Quadratic Equation Class 10 Maths Notes PDF Download

Quadratic equation

If we consider a quadratic polynomial :

⇒ $p(x) = ax^2 + bx + c$

$p(x) = 0$ → quadratic polynomial.

If $p(x) = 0$ is quadratic equation, then zeroes of polynomial are known as roots of quadratic equation.

⇒ Working rule to identify quadratic equation-

Step: Write the equation in the form of $f(x) = 0$ or $p(x) = 0$.

Step : (i) If $f(x)$ is polynomial, then find its degree.

(ii) If $f(x)$ is not polynomial, then first make it polynomial

Step: If Degree of polynomial is 2.

Ex ⇒ Check whether the following are quadratic equation.

(i) $(x+1)^2 = 2(x-3)$

$\Rightarrow x^2 + 1 + 2x = 2x - 6$

$\Rightarrow x^2 + 7 = 0$ yes

(ii) $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x(x+1) - 2(x+1) = x(x+3) - 1(x+3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow -x + x - 2 + 3 - 3x = 0$

$\Rightarrow 3x - 1 = 0$ No

(iii) $x^2 + 3x + 1 = (x-2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow 3x + 4x = 4 - 1$

$\Rightarrow 7x = 3$

$\Rightarrow 7x - 3 = 0$ No

(iv) $x^2 + \frac{1}{x^2} = 2 ; x \neq 0$

$\Rightarrow x^4 + 1 = 2x^2$

$\Rightarrow (x^4 - 2x^2 + 1 = 0)$ No

(v) $x^2+2\sqrt{x}+1=0$
Not polynomial:

(vi) $x+\frac{1}{x}=1$

$\Rightarrow x^2+1=x$
$\Rightarrow x^2-x+1=0$ Yes.

Ex: In each of the following quadratic equation the value of x are given. Determine wether these value are roots of quadratic equation (or) Not.

(i) $2x^2+3x+1=0$; $x=-1$ put
$\Rightarrow 2(-1)^2+3(-1)+1=0$
$\Rightarrow 2-3+1=0$
$\Rightarrow 0=0$
$x=-1$ is roots of quadratic equ.

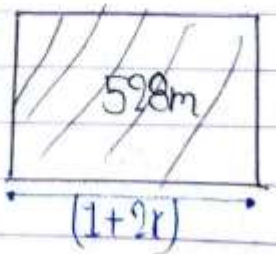
(ii) $2x^2+(k-6)x-3k=0$, $x=\frac{-k}{2}$
$\Rightarrow 2\left(\frac{-k}{2}\right)^2+(k-6)\left(\frac{-k}{2}\right)-3k=0$
$\Rightarrow 2\times\frac{k^2}{4}+k-6\times\frac{-k}{2}-3k=0$
$\Rightarrow \frac{k^2}{2}+k-\frac{k^2}{2}+6k-3k=0$
$\Rightarrow \frac{k^2+2k-k^2+6k-6k}{2}=\frac{2k}{2}$
$\Rightarrow k=0$, $x=-k$ is roots of quadratic equ.

(iii) $3x^2-x-1=0$, $x=-1$ put
$\Rightarrow 3(-1)^2+1-1=0$
$\Rightarrow 3+1-1=3$
$x=-1$ is not roots of quadratic equ.

To find - Quadratic equation -

Ex:- The area of rectangular plot is 528 m². the lenght of the plot (in metres) is one more then twice its breath. We need to find the lenghth and breath of the plot.

Solve: lenght = let. lenght is x
its breath is $1+2x$



$\Rightarrow x(1+2x) = 528$

$\Rightarrow x + 2x^2 = 528$

$\Rightarrow \boxed{2x^2 + x - 528 = 0}$ Ans

Ex: The product of two consecutive positive integers is 306. We need to find the integers.

Solve $\Rightarrow$ first integer $= x$

Second integer $= x+1$

$\Rightarrow (x+1)x = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow \boxed{x^2 + x - 306 = 0}$ Ans.

