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MATHEMATICS

Class 10th (KPK)

Chapter # 4 Partial Fraction

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Exercise # 4.1

UNIT # 4

PARTIAL FRACTIONS

Partial Fraction:

A procedure which does splitting up a fraction into two or more fractions with only one factors in the denominator is called partial fraction.

In other words, a set of fractions whose algebraic sum is a given fraction is called partial fraction.

Rational Fraction:

A rational function can be written in the form of:

$$f(x) = \frac{P(x)}{Q(x)}$$

Where P (x) and Q (x) are polynomials, where Q $(x) \neq 0$

Proper rational fraction:

A rational fraction is proper fraction, if degree of numerator P (x) is less than the degree of denominator Q (x).

Example

$$\frac{1}{x+1}, \frac{2x}{x^2+2}, \frac{x^2+x-3}{x^3+x^2-x+1}$$

Improper rational fraction

A rational fraction is an improper fraction, if degree of numerator P (x) is greater than or equal to the degree of denominator Q (x).

Example

$$\frac{x^3+4}{(x+1)(x+2)}, \frac{x}{2x+2}, \frac{x^2+x-3}{x^2-x+1}, \frac{x^3+x^2+x-3}{x^2-x+1}$$

Note:

Any improper rational fraction can be reduced into sum of polynomials and rational fraction by large division.

Example:

$$\frac{2x^2+1}{x-1}$$

Solution:

$$\begin{array}{r|l} & 2x+2 \\ \hline x-1 & 2x^2+1 \\ & \pm 2x^2 \mp 2x \\ \hline & 2x+1 \\ & \pm 2x \mp 2 \\ \hline & 3 \end{array}$$

$$\frac{2x^2+1}{x-1} = 2x+2+\frac{3}{x-1}$$

Exercise # 4.1

Resolution of fraction into partial fraction

Resolution of rational fraction $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$ into partial fraction depends upon the factors of denominator $Q(x)$

Case # 1:

Let proper fraction $\frac{P(x)}{Q(x)}$ given

Factorize the polynomial $Q(x)$ in the denominator if it is not factorized.

$$\frac{P(x)}{Q(x)} = \frac{A}{x+a} + \frac{B}{x+b}$$

Example # 1:

Resolve $\frac{1}{(x+1)(x+2)}$ into partial fraction.

Solution:

$$\frac{1}{(x+1)(x+2)}$$

Let

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x+2)$

$$\frac{1}{(x+1)(x+2)} \times (x+1)(x+2) = \frac{A}{x+1} \times (x+1)(x+2) + \frac{B}{x+2} \times (x+1)(x+2)$$

$1 = A(x+2) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$1 = A(-1+2) + B(0)$

$1 = A(1) + 0$

$1 = A$

$A = 1$

Put $x+2=0 \Rightarrow x=-2$ in equ (ii)

$1 = A(0) + B(-2+1)$

$1 = 0 + B(-1)$

$1 = -B$

$-B = 1$

$B = -1$

Put the values of A and B in equ (i)

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} + \frac{-1}{x+2}$$

$$\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$$

Exercise # 4.1

Example # 2: Find partial fraction of $\frac{3x+2}{x^2-x-2}$

Solution:

$$\frac{3x+2}{x^2-x-2}$$

Now

Let

$$\frac{3x+2}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-2)$

$$\frac{3x+2}{(x+1)(x-2)} \times (x+1)(x-2) = \frac{A}{x+1} \times (x+1)(x-2) + \frac{B}{x-2} \times (x+1)(x-2)$$

$$3x+2 = A(x-2) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$$3(-1)+2 = A(-1-2) + B(0)$$

$$-3+2 = A(-3) + 0$$

$$-1 = -3A$$

$$\frac{-1}{-3} = A$$

$$\frac{1}{3} = A$$

$$A = \frac{1}{3}$$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$$3(2)+2 = A(0) + B(2+1)$$

$$6+2 = 0 + B(3)$$

$$8 = 3B$$

$$\frac{8}{3} = B$$

$$B = \frac{8}{3}$$

Put the values of A and B in equ (i)

$$\frac{3x+2}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$$\frac{3x+2}{(x+1)(x-2)} = \frac{\frac{1}{3}}{x+1} + \frac{\frac{8}{3}}{x-2}$$

