longrightarrow (a)$

Now R.H.S

$z_1z_2 = (\sqrt{3}+\sqrt{2}\,i)\cdot(\sqrt{3}-\sqrt{2}\,i)$

$= (\sqrt{3})^2-(\sqrt{2}\,i)^2$

$= 3-2i^2 = 3-2(-1) = 3+2 = 5$

and $z_1z_3 = (\sqrt{3}+\sqrt{2}\,i)\cdot(2-2i)$

$= 2\sqrt{3}-2\sqrt{3}\,i+2\sqrt{2}\,i-2\sqrt{2}\,i^2$

$= 2\sqrt{3}-2\sqrt{3}\,i+2\sqrt{2}\,i+2\sqrt{2}$

$= 2\sqrt{3}+2\sqrt{2}-2\sqrt{3}\,i+2\sqrt{2}\,i \longrightarrow (b)$

Then $z_1z_2+z_1z_3 = (5)+(2\sqrt{3}+2\sqrt{2}-2\sqrt{3}\,i+2\sqrt{2}\,i)$

$= 5+2\sqrt{3}+2\sqrt{2}-2\sqrt{3}\,i+2\sqrt{2}\,i \longrightarrow (b)$

From eqns (a) and (b), it is verified that

$z_1(z_2+z_3) = z_1z_2+z_1z_3$ (It is called distributive property of multiplication over addition)

Q:4 Find the addative inverse of the following

(i) $2+3i$

Short cut method (for M.C.Qs)

Sol let $z = 2+3i$

Then $-z$ is the addative inverse

and $-z = -(2+3i)$

$= -2-3i$ Ans

Method # 02: (Recommended for questions)

let $a+bi$ is the addative inverse of $2+3i$,

Then $(a+bi)+(2+3i) = 0+0i$

Because when a # is added with its addative inverse the result is addative identity $0+0i$

$\Rightarrow a+2 + bi+3i = 0+0i$

$(a+2)+(b+3)i = 0+0i$

compare the real and img parts, get

$a+2=0$ and $b+3=0$

$\Rightarrow a=-2$ $\Rightarrow b=-3$

Hence $a+bi = -2-3i$

is the addative inverse.

(ii) $z = (2,-3)$

Method #01:

$-z$ is the addative inverse of z

and $-z = -(2,-3)$

$= (-2,3)$ Ans

Method #02:

Let (a,b) is the addative inverse of $(2,-3)$

Then $(a,b)+(2,-3) = (0,0)$

$\Rightarrow a+bi + 2-3i = 0+0i$

$\Rightarrow a+2 + (b-3)i = 0+0i$

Compare the real and imagnary parts, we get

$a+2=0$ and $b-3=0$

$\Rightarrow a=-2$ $b=3$

Hence $a+bi = -2+3i$ or $(a,b) = (-2,3)$

is the addative inverse of $(2,-3)$.

Q:5 Find the multiplicative inverse of

(i) $1+2i$

Method # 01: For M.C.Qs : let $z = 1+2i = (1,2)$ $a=1$ & $b=2$

By formula

Multiplicative inverse is $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

So M. Inverse $= \left(\frac{1}{1^2+2^2}, \frac{-2}{1^2+2^2} \right)$

$= \left(\frac{1}{5}, \frac{-2}{5} \right)$ Ans

method # 02 :

$z = 1+2i$

As $\frac{1}{z}$ is the M. Inverse of z

$\Rightarrow \frac{1}{z} = \frac{1}{1+2i}$

$\Rightarrow z^{-1} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$

$= \frac{1-2i}{(1)^2-(2i)^2}$

$= \frac{1-2i}{1-4i^2}$

$= \frac{1-2i}{1+4}$

$= \frac{1-2i}{5}$

$\Rightarrow z^{-1} = \frac{1}{5} - \frac{2}{5}i$ Ans

or

$z^{-1} = \left(\frac{1}{5}, -\frac{2}{5}\right)$ Ans

Method # 03 :

$z = 1+2i$

let $a+bi$ is the M. Inverse, then

$(a+bi)(1+2i) = 1+0i$ (Because when a # is multiplied with its M.Inverse the result is M.identity.