Ex: Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

Solve $\Rightarrow$ let rohan age is $(x+3)$

Rohan mother age is $(x+26)$

$\Rightarrow (x+3) \Rightarrow (x+26+3) = (x+29)$

$\Rightarrow (x+3)(x+29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x = 360 - 87$

$\Rightarrow x^2 + 32x = 273$

$\Rightarrow \boxed{x^2 + 32x - 273 = 0}$ Ans

Ex: A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Quadratic Equation Class 10 Maths Notes PDF Download

Solve :- let speed of train = x km/h

$\Rightarrow$ time taken to cover 480 km distance = $\left(\frac{480}{x}\ h\right)$

Now, (New speed of train = $(x-8)$ km/h

time taken $\Rightarrow \left(\frac{480}{x-8}\right)$ h

$\Rightarrow \frac{480}{x} - 3 = \frac{480}{x-8}$

$\Rightarrow \frac{480-3x}{x} = \frac{480}{x-8} \Rightarrow (x^2-8x-1280=0)$ Ans.

* Solving A quadratic equation $\rightarrow ax^2+bx+c=0$ by factorisation

(i) $2x^2-11x+12=0$

$\Rightarrow 2x^2-8x-3x+12$

$\Rightarrow 2x(x-4)-3(x-4)$

$\Rightarrow (2x-3)(x-4)$

$\Rightarrow \boxed{x = 3/2, 4}$ Ans

(ii) $\sqrt{3}x^2+10x+7\sqrt{3}=0$

$\Rightarrow \sqrt{3}x^2+7x+3x+7\sqrt{3}=0$

$\Rightarrow x(\sqrt{3}x+7)+\sqrt{3}\sqrt{3}x+7\sqrt{3}=0$

$\Rightarrow x(\sqrt{3}x+7)+\sqrt{3}(\sqrt{3}x+7)=0$

$\Rightarrow (x+\sqrt{3})(\sqrt{3}x+7)$

$\Rightarrow \boxed{x = -\sqrt{3}, \frac{-7}{\sqrt{3}}}$ Ans

(iii) $2x^2-x+1/8=0$

$\Rightarrow 16x^2-8x+1=0$

$\Rightarrow 16x^2-4x-4x+1=0$

$\Rightarrow 4x(4x-1)-1(4x-1)=0$

$\Rightarrow (4x-1)(4x-1) \Rightarrow (4x-1)^2=0$

$\Rightarrow \boxed{x = 1/2, 1/2}$ Ans

Ex:- Solve for x : $12abx^2-(9a^2-8b^2)x-6ab=0$ [CBSE 2006]

$\Rightarrow 12abx^2-9ax+8b^2x-6ab=0$

$\Rightarrow 3ax(4bx-3a)+2b(4bx-3a)=0$

$\Rightarrow (3ax+2b)(4bx-3a) \Rightarrow x = -2b/3a, 3a/4b$ Ans

Quadratic Equation Class 10 Maths Notes PDF Download

Ex:- Solve for x: $4x^2 - 4a^2x + (a^4 - b^4) = 0$ [CBSE 2006]

$\Rightarrow 4x^2 - 4a^2x + a^4 - b^4 = 0$

$\Rightarrow (2x)^2 + (a^2)^2 - 2 \times 2x \times a - b^4 = 0$

$\Rightarrow (2x - a^2)^2 - b^4 = 0$

$\Rightarrow (2x - a^2)^2 - (b^2)^2 = 0$

$\Rightarrow (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\Rightarrow x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$ Ans

* Shri dharacharyya - formula $\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

a = cofficent of x^2

b = cofficent of x

c = constant

Examples:- (i) $6x^2 + 7x - 10 = 0$

$\Rightarrow a = 6$

$\Rightarrow b = 7$

$\Rightarrow c = -10$

$\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow \frac{-7 + \sqrt{(7)^2 - 4 \times 6 \times -10}}{2 \times 6}, \frac{-7 - \sqrt{(7)^2 - 4 \times 6 \times -10}}{2 \times 6}$