$$\frac{3x+2}{(x+1)(x-2)} = \frac{1}{3(x+1)} + \frac{8}{2(x-2)}$$

R.W

$$\frac{3x+2}{x^2-x-2} = \frac{3x+2}{(x+1)(x-2)}$$

Example # 3: Find partial fraction of $\frac{x}{(x+1)^2}$

Solution:

$$\frac{x}{(x+1)^2}$$

Let

Exercise # 4.1

$$\frac{x}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)^2$

$$\frac{x}{(x+1)^2} \times (x+1)^2 = \frac{A}{x+1} \times (x+1)^2 + \frac{B}{(x+1)^2} \times (x+1)^2$$

$x = A(x+1) + B \quad \ldots\ldots \text{equ(ii)}$

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$-1 = A(0) + B$

$-1 = B$

$B = -1$

equ (ii) $\Rightarrow$

$x = A(x+1) + B$

$x = Ax + A + B$

$x = Ax + (A + B)$

By comparing the coefficients of x, we get

$A = 1$

Put the values of A and B in equ (i)

$$\frac{x}{(x+1)^2} = \frac{1}{x+1} + \frac{-1}{(x+1)^2}$$

$$\frac{x}{(x+1)^2} = \frac{1}{x+1} - \frac{1}{(x+1)^2}$$

Example # 4: Find partial fraction of $\mathbf{\dfrac{2x^2+1}{(x-2)^2(x+3)}}$

<u>Solution</u>:

$$\frac{2x^2+1}{(x-2)^2(x+3)}$$

Let

$$\frac{2x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)^2$, we get

$2x^2 + 1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x - 2 = 0 \Rightarrow x = 2$ in equ (ii)

$2(2)^2 + 1 = A(0)(2+3) + B(2+3) + C(0)^2$

$2(4) + 1 = 0 + B(5) + 0$

$8 + 1 = 5B$

$9 = 5B$

$\frac{9}{5} = B$

$B = \frac{9}{5}$

Put $x + 3 = 0 \Rightarrow x = -3$ in equ (ii)

$2(-3)^2 + 1 = A(-3-2)(0) + B(0) + C(-3-2)^2$

$2(9) + 1 = 0 + 0 + C(-3-2)^2$

$18 + 1 = C(-5)^2$

$19 = C(25)$

Exercise # 4.1

$\frac{19}{25} = C$

$\mathrm{C} = \frac{19}{25}$

equ (ii) $\Rightarrow$

$2x^2 + 1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$

$2x^2 + 1 = A(x^2 + 3x - 2x - 6) + Bx + 3B + C(x^2 - 4x + 2)$

$2x^2 + 1 = A(x^2 + x - 6) + Bx + 3B + C(x^2 - 4x + 2)$

$2x^2 + 1 = Ax^2 + Ax - 6A + Bx + 3B + Cx^2 - 4Cx + 2C$

$2x^2 + 1 = Ax^2 + Cx^2 + Ax + Bx - 4Cx - 6A + 3B + 2C$

$2x^2 + 1 = (A + C)x^2 + (A + B - 4C)x + (-6A + 3B + 2C)$

By comparing the coefficients of x^2, we get

$A + C = 2$

Put $\mathrm{C} = \frac{19}{25}$

$A + \frac{19}{25} = 2$

$A = 2 - \frac{19}{25}$

$A = \frac{50 - 19}{25}$

$A = \frac{31}{25}$

Put the values of A, B and C in equ (i)

$$\frac{2x^2 + 1}{(x-2)^2(x+3)} = \frac{\frac{31}{25}}{x-2} + \frac{\frac{9}{5}}{(x-2)^2} + \frac{\frac{19}{25}}{x+3}$$

$$\frac{2x^2 + 1}{(x-2)^2(x+3)} = \frac{31}{25(x-2)} - \frac{9}{5(x-2)^2} + \frac{19}{25(x+3)}$$

Exercise # 4.1

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Resolve the following fractions into partial fraction.