$\Rightarrow a+2ai+bi+2bi^2 = 1+0i$

$\Rightarrow a+2ai+bi-2b = 1+0i$

$\Rightarrow (a-2b)+(2a+b)i = 1+0i$

compare the real and img parts, we get

$a-2b = 1 \longrightarrow (1)$

$2a+b = 0 \longrightarrow (2)$

From eqn (1) we get $a = 1+2b$. P.T.V in eqn (2)

eqn (2) $\Rightarrow$ $2a+b=0$

$2(1+2b)+b=0$

$2+4b+b=0$

$2+5b=0$

$\Rightarrow 5b=-2$

$\boxed{b=-2/5}$

Finally $a = 1+2b$

$= 1+2\left(\frac{-2}{5}\right) = \frac{5-4}{5} = 1/5$

Hence $(a,b) = (1/5, -2/5)$ is the M. Inverse of $(1,2)$

1-5a

(ii) $z = (-1,2) \Rightarrow a = -1$ & $b = 2$

Method # 01: By formula

$$M.\text{Inverse} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

$$= \left(\frac{-1}{(-1)^2+2^2}, \frac{-2}{(-1)^2+2^2}\right)$$

$$= \left(\frac{-1}{1+4}, \frac{-2}{1+4}\right)$$

$$= \left(\frac{-1}{5}, \frac{-2}{5}\right) \text{ Ans}$$

Method # 02:

$\frac{1}{z}$ is the M. Inverse of $z = (-1,2)$ or $-1+2i$

So $\frac{1}{z} = \frac{1}{-1+2i}$

$$z^{-1} = \frac{1}{-1+2i} \times \frac{-1-2i}{-1-2i}$$

$$= \frac{-1-2i}{(-1)^2-(2i)^2}$$

$$\frac{-1-2i}{1-4i^2} = \frac{-1-2i}{1+4} = \frac{-1-2i}{5} = \frac{-1}{5} - \frac{2}{5}i$$

$$= \left(\frac{-1}{5}, -\frac{2}{5}\right) \text{ Ans}$$

Method # 03:

$z = (-1,2) = -1+2i$

Let (a,b) is the M. Inverse of $(-1,2)$

$\Rightarrow (a,b)(-1,2) = (1,0)$

$\Rightarrow (a+bi)(-1+2i) = (1,0)$

$\Rightarrow -a+2ai-bi+2bi^2 = 1+0i$

$\Rightarrow -a+2ai-bi-2b = 1+0i$

$\Rightarrow (-a-2b)+(2a-b)i = 1+0i$

compare the real and img parts, we get

$-a-2b = 1 \;\text{(i)}$ & $2a-b = 0 \;\text{(ii)}$

From equ (i)

$a = -1-2b$

Equ (ii) $\Rightarrow 2a-b = 0$

$\Rightarrow 2(-1-2b)-b = 0$

$\Rightarrow -2-4b-b = 0$

$\Rightarrow -2-5b = 0$

$\Rightarrow 5b = -2$

$\boxed{b = -2/5}$

Now $a = -1-2b$

$a = -1-2\left(\frac{-2}{5}\right)$

$a = -1+\frac{4}{5}$

$a = \frac{-5+4}{5} = -1/5$

Hence $(a,b) = \left(\frac{-1}{5}, -2/5\right)$ is the M. Inverse

Q:6 $z_1 = -3-\sqrt{-3}$, $z_2 = 4+\sqrt{-4}$

$\Rightarrow z_1 = -3-\sqrt{3}\,i$, $z_2 = 4+2i$

verify that $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

$\sqrt{-4}$
$= \sqrt{4\times -1}$
$= \sqrt{4}\sqrt{-1}$
$= 2i$

Sol L.H.S

$z_1+z_2 = (-3-\sqrt{3}i) + (4+2i)$

$= -3+4+2i-\sqrt{3}i$

$z_1+z_2 = 1+(2-\sqrt{3})i$

Take conjugate

$\Rightarrow \overline{z_1+z_2} = 1-(2-\sqrt{3})i$

$= 1+(-2+\sqrt{3})i$

$= 1+(\sqrt{3}-2)i \longrightarrow (1)$

OR

$z_1+z_2 = 1+2i-\sqrt{3}i$

$\overline{z_1+z_2} = 1-2i+\sqrt{3}i$

R.H.S $z_1 = -3-\sqrt{3}i$ & $z_2 = 4+2i$

$\Rightarrow \overline{z_1} = -3+\sqrt{3}i$ & $\overline{z_2} = 4-2i$

$\overline{z_1}+\overline{z_2} = (-3+\sqrt{3}i)+(4-2i)$

$= -3+4+\sqrt{3}i-2i$

$= 1+(\sqrt{3}-2)i \longrightarrow (2)$

From eqns (1) & (2), it is proved that

$\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$

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Q:7 $z_1 = -a-3bi$, $z_2 = 2a+bi$