$\Rightarrow \frac{-7 + \sqrt{49 + 240}}{12}, \frac{-7 - \sqrt{49 + 240}}{12}$

$\Rightarrow \frac{-7 + 17}{12}, \frac{-7 - 17}{12} \Rightarrow \frac{10}{12}, \frac{-24}{12}$

$\Rightarrow x = 5/6, -2$ Ans

(iii) $x - 1/x = 3$

$\Rightarrow x^2 - 3x - 1 = 0 \quad a=1, b=-3, c=-1$

$\Rightarrow \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{3+\sqrt{(-3)^2-4\times1\times-1}}{2\times1}, \frac{3-\sqrt{(-3)^2-4\times1\times-1}}{2\times1}$

$\Rightarrow \frac{3+\sqrt{9+4}}{2}, \frac{3-\sqrt{9+4}}{2}$

$x \Rightarrow \frac{3+\sqrt{13}}{2}, \frac{3-\sqrt{13}}{2}$ Ans

Quadratic Equation Class 10 Maths Notes PDF Download

* Nature of the roots →

Lets consider an quadratic equation $ax^2+bx+c=0$.

then, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2}$

$\Rightarrow D = b^2-4ac$, D → Discriminent.

(I) If $D>0$ (or) $b^2-4ac>0$, then, has 2 distinct real roots.
(II) If $D=0$ (or) $b^2-4ac=0$, then equ has 2-equall roots.
(III) If $D<0$ (or) $b^2-4ac<0$, then equ has no real roots.

Ex: find the discriminat of the quadratic equation.

(i) $x^2-4x+3=0$

$a=1, b=-4, c=3$

$\Rightarrow D = b^2-4ac$

$\Rightarrow D = (-4)^2-4\times1\times3$

$D = 16-12$

$\boxed{D=4}$

$(D>0)$ → two distinat real roots.

(ii) $2x^2-3x+5=0$

$a=2, b=-3, c=5$

$\Rightarrow D = (b)^2-4ac$

$\Rightarrow (-3)^2-4\times2\times5$

$\Rightarrow 9-40$

$\Rightarrow D=-31$

$(D<0)$ → # No real root.

(iii) $3x^2-4\sqrt{3}x+4=0$

$a=3, b=-4\sqrt{3}, c=4$

$\Rightarrow D = b^2-4ac$

$\Rightarrow (-4\sqrt{3})^2-4\times3\times4$

$\Rightarrow 16\times3-48=0$

$\Rightarrow 48-48=0$

$\Rightarrow (D=0)$ # two equal roots.

Quadratic Equation Class 10 Maths Notes PDF Download

Ex: find the value of 'k' If the equation $x^2-2(k+1)x+k^2=0$ has equall roots.

$\Rightarrow x^2-2(k+1)x+k^2=0$

$a=1, b=-2(k+1), c=k^2$

$\Rightarrow D=0$

$\Rightarrow b^2-4ac=0$

$\Rightarrow (-2(k+1))^2-4\times1\times k^2=0$

$\Rightarrow 4(k+1)^2-4k^2=0$

$\Rightarrow 4(k^2+1+2k)-4k^2$

$\Rightarrow 4k^2+4+8k-4k^2$

$\Rightarrow 8k=-4$

$(k=-1/2)$ Ans

Ex: The difference of two numbers is 5 and the difference of their product is 1/10.

Solve - Let first number = x and second number = x+5

$\Rightarrow \frac{1}{x}-\frac{1}{x+5}=\frac{1}{10}$

$\Rightarrow \frac{(x+5)-x}{x(x+5)}=\frac{1}{10} \Rightarrow 50=x(x+5)$

$50=x^2+5x$

$x^2+5x-50=0$ factor.

$\Rightarrow x^2+10x-5x-50=0$

$\Rightarrow x(x+10)-5(x+10)=0$

$\Rightarrow (x-5)(x+10)$

$\Rightarrow x=5,-10$ Ans