(1) $\frac{3x-2}{2x^2-x}$

Solution:

$\frac{3x-2}{2x^2-x}$

$\frac{3x-2}{2x^2-x} = \frac{3x-2}{x(2x-1)}$

Let

$\frac{3x-2}{2x^2-x} = \frac{\mathrm{A}}{x} + \frac{\mathrm{B}}{2x-1}$ ..... equ(i)

Multiply equ (i) by $x(2x-1)$

$\frac{3x-2}{2x^2-x} \times x(2x-1) = \frac{\mathrm{A}}{x} \times x(2x-1) + \frac{\mathrm{B}}{2x-1} \times x(2x-1)$

$3x - 2 = \mathrm{A}(2x-1) + \mathrm{B}x$ ..... equ(ii)

Put $x = 0$ in equ (ii)

Exercise # 4.1

$3(0) - 2 = A(2(0) - 1) + B(0)$
$0 - 2 = A(0 - 1) + 0$
$-2 = A(-1)$
$-2 = -A$
$2 = A$
$A = 2$

Put $2x - 1 = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$ in equ (ii)

$3\left(\frac{1}{2}\right) - 2 = A(0) + B\left(\frac{1}{2}\right)$
$\frac{3}{2} - 2 = 0 + \frac{B}{2}$
$\frac{3-4}{2} = \frac{B}{2}$
$\frac{-1}{2} = \frac{B}{2}$
$-1 = B$
$B = -1$

Put the values of A and B in equ (i)

$\frac{3x - 2}{2x^2 - x} = \frac{2}{x} + \frac{-1}{2x - 1}$
$\frac{3x - 2}{2x^2 - x} = \frac{2}{x} - \frac{1}{2x - 1}$

(2) $\boldsymbol{\frac{x - 1}{x^2 + 6x + 5}}$

Solution:

$\frac{x - 1}{x^2 + 6x + 5}$
$\frac{x - 1}{x^2 + 6x + 5} = \frac{x - 1}{(x + 1)(x + 5)}$

R.W
$x^2 + 6x + 5 = x^2 + 1x + 5x + 5$
$x^2 + 6x + 5 = x(x + 1) + 5(x + 1)$
$x^2 + 6x + 5 = (x + 1)(x + 5)$

Let

$\frac{x - 1}{(x + 1)(x + 5)} = \frac{A}{x + 1} + \frac{B}{x + 5}$ .....equ(i)

Multiply equ (i) by $(x + 1)(x + 5)$

$\frac{x - 1}{(x + 1)(x + 5)} \times (x + 1)(x + 5) = \frac{A}{x + 1} \times (x + 1)(x + 5) + \frac{B}{x + 5} \times (x + 1)(x + 5)$
$x - 1 = A(x + 5) + B(x + 1)$ .....equ(ii)

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$-1 - 1 = A(-1 + 5) + B(0)$
$-2 = A(4) + 0$
$-2 = 4A$
$\frac{-2}{4} = A$
$\frac{-1}{2} = A$

Exercise # 4.1

$A = \frac{-1}{2}$

Put $x + 5 = 0 \Rightarrow x = -5$ in equ (ii)

$-5 - 1 = A(0) + B(-5 + 1)$

$-6 = 0 + B(-4)$

$-6 = -4B$

$6 = 4B$

$\frac{6}{4} = B$

$\frac{3}{2} = B$

$B = \frac{3}{2}$

Put the values of A and B in equ (i)

$$\frac{x-1}{(x+1)(x+5)} = \frac{\frac{-1}{2}}{x+1} + \frac{\frac{3}{2}}{x+5}$$

$$\frac{x-1}{(x+1)(x+5)} = \frac{-1}{2(x+1)} + \frac{3}{2(x+5)}$$

OR

$$\frac{x-1}{x^2+6x+5} = \frac{-1}{2(x+1)} + \frac{3}{2(x+5)}$$

(3) $\boldsymbol{\frac{1}{x^2-1}}$

Solution:

$$\frac{1}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{1}{x^2-1^2}$$

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$$

Now

Let

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)$, we get

$1 = A(x-1) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$1 = A(-1-1) + B(0)$

$1 = A(-2) + 0$

$1 = -2A$

$\frac{1}{-2} = A$

$A = \frac{1}{-2}$

Exercise # 4.1

$A = -\frac{1}{2}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A(0) + B(1+1)$