verify that $\overline{z_1 \cdot z_2} = \overline{z_1}\cdot\overline{z_2}$

Sol L.H.S $z_1\cdot z_2 = (-a-3bi)\cdot(2a+bi)$

$= -2a^2-abi-6abi-3b^2i^2$

$= -2a^2-7abi+3b^2$

$z_1\cdot z_2 = -2a^2+3b^2-7abi$

$\Rightarrow \overline{z_1\cdot z_2} = -2a^2+3b^2+7abi \longrightarrow (i)$

R.H.S $z_1 = -a-3bi$ & $z_2 = 2a+bi$

$\Rightarrow \overline{z_1} = -a+3bi$ $\overline{z_2} = 2a-bi$

Now $\overline{z_1}\cdot\overline{z_2} = (-a+3bi)\cdot(2a-bi)$

$= -2a^2+abi+6abi-3b^2i^2$

$= -2a^2+7abi+3b^2$

$= -2a^2+3b^2+7abi \longrightarrow (ii)$

From eqns (i) and (ii) it is verified that

$\overline{z_1\cdot z_2} = \overline{z_1}\cdot\overline{z_2}$

Q:8 If $z_1 = -a-3bi$ & $z_2 = 2a-3bi$

verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

L.H.S $\frac{z_1}{z_2} = \frac{-a-3bi}{2a-3bi}$

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$\times$ and $\div$ by $2a+3bi$, we get

$$\Rightarrow \frac{z_1}{z_2} = \frac{-a-3bi}{2a-3bi} \times \frac{2a+3bi}{2a+3bi}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{(-a-3bi)(2a+3bi)}{(2a-3bi)(2a+3bi)}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{-2a^2-3abi-6abi-9b^2i^2}{(2a)^2-(3bi)^2}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{-2a^2-9abi+9b^2}{4a^2-9b^2i^2}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{-2a^2+9b^2-9abi}{4a^2+9b^2}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{-9ab}{4a^2+9b^2}\,i$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{-2a^2+9b^2}{4a^2+9b^2} - \frac{9ab}{4a^2+9b^2}\,i \;\cancel{\longrightarrow}\; ①$$

Take conjugate, we get

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9ab}{4a^2+9b^2}\,i \longrightarrow ①$$

R.H.S $z_1 = -a-3bi$ & $z_2 = 2a-3bi$

$\Rightarrow \overline{z_1} = -a+3bi$ $\Rightarrow \overline{z_2} = 2a+3bi$

Then
$$\frac{\overline{z_1}}{\overline{z_2}} = \frac{-a+3bi}{2a+3bi}$$

$$= \frac{-a+3bi}{2a+3bi} \times \frac{2a-3bi}{2a-3bi}$$

$$= \frac{(-a+3bi)(2a-3bi)}{(2a+3bi)(2a-3bi)}$$

$$= \frac{-2a^2+3abi+6abi-9b^2i^2}{(2a)^2-(3bi)^2}$$

$$= \frac{-2a^2+9abi+9b^2}{4a^2-9b^2i^2}$$

$$= \frac{-2a^2+9b^2+9abi}{4a^2+9b^2}$$

$$= \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9ab}{4a^2+9b^2}\,i \longrightarrow ②$$

From equs ① and ② it is verified that

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}.$$

Q:9 Show that for any complex numbers z_1, z_2

$$|z_1 . z_2| = |z_1||z_2|$$

Sol Let $z_1 = a + bi$, $z_2 = c + di$

L.H.S $z_1 . z_2 = (a + bi) . (c + di)$

$$= ac + adi + bci + bdi^2$$

$$= ac + adi + bci - bd$$

$$z_1 . z_2 = (ac - bd) + (ad + bc)i$$

Take absolute, we get

$$\Rightarrow |z_1 . z_2| = \sqrt{(ac - bd)^2 + (ad + bc)^2}$$

$$= \sqrt{a^2c^2 + b^2d^2 - 2abcd + a^2d^2 + b^2c^2 + 2abcd}$$

$$\Rightarrow |z_1 . z_2| = \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2} \longrightarrow (i)$$

R.H.S $z_1 = a + bi$ & $z_2 = c + di$

$$\Rightarrow |z_1| = \sqrt{a^2 + b^2} \qquad \Rightarrow |z_2| = \sqrt{c^2 + d^2}$$

Now $|z_1| . |z_2| = \sqrt{a^2 + b^2}\sqrt{c^2 + d^2}$

$$= \sqrt{(a^2 + b^2) . (c^2 + d^2)}$$

$$= \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2} \longrightarrow (ii)$$