$1 = 0 + B(2)$

$1 = 2B$

$\frac{1}{2} = B$

$B = \frac{1}{2}$

Put the values of A and B in equ (i)

$$\frac{1}{(x+1)(x-1)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$\frac{1}{(x+1)(x-1)} = \frac{-1}{2(x+1)} + \frac{1}{2(x-1)}$$

OR

$$\frac{1}{x^2-1} = \frac{-1}{2(x+1)} + \frac{1}{2(x-1)}$$

(4) $\boldsymbol{\frac{x}{x^2+4x-5}}$

Solution:

$$\frac{x}{x^2+4x-5}$$

$$\frac{x}{x^2+4x-5} = \frac{x}{(x-1)(x+5)}$$

R.W

$x^2 + 4x - 5 = x^2 - 1x + 5x - 5$

$x^2 + 4x - 5 = x(x-1) + 5(x-1)$

$x^2 + 4x - 5 = (x-1)(x+5)$

Let

$$\frac{x}{(x-1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+5} \quad \text{.....equ(i)}$$

Multiply equ (i) by $(x-1)(x+5)$

$$\frac{x}{(x-1)(x+5)} \times (x-1)(x+5) = \frac{A}{x-1} \times (x-1)(x+5) + \frac{B}{x+5} \times (x-1)(x+5)$$

$x = A(x+5) + B(x-1) \quad \text{.....equ(ii)}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A(1+5) + B(0)$

$1 = A(6) + 0$

$1 = 6A$

$\frac{1}{6} = A$

$A = \frac{1}{6}$

Put $x + 5 = 0 \Rightarrow x = -5$ in equ (ii)

$-5 = A(0) + B(-5-1)$

$-5 = 0 + B(-6)$

$-5 = -6B$

$5 = 6B$

Exercise # 4.1

$\frac{5}{6} = \mathrm{B}$

$\mathrm{B} = \frac{5}{6}$

Put the values of A and B in equ (i)

$$\frac{x}{(x-1)(x+5)} = \frac{\frac{1}{6}}{x+1} + \frac{\frac{5}{6}}{x+5}$$

$$\frac{x}{(x-1)(x+5)} = \frac{1}{6(x-1)} + \frac{5}{6(x+5)}$$

OR

$$\frac{x}{x^2+4x-5} = \frac{1}{6(x-1)} + \frac{5}{6(x+5)}$$

(5) $\frac{\mathbf{4x+2}}{\mathbf{(x+2)(2x-1)}}$

Solution:

$$\frac{4x+2}{(x+2)(2x-1)}$$

Let

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{\mathrm{A}}{x+2} + \frac{\mathrm{B}}{2x-1} \quad \ldots\ldots \mathrm{equ(i)}$$

Multiply equ (i) by $(x+2)(2x-1)$

$$\frac{4x+2}{(x+2)(2x-1)} \times (x+2)(2x-1) = \frac{\mathrm{A}}{x+2} \times (x+2)(2x-1) + \frac{\mathrm{B}}{2x-1} \times (x+2)(2x-1)$$

$4x+2 = \mathrm{A}(2x-1) + \mathrm{B}(x+2) \quad \ldots\ldots \mathrm{equ(ii)}$

Put $x+2=0 \Rightarrow x=-2$ in equ (ii)

$4(-2)+2 = \mathrm{A}(2(-2)-1) + \mathrm{B}(0)$

$-8+2 = \mathrm{A}(-4-1) + 0$

$-6 = \mathrm{A}(-5)$

$-6 = -5\mathrm{A}$

$6 = 5\mathrm{A}$

$\frac{6}{5} = \mathrm{A}$

$\mathrm{A} = \frac{6}{5}$

Put $2x-1=0 \Rightarrow 2x=1 \Rightarrow x=\frac{1}{2}$ in equ (ii)

$4\left(\frac{1}{2}\right)+2 = \mathrm{A}(0) + \mathrm{B}\left(\frac{1}{2}+2\right)$

$2+2 = 0 + \mathrm{B}\left(\frac{1+4}{2}\right)$

$4 = \mathrm{B}\left(\frac{5}{2}\right)$

$4 \times \frac{2}{5} = \mathrm{B}$

Exercise # 4.1

$\frac{8}{5} = \text{B}$

$\text{B} = \frac{8}{5}$

Put the values of A and B in equ (i)

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{\frac{6}{5}}{x+2} + \frac{\frac{8}{5}}{2x-1}$$

$$\frac{4x+2}{(x+2)(2x-1)} = \frac{6}{5(x+2)} + \frac{8}{5(2x-1)}$$

(7) $\boldsymbol{\frac{x^2+5x+3}{(x^2-1)(x+1)}}$

Solution:

$$\frac{x^2+5x+3}{(x^2-1)(x+1)} = \frac{x^2+5x+3}{(x-1)(x+1)(x+1)}$$

$$\frac{x^2+5x+3}{(x^2-1)(x+1)} = \frac{x^2+5x+3}{(x-1)(x+1)^2}$$

Now

Let

$$\frac{x^2+5x+3}{(x-1)(x+1)^2} = \frac{\text{A}}{x-1} + \frac{\text{B}}{x+1} + \frac{\text{C}}{(x+1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)(x+1)^2$

$$\frac{x^2+5x+3}{(x-1)(x+1)^2} \times (x-1)(x+1)^2 = \frac{\text{A}}{x-1} \times (x-1)(x+1)^2 + \frac{\text{B}}{x+1} \times (x-1)(x+1)^2 + \frac{\text{C}}{(x+1)^2} \times (x-1)(x+1)^2$$

$x^2+5x+3 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$(1)^2 + 5(1) + 3 = A(1+1)^2 + B(0)(x+1) + C(0)$

$1+5+3 = A(2)^2 + 0 + 0$

$9 = \text{A}(4)$

$9 = 4\text{A}$

$\frac{9}{4} = \text{A}$

$\text{A} = \frac{9}{4}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$(-1)^2 + 5(-1) + 3 = A(0)^2 + B(x-1)(0) + C(-1-1)$

$1-5+3 = A(0) + B(0) + C(-2)$

$-4+3 = 0+0-2C$

$-1 = -2C$

$1 = 2C$

$\frac{1}{2} = C$

Exercise # 4.1

$C = \frac{1}{2}$

equ (ii) $\Rightarrow$

$x^2 + 5x + 3 = A(x^2 + 2x + 1) + B(x^2 - 1) + Cx - C$

$x^2 + 5x + 3 = Ax^2 + 2Ax + A + Bx^2 - B + Cx - C$

$x^2 + 5x + 3 = Ax^2 + Bx^2 + 2Ax + Cx + A - B - C$

$x^2 + 5x + 3 = (A + B)x^2 + (2A + C)x + (A - B - C)$

By comparing the coefficients of x^2, we get

$A + B = 1$

Put $A = \frac{9}{4}$

$\frac{9}{4} + B = 1$

$B = 1 - \frac{9}{4}$

$B = \frac{4 - 9}{4}$

$B = \frac{-5}{4}$

Put the values of A, B and C in equ (i)

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{\frac{9}{4}}{x-1} + \frac{\frac{-5}{4}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2}$$

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{9}{4(x-1)} - \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}$$

(8) $\frac{\mathbf{x^2 + 2}}{\mathbf{(x+2)(x^2+5x+6)}}$

Solution:

$$\frac{x^2 + 2}{(x+2)(x^2+5x+6)} = \frac{x^2 + 2}{(x+2)(x+3)(x+2)}$$

$$\frac{x^2 + 2}{(x+2)(x^2+5x+6)} = \frac{x^2 + 2}{(x+3)(x+2)^2}$$

$R.W$

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

$x^2 + 5x + 6 = x(x+2) + 3(x+2)$

$x^2 + 5x + 6 = (x+3)(x+2)$

Let

$$\frac{x^2 + 2}{(x+2)(x^2+5x+6)} = \frac{A}{x+3} + \frac{B}{x+2} + \frac{C}{(x+2)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+3)(x+2)^2$

$$\frac{x^2 + 2}{(x+2)(x^2+5x+6)} \times (x+3)(x+2)^2 = \frac{A}{x+3} \times (x+3)(x+2)^2 + \frac{B}{x+2} \times (x+3)(x+2)^2 + \frac{C}{(x+2)^2} \times (x+3)(x+2)^2$$