From eqns (i) and (ii), it is proved that

$$|z_1 . z_2| = |z_1| . |z_2|$$

(ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

Sol Let $z_1 = a + bi$ & $z_2 = c + di$

L.H.S $\frac{z_1}{z_2} = \frac{a + bi}{c + di} \times \frac{c - di}{c - di}$

$$= \frac{(a + bi)(c - di)}{(c + di)(c - di)}$$

$$= \frac{ac - adi + bci - bdi^2}{(c)^2 - (di)^2}$$

$$= \frac{ac + bci - adi + bd}{c^2 - d^2i^2}$$

$$= \frac{ac + bd + (bc - ad)i}{c^2 + d^2}$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}\, i$$

Now take absolute

$$\Rightarrow \left|\frac{z_1}{z_2}\right| = \sqrt{\left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{bc - ad}{c^2 + d^2}\right)^2}$$

$$\Rightarrow \left|\frac{z_1}{z_2}\right| = \sqrt{\frac{(ac+bd)^2}{(c^2+d^2)^2} + \frac{(bc-ad)^2}{(c^2+d^2)^2}}$$

$$= \sqrt{\frac{(ac+bd)^2 + (bc-ad)^2}{(c^2+d^2)^2}}$$

$$= \sqrt{\frac{a^2c^2+b^2d^2+2abcd + b^2c^2+a^2d^2-2abcd}{(c^2+d^2)^2}}$$

$$= \sqrt{\frac{a^2c^2+b^2d^2 + b^2c^2+ a^2d^2}{(c^2+d^2)^2}}$$

$$= \sqrt{\frac{a^2(c^2+d^2) + b^2(c^2+d^2)}{(c^2+d^2)^2}} = \sqrt{\frac{(c^2+d^2)(a^2+b^2)}{(c^2+d^2)^2}}$$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\frac{a^2+b^2}{c^2+d^2}} \longrightarrow (i)$$

Now R.H.S

$z_1 = a+bi$ & $z_2 = c+di$

$\Rightarrow |z_1| = \sqrt{a^2+b^2}$ $\quad |z_2| = \sqrt{c^2+d^2}$

Then $\frac{|z_1|}{|z_2|} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} = \sqrt{\frac{a^2+b^2}{c^2+d^2}} \longrightarrow (ii)$

From Eqs (i) and (ii), it is proved that

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

Q.10 Separate into real and imaginary parts.

(i) $z = 2+3i$

$\Rightarrow$ Real part $= 2$

Img $\quad$ " $= 3$

(ii) $(3-2i)^2$

Sol $(3-2i)^2 = (3)^2 + (2i)^2 - 2(3)(2i)$

$= 9 + 4i^2 - 12i$

$= 9-4-12i$

$= 5-12i$

Hence real part $=5$, Img " $=-12$ } Ans

Formula

$(a-b)^2 = a^2+b^2-2ab$

(iii) $(3-4i)^{-1}$

Sol $(3-4i)^{-1} = \frac{1}{3-4i}$ $\quad$ × & ÷ by $3+4i$

$= \frac{1}{3-4i} \times \frac{3+4i}{3+4i}$

$= \frac{3+4i}{(3)^2-(4i)^2}$

$= \frac{3+4i}{9-16i^2} = \frac{3+4i}{9+16} = \frac{3+4i}{25}$

$= 3/25 + 4/25\, i$

Hence real part $= 3/25$ and Img part $= 4/25$ } Ans

(iv) $(2a-bi)^{-2}$

Sol

$$(2a-bi)^{-2} = \frac{1}{(2a-bi)^2}$$

$$= \frac{1}{(2a)^2+(bi)^2-2(2a)(bi)}$$

$$= \frac{1}{4a^2+b^2i^2-4abi}$$

$$= \frac{1}{4a^2-b^2-4abi} \quad \times \text{ and } \div \text{ by } 4a^2-b^2+4abi$$

$$= \frac{1}{4a^2-b^2-4abi} \times \frac{4a^2-b^2+4abi}{4a^2-b^2+4abi}$$

$$= \frac{4a^2-b^2+4abi}{(4a^2-b^2)^2-(4abi)^2}$$

$$= \frac{4a^2-b^2+4abi}{16a^4+b^4-8a^2b^2-16a^2b^2i^2}$$

$$= \frac{4a^2-b^2+4abi}{16a^4+b^4-8a^2b^2+16a^2b^2}$$

$$= \frac{4a^2-b^2+4abi}{16a^4+b^4+8a^2b^2} = \frac{4a^2-b^2+4abi}{(4a^2)^2+(b^2)^2+2(4a^2)(b^2)}$$

$$= \frac{4a^2-b^2+4abi}{(4a^2+b^2)^2}$$

$$= \frac{4a^2-b^2}{(4a^2+b^2)^2} + \frac{4ab}{(4a^2+b^2)^2}\,i$$

Here real part $= \dfrac{4a^2-b^2}{(4a^2+b^2)^2}$
and Img part $= \dfrac{4ab}{(4a^2+b^2)^2}$ } Ans

(v) $\dfrac{3-2i}{-1+i}$

Sol $\times$ and $\div$ by $-1-i$ to get

$$\frac{3-2i}{-1+i} = \frac{3-2i}{-1+i} \times \frac{-1-i}{-1-i}$$

$$= \frac{(3-2i)(-1-i)}{(-1+i)(-1-i)}$$

$$= \frac{-3-3i+2i+2i^2}{(-1)^2-(i)^2}$$

$$= \frac{-3-i-2}{1-i^2}$$

$$= \frac{-5-i}{1+1} = \frac{-5-i}{2} = -\frac{5}{2} - \frac{1}{2}i$$

Here real part $= -5/2$
and img part $= -1/2$ } Ans

(vi) $\left(\dfrac{5-2i}{2+3i}\right)^{-1}$

Sol $\left(\dfrac{5-2i}{2+3i}\right)^{-1} = \left(\dfrac{2+3i}{5-2i}\right)^{1}$ Rationalizing the denominator, we get

$$= \frac{2+3i}{5-2i} \times \frac{5+2i}{5+2i}$$

$$= \frac{10+4i+15i+6i^2}{(5)^2-(2i)^2}$$

$$= \frac{10+19i-6}{25-4i^2}$$

$$= \frac{4+19i}{25+4}$$

$$= \frac{4+19i}{29}$$

$$= \frac{4}{29} + \frac{19}{29}i$$

Hence real part $= 4/29$

and img part $= 19/29$

(vii) $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2$

Sol $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^2 = \frac{(\sqrt{3}-i)^2}{(\sqrt{3}+i)^2}$

$$= \frac{(\sqrt{3})^2 + i^2 - 2\sqrt{3}\,i}{(\sqrt{3})^2 + i^2 + 2\sqrt{3}\,i}$$

$$= \frac{3 - 1 - 2\sqrt{3}\,i}{3 - 1 + 2\sqrt{3}\,i}$$

$$= \frac{2 - 2\sqrt{3}\,i}{2 + 2\sqrt{3}\,i}$$

$$= \frac{2(1-\sqrt{3}\,i)}{2(1+\sqrt{3}\,i)}$$

$$= \frac{1-\sqrt{3}\,i}{1+\sqrt{3}\,i}$$

Quote:- Fathers send their sons to school/college either because they went to school/college or because they didn't.

Rationalizing the denominator, we get

$$= \frac{1-\sqrt{3}\,i}{1+\sqrt{3}\,i} \times \frac{1-\sqrt{3}\,i}{1-\sqrt{3}\,i}$$

$$= \frac{(1-\sqrt{3}\,i)^2}{(1)^2-(\sqrt{3}\,i)^2}$$

$$= \frac{(1)^2 + (\sqrt{3}\,i)^2 - 2(1)(\sqrt{3}\,i)}{1 - 3i^2}$$

$$= \frac{1 + 3i^2 - 2\sqrt{3}\,i}{4} = \frac{1-3-2\sqrt{3}\,i}{4}$$

$$= \frac{-2-2\sqrt{3}\,i}{4}$$

$$= \frac{2(-1-\sqrt{3}\,i)}{4} = \frac{-1-\sqrt{3}\,i}{2} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Here real part $= -1/2$
and img part $= -\sqrt{3}/2$ Ans

(viii) $\left(\frac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}\right)^{-2}$

Sol $\left(\frac{1+\sqrt{3}\,i}{1-\sqrt{3}\,i}\right)^{-2} = \left(\frac{1-\sqrt{3}\,i}{1+\sqrt{3}\,i}\right)^2$

$$= \frac{(1-\sqrt{3}\,i)^2}{(1+\sqrt{3}\,i)^2}$$

$$= \frac{(1)^2 + (\sqrt{3}\,i)^2 - 2\sqrt{3}\cdot i}{(1)^2 + (\sqrt{3}\,i)^2 + 2\sqrt{3}\,i}$$

$$= \frac{1 + 3i^2 - 2\sqrt{3}\,i}{1 + 3i^2 + 2\sqrt{3}\,i}$$

$$= \frac{1 - 3 - 2\sqrt{3}\,i}{1 - 3 + 2\sqrt{3}\,i}$$

$$= \frac{-2 - 2\sqrt{3}\,i}{-2 + 2\sqrt{3}\,i} = \frac{2(-1 - \sqrt{3}\,i)}{2(-1 + \sqrt{3}\,i)}$$