$x^2 + 2 = A(x+2)^2 + B(x+3)(x+2) + C(x+3) \quad \ldots\ldots$ equ(ii)

Put $x + 3 = 0 \Rightarrow x = -3$ in equ (ii)

$(-3)^2 + 2 = A(-3+2)^2 + B(0)(x+2) + C(0)$

$9 + 2 = A(-1)^2 + 0 + 0$

$11 = A(1)$

Exercise # 4.1

$11 = A$
$A = 11$
Put $x + 2 = 0 \Rightarrow x = -2$ in equ (ii)
$(-2)^2 + 2 = A(0)^2 + B(x+3)(0) + C(-2+3)$
$4 + 2 = 0 + 0 + C(1)$
$6 = C$
$C = 6$
equ (ii) $\Rightarrow$
$x^2 + 2 = A(x+2)^2 + B(x+3)(x+2) + C(x+3)$
$x^2 + 2 = A(x^2 + 2x + 1) + B(x+3)(x+2) + C(x+3)$
$x^2 + 5x + 3 = A(x^2 + 2x + 1) + B(x^2 - 1) + Cx - C$
$x^2 + 5x + 3 = Ax^2 + 2Ax + A + Bx^2 - B + Cx - C$
$x^2 + 5x + 3 = Ax^2 + Bx^2 + 2Ax + Cx + A - B - C$
$x^2 + 5x + 3 = (A+B)x^2 + (2A+C)x + (A - B - C)$
By comparing the coefficients of x^2, we get
$A + B = 1$
Put $\text{A} = \frac{9}{4}$
$\frac{9}{4} + B = 1$
$B = 1 - \frac{9}{4}$
$B = \frac{4-9}{4}$
$B = \frac{-5}{4}$
Put the values of A, B and C in equ (i)

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{\frac{9}{4}}{x-1} + \frac{\frac{-5}{4}}{x+1} + \frac{\frac{1}{2}}{(x+1)^2}$$

$$\frac{x^2 + 5x + 3}{(x-1)(x+1)^2} = \frac{9}{4(x-1)} - \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}$$

OR

$$\frac{x^2 + 2}{(x+2)(x^2 + 5x + 6)} = \frac{9}{4(x-1)} - \frac{5}{4(x+1)} + \frac{1}{2(x+1)^2}$$

(8) $\frac{\mathbf{2x - 1}}{\boldsymbol{x(x-3)^2}}$

Solution:

$$\frac{2x-1}{x(5x-3)^2}$$

Let

$$\frac{2x-1}{x(x-3)^2} = \frac{\text{A}}{x} + \frac{\text{B}}{x-3} + \frac{\text{C}}{(x-3)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $x(x-3)^2$

Exercise # 4.1

$$\frac{2x-1}{x(x-3)^2}\times x(x-3)^2=\frac{A}{x}\times x(x-3)^2+\frac{B}{x-3}\times x(x-3)^2+\frac{C}{(x-3)^2}\times x(x-3)^2$$

$2x-1=A(x-3)^2+Bx(x-3)+Cx \quad .....\text{equ(ii)}$

Put $x=0$ in equ (ii)

$2(0)-1=A(0-3)^2+B(0)(0-3)+C(0)$

$0-1=A(-3)^2+0+0$

$-1=A(9)$

$\frac{-1}{9}=A$

$A=\frac{-1}{9}$

Put $x-3=0\Rightarrow x=3$ in equ (ii)

$2(3)-1=A(0)^2+B(3)(0)+C(3)$

$6-1=0+0+3C$

$5=3C$

$\frac{5}{3}=C$

$C=\frac{5}{3}$

equ (ii) $\Rightarrow$

$2x-1=A(x-3)^2+Bx(x-3)+Cx$

$2x-1=A(x^2-6x+9)+Bx^2-3Bx+Cx$

$2x-1=Ax^2-6Ax+9A+Bx^2-3Bx+Cx$

$2x-1=Ax^2+Bx^2-6Ax-3Bx+Cx+9A$

$2x-1=(A+B)x^2+(-6A-3B+C)x+9A$

By comparing the coefficients of x^2, we get

$A+B=0$

Put $A=\frac{-1}{9}$

$\frac{-1}{9}+B=0$

$B=\frac{1}{9}$

Put the values of A, B and C in equ (i)

$$\frac{2x-1}{x(x-3)^2}=\frac{\frac{-1}{9}}{x}+\frac{\frac{1}{9}}{x-3}+\frac{\frac{5}{3}}{(x-3)^2}$$

$$\frac{2x-1}{x(x-3)^2}=\frac{-1}{9x}+\frac{1}{9(x-3)}+\frac{5}{3(x-3)^2}$$

(9) $\boldsymbol{\frac{x^2}{x^2+2x+1}}$

<u>Solution</u>:

$\frac{x^2}{x^2+2x+1}$

As $\frac{x^2}{x^2+2x+1}$ is improper

Exercise # 4.1

So

$$\begin{array}{r|l} & 1 \\ \hline x^2+2x+1 & x^2 \\ & \pm x^2 \pm 2x \pm 1 \\ \hline & -2x-1 \end{array}$$

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2x-1}{(x)^2+2(x)(1)+(1)^2}$$

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2x-1}{(x+1)^2} \quad \ldots\ldots \text{equ}(\mathbf{A})$$

Now

Let

$$\frac{-2x-1}{(x+1)^2} = \frac{\text{A}}{x+1} + \frac{\text{B}}{(x+1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+3)(x+2)^2$

$$\frac{-2x-1}{(x+1)^2} \times (x+1)^2 = \frac{\text{A}}{x+1} \times (x+1)^2 + \frac{\text{B}}{(x+1)^2} \times (x+1)^2$$

$-2x-1 = A(x+1)+B \quad \ldots\ldots \text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$-2(-1)-1 = A(0)+B$

$2-1 = 0+B$

$1 = B$

equ (ii) $\Rightarrow$

$-2x-1 = A(x+1)+B$

$-2x-1 = Ax+A+B$

$-2x-1 = Ax+(A+B)$

By comparing the coefficients of x, we get

$A = -2$

Put the values of A and B in equ (i)

$$\frac{-2x-1}{(x+1)^2} = \frac{-2}{x+1} + \frac{1}{(x+1)^2}$$

Put the above in equ (**A**)

$$\frac{x^2}{x^2+2x+1} = 1 + \frac{-2}{x+1} + \frac{1}{(x+1)^2}$$

$$\frac{x^2}{x^2+2x+1} = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

(10) $\boldsymbol{\dfrac{x^2}{(x-1)^2(x+1)}}$

<u>Solution</u>:

$$\frac{x^2}{(x-1)^2(x+1)}$$

Let

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{\text{A}}{x-1} + \frac{\text{B}}{(x-1)^2} + \frac{\text{C}}{x+1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)^2(x+1)$

Exercise # 4.1

$$\frac{x^2}{(x-1)^2(x+1)}\times(x-1)^2(x+1)=\frac{A}{x-1}\times(x-1)^2(x+1)+\frac{B}{(x-1)^2}\times(x-1)^2(x+1)+\frac{C}{x+1}\times(x-1)^2(x+1)$$

$x^2 = A(x-1)(x+1)+B(x+1)+C(x-1)^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$(1)^2 = A(0)(1+1)+B(1+1)+C(0)^2$

$1 = 0+B(2)+0$

$1 = 2B$

$\frac{1}{2}=B$

$B=\frac{1}{2}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$(-1)^2 = A(-1-1)(0)+B(0)+C(-1-1)^2$

$1 = 0+0+C(-2)^2$

$1 = C(4)$

$\frac{1}{4}=C$

$C=\frac{1}{4}$

equ (ii) $\Rightarrow$

$x^2 = A(x-1)(x+1)+B(x+1)+C(x-1)^2$

$x^2 = A(x^2-1)+Bx+B+C(x^2-2x+1)$

$x^2 = Ax^2-A+Bx+B+Cx^2-2Cx+C$

$x^2 = Ax^2+Cx^2+Bx-2Cx-A+B+C$

$x^2 = (A+C)x^2+(B-2C)x+(-A+B+C)$

By comparing the coefficients of x^2, we get

$A+C=1$

Put $C=\frac{1}{4}$

$A+\frac{1}{4}=1$

$A=1-\frac{1}{4}$

$A=\frac{4-1}{4}$

$A=\frac{3}{4}$

Put the values of A, B and C in equ (i)