$$= \frac{-1 - \sqrt{3}\,i}{-1 + \sqrt{3}\,i}$$

$\times$ and $\div$ by $-1 - \sqrt{3}\,i$

$$= \frac{-1 - \sqrt{3}\,i}{-1 + \sqrt{3}\,i} \times \frac{-1 - \sqrt{3}\,i}{-1 - \sqrt{3}\,i}$$

$$= \frac{(-1 - \sqrt{3}\,i)^2}{(-1)^2 - (\sqrt{3}\,i)^2}$$

$$= \frac{(-1)^2 + (\sqrt{3}\,i)^2 + 2(-1)(-\sqrt{3}\,i)}{1 - 3i^2}$$

$$= \frac{1 + 3i^2 + 2\sqrt{3}\,i}{1 + 3} = \frac{1 - 3 + 2\sqrt{3}\,i}{4}$$

$$= \frac{-2 + 2\sqrt{3}\,i}{4}$$

$$= \frac{2(-1 + \sqrt{3}\,i)}{4}$$

$$= \frac{-1 + \sqrt{3}\,i}{2} \quad \text{Ans}$$

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Exercise # 1.3

Q:1 Solve the systems of eqns

(1)

$$z + w = 3i \longrightarrow (1)$$
$$2z + 3w = 2 \longrightarrow (2)$$

multiply eqn (1) by 2, ~~we get~~ and then subtract, we get

$$\Rightarrow 2z + 2w = 6i$$
$$\underline{-2z \pm 3w = -2}$$
$$-w = 6i - 2$$
$$w = -6i + 2 \quad \text{or} \quad \boxed{w = 2 - 6i}$$

Eqn (1) $\Rightarrow$

$$z + w = 3i$$
$$\Rightarrow z = 3i - w$$
$$\Rightarrow z = 3i - (2 - 6i)$$
$$\Rightarrow z = 3i - 2 + 6i$$
$$\Rightarrow z = 9i - 2 \quad \text{or} \quad \boxed{z = -2 + 9i}$$

Hence $\text{S.set} = \{z, w\}$

$$= \{-2 + 9i,\ 2 - 6i\} \quad \text{Ans}$$

(2)

$$z - 4w = 3i \longrightarrow (1)$$
$$2z + 3w = 11 - 5i \longrightarrow (2)$$

Sol: Multiply eqn (1) by 2 and then subtract, we get

$$\Rightarrow 2z - 8w = 6i$$
$$\underline{-2z - 3w = -11 + 5i}$$
$$\Rightarrow -11w = 6i - (11 - 5i)$$
$$\Rightarrow -11w = 6i - 11 + 5i$$
$$\Rightarrow -11w = 11i - 11$$

$\div$ by 11

$$\Rightarrow -w = i - 1$$
$$\Rightarrow \boxed{w = 1 - i}$$

Eqn (1) $\Rightarrow$

$$z - 4w = 3i$$
$$\Rightarrow z - 4(1 - i) = 3i$$
$$\Rightarrow z - 4 + 4i = 3i$$
$$\Rightarrow z - 4 = -i$$
$$\Rightarrow \boxed{z = 4 - i}$$

Hence $\text{S.set} = \{z, w\}$

$$= \{4 - i,\ 1 - i\} \quad \text{Ans}$$

③ $3z + (2+i)w = 11-i \longrightarrow ①$

$(2-i)z - w = -1+i \longrightarrow ②$

Sol

Multiply eqn ② by $(2+i)$, we get

$(2+i)(2-i)z - (2+i)w = (2+i)(-1+i)$

$\Rightarrow (2^2-i^2)z - (2+i)w = -2+2i-i+i^2$

$\Rightarrow (4-(-1))z - (2+i)w = -2+i-1$

$\Rightarrow 5z - (2+i)w = -3+i \longrightarrow ③$

Eqn ① + Eqn ③, we get

$$\begin{aligned} 3z + (2+i)w &= 11-i \\ 5z - (2+i)w &= -3+i \\ \hline 8z &= 8 \end{aligned}$$

$\Rightarrow z = 8/8 \Rightarrow \boxed{z=1}$

Eqn ① $\Rightarrow 3z + (2+i)w = 11-i$

$\Rightarrow 3(1) + (2+i)w = 11-i$

$\Rightarrow 3 + (2+i)w = 11-i$

$\Rightarrow (2+i)w = 8-i$

$\boxed{w = \frac{8-i}{2+i}}$

Hence S.Set $= \{z, w\} \Rightarrow$ S.S $= \left\{1, \frac{8-i}{2+i}\right\}$ or S.Set $= \{1, 3-2i\}$

Note

$\frac{8-i}{2+i}$

$= \frac{8-i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{16-8i-2i+i^2}{(2)^2-i^2}$

$= \frac{16-10i-1}{5}$

$= \frac{15-10i}{5} = 3-2i$

Q:- Factorize the polynomials $P(z)$ into linear factors

④ $P(z) = z^2+4$

$= z^2-(-4)$ As $i^2 = -$

$= z^2-(i^2 4)$

$= z^2-(i^2 2^2)$

$= z^2-(2i)^2$ $a^2-b^2 = (a+b)(a-b)$ formula

$= (z+2i)(z-2i)$ Ans

⑤ $P(z) = 3z^2+7$

$= 3z^2-(-7)$

$= 3z^2-(i^2 7)$

$= 3z^2-(i^2(\sqrt{7})^2)$

$= 3z^2-(\sqrt{7}i)^2$

$= (\sqrt{3}z)^2-(\sqrt{7}i)^2$

$= (\sqrt{3}z+\sqrt{7}i)(\sqrt{3}z-\sqrt{7}i)$ Ans

⑥ $P(z) = z^3 - 2z^2 + z - 2$

Method #01

Since $P(2) = 2^3 - 2(2)^2 + 2 - 2$

$= 8 - 8 + 2 - 2$

$P(2) = 0$

$\Rightarrow$ 2 is root

$\Rightarrow$ $x - 2$ is factor of $z^3 - 2z^2 + z - 2$

Then $z^3 - 2z^2 + z - 2 = z^3 - 2z^2 + z - 8 + 6$

$= z^3 - 8 - 2z^2 + z + 6$

$= (z^3 - 2^3) - (2z^2 - z - 6)$

$= (z-2)(z^2 + 2^2 + 2z) - (2z^2 - 4z + 3z - 6)$

$= (z-2)(z^2 + 2z + 4) - \{2z(z-2) + 3(z-2)\}$

$= (z-2)(z^2 + 2z + 4) - \{(z-2)(2z+3)\}$

$= (z-2)(z^2 + 2z + 4) - (z-2)(2z+3)$

$= (z-2)\{(z^2 + 2z + 4) - (2z+3)\}$

$= (z-2)\{z^2 + 2z + 4 - 2z - 3\}$

$= (z-2)(z^2 + 1)$

$= (z-2)\{z^2 - (-1)\}$

$= (z-2)\{z^2 - i^2\}$

$= (z-2)(z+i)(z-i)$ Ans

Method #02 By Synthetic Division

$P(z) = z^3 - 2z^2 + z - 2$

By synthetic division

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 1 & -2 \\ & \downarrow & +2 & 0 & 2 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$R = 0$

$\Rightarrow$ 2 is root

$\Rightarrow$ $z - 2$ is factor

The depressed factor is

$1z^2 + 0z + 1$

$= z^2 + 1$

$= z^2 - (-1)$

$= z^2 - i^2$

$= (z+1)(z-1)$

Hence

$P(z) = z^3 - 2z^2 + z - 2$

$= (z-2)(z+1)(z-1)$ Ans

(7) $P(z) = z^3 + 6z + 20$

Sol $P(z) = z^3 + 0z^2 + 6z + 20$

By synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 6 & 20 \\ & & -2 & 4 & -20 \\ \hline & 1 & -2 & 10 & \boxed{0} = R \end{array}$$

so -2 is root

$\Rightarrow z-(-2)$ is factor

$\Rightarrow z+2$ is factor

and the depressed factor will be

$1z^2 - 2z + 10$

$= 1z^2 - 2z + 1 + 9$

$= (z-1)^2 + 9$

$= (z-1)^2 - (-9)$

$= (z-1) - (i^2 3^2)$

$= (z-1) - (3i)^2$

$= (z-1+3i)(z-1-3i)$

Hence factors of $z^3 + 6z + 20$ are $(z+2)$, $(z-1+3i)$, $(z-1-3i)$

Sol $z^3 + 6z + 20 = (z+2)(z-1+3i)(z-1-3i)$

Q: Solve the eqns

(8) $z^2 + 6z + 13 = 0$

Sol $z^2 + 6z = -13$

$\Rightarrow z^2 + 2(3)z = -13$

Add 9 to b.s

$\Rightarrow z^2 + 2(3)z + 9 = -13 + 9$

$\Rightarrow (z+3)^2 = -4$

Take sq. root, we get

$\Rightarrow \sqrt{(z+3)^2} = \pm\sqrt{-4}$

$\Rightarrow z+3 = \pm 2i$

$\Rightarrow z = -3 \pm 2i$

Hence S.Set $= \{-3 \pm 2i\}$ Ans

OR

$1z^2 + 6z + 13 = 0$

$a=1, \quad b=6, \quad c=13$

By quadratic formula

$z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm \sqrt{6^2 - 4(1)(13)}}{2(1)}$

$= \dfrac{-6 \pm \sqrt{-16}}{2} = \dfrac{-6 \pm 4i}{2}$

$= -3 \pm 2i$

S.Set $= \{-3 \pm 2i\}$

(9) $z + \frac{2}{z} = 2$

Sol $\Rightarrow \dfrac{z^2 + 2}{z} = 2$

$\Rightarrow z^2 + 2 = 2z$

$\Rightarrow z^2 - 2z = -2$

Add 1 to b.s

$\Rightarrow z^2 - 2z + 1 = -2 + 1$

$\Rightarrow z^2 - 2z + 1^2 = -1$

$\Rightarrow (z-1)^2 = -1$

Take square root

$\sqrt{(z-1)^2} = \pm\sqrt{-1}$

$z - 1 = \pm i$

$\Rightarrow z = 1 \pm i$

S.Set $= \{1 \pm i\}$ Ans

(10) $2z^2+15=4z$

Sol $2z^2-4z=-15$

Divide by 2 we get

$\Rightarrow z^2-2z=-\frac{15}{2}$ Add 1^2 to b.s

$\Rightarrow z^2-2z+1^2=-\frac{15}{2}+1^2$

$\Rightarrow (z-1)^2=\frac{-15+2}{2}$

$\Rightarrow (z-1)^2=\frac{-13}{2}$

$\Rightarrow (z-1)^2=i^2\left(\sqrt{\frac{13}{2}}\right)^2$

$\Rightarrow (z-1)^2=\left(\sqrt{\frac{13}{2}}\,i\right)^2$

Take sq. root, we get

$\Rightarrow \sqrt{(z-1)^2}=\pm\sqrt{\left(\sqrt{\frac{13}{2}}\,i\right)^2}$

$\Rightarrow z-1=\pm\sqrt{\frac{13}{2}}\,i$

$\Rightarrow z=1\pm\sqrt{\frac{13}{2}}\,i$

S. Set $=\left\{1\pm\sqrt{\frac{13}{2}}\,i\right\}$ Ans

Method # 02

OR $2z^2-4z+15=0$

By quadratic formula

$z=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(15)}}{2(2)}$

$z=\frac{4\pm\sqrt{16-120}}{4}$

$z=\frac{4\pm\sqrt{-104}}{4}$

$z=\frac{4\pm\sqrt{4\times-26}}{4}$

$z=\frac{4\pm2\sqrt{-26}}{4}$

$\Rightarrow z=\frac{4\pm2\sqrt{26}\,i}{4}$

$\Rightarrow z=1\pm\frac{\sqrt{26}}{2}\,i$

$\Rightarrow z=1\pm\sqrt{\frac{26}{4}}\,i$

$z=1\pm\sqrt{\frac{13}{2}}\,i$

S. Set $=\left\{1\pm\sqrt{\frac{13}{2}}\,i\right\}$ Ans

Q:11 Show that $z_1=-1+i$ and $z_2=-1-i$ satisfies the eqn $z^2+2z+2=0$

Sol $z^2+2z+2=0$

put $-1+i$ in place of z

$\Rightarrow (-1+i)^2+2(-1+i)+2=0$

$\Rightarrow (-1)^2+i^2-2i-2+2i+2=0$

$\Rightarrow 1-1-2i-2+2i+2=0$

$\Rightarrow 0=0$

Hence $z_1=-1+i$ satisfies the eqn $z^2+2z+2=0$

, Now put $-1-i$ in place of z

$\Rightarrow (-1-i)^2+2(-1-i)+2=0$

$\Rightarrow (-1)^2+(-i)^2+2(-1)(-i)-2-2i+2=0$

$\Rightarrow 1+i^2+2i-2-2i+2=0$

$\Rightarrow 1-1+2i-2-2i+2=0$

$0=0$

Hence $z_2=-1-i$ satisfies the eqn $z^2+2z+2=0$

Q:12 Determine whether $1+2i$ is solution of $z^2-2z+5=0$

Sol $z^2-2z+5=0$

Put $z=1+2i$

$\Rightarrow (1+2i)^2-2(1+2i)+5=0$

$\Rightarrow 1^2+(2i)^2+2(1)(2i)-2(1+2i)+5=0$

$\Rightarrow 1+4i^2+4i-2-4i+5=0$

$\Rightarrow 1-4+4i-2-4i+5=0$

$\Rightarrow -3+4i-2-4i+5=0$

$0=0$

Hence $1+2i$ is solution of $z^2-2z+5=0$

Hurrah! That's the end of chapter # 07