$=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$

$$\frac{x^2}{(x-1)^2(x+1)}=\frac{\frac{3}{4}}{x-1}+\frac{\frac{1}{2}}{(x-1)^2}+\frac{\frac{1}{4}}{x+1}$$

$$\frac{x^2}{(x-1)^2(x+1)}=\frac{3}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}$$

Exercise # 4.1

Exercise # 4.2

Example # 5: Find partial fraction of $\dfrac{1}{(x+1)(x^2+2)}$

Solution:

$$\frac{1}{(x+1)(x^2+2)}$$

Let

$$\frac{1}{(x+1)(x^2+2)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2} \quad \ldots.. \text{equ(i)}$$

Multiply equ (i) by $(x-1)(x^2+3)$

$$\frac{1}{(x+1)(x^2+2)} \times (x+1)(x^2+2) = \frac{A}{x+1} \times (x+1)(x^2+2) + \frac{Bx+C}{x^2+2} \times (x+1)(x^2+2)$$

$$1 = A(x^2+2) + (Bx+C)(x+1) \quad \ldots.. \text{equ(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$$1 = A((-1)^2+2) + (B(-1)+C)(0)$$

$$1 = A(1+2) + 0$$

$$1 = A(3)$$

$$1 = 3A$$

$$\frac{1}{3} = A$$

$$A = \frac{1}{3}$$

equ (ii) $\Rightarrow$

$$1 = A(x^2+2) + (Bx+C)(x+1)$$

$$1 = Ax^2 + 2A + Bx^2 + Bx + Cx + C$$

$$1 = Ax^2 + Bx^2 + Bx + Cx + 2A + C$$

$$1 = (A+B)x^2 + (B+C)x + (2A+C)$$

Compare the coefficients of x^2, x *and constant* we get

$$A + B = 0 \quad \ldots.. \text{equ}(\boldsymbol{a})$$

$$B + C = 0 \quad \ldots.. \text{equ}(\boldsymbol{b})$$

$$2A + C = 1 \quad \ldots.. \text{equ}(\boldsymbol{c})$$

Put $A = \frac{1}{3}$ in equ $(\boldsymbol{a})$

$$\frac{1}{3} + B = 0$$

$$B = -\frac{1}{3}$$

Put $B = -\frac{1}{3}$ in equ $(\boldsymbol{b})$

$$-\frac{1}{3} + C = 0$$

$$C = \frac{1}{3}$$

Put the values of A, B and C in equ (i)

$$\frac{1}{(x+1)(x^2+2)} = \frac{\frac{1}{3}}{x+1} + \frac{-\frac{1}{3}x + \frac{1}{3}}{x^2+2}$$

Exercise # 4.2

$$\frac{1}{(x+1)(x^2+2)} = \frac{\frac{1}{3}}{x+1} + \frac{\frac{-1x+1}{3}}{x^2+2}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} + \frac{-1x+1}{3(x^2+2)}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} + \frac{-(x-1)}{3(x^2+2)}$$
$$\frac{1}{(x+1)(x^2+2)} = \frac{1}{3(x+1)} - \frac{x-1}{3(x^2+2)}$$

Example 6: Find partial fraction of $\frac{4x^2-28}{x^4-x^2-6}$

Solution:

$$\frac{4x^2-28}{x^4-x^2-6}$$
$$\frac{4x^2-28}{x^4-x^2-6} = \frac{4x^2-28}{(x^2-3)(x^2+2)}$$

R.W

$x^4-x^2-6 = x^4-3x^2+2x^2-6$

$x^4-x^2-6 = x^2(x^2-3)+2(x^2-3)$

$x^4-x^2-6 = (x^2-3)(x^2+2)$

Let

$$\frac{4x^2-28}{(x^2-3)(x^2+2)} = \frac{Ax+B}{x^2-3} + \frac{Cx+D}{x^2+2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x^2-3)(x^2+2)$

$$\frac{4x^2-28}{(x^2-3)(x^2+2)} \times (x^2-3)(x^2+2) = \frac{Ax+B}{x^2-3} \times (x^2-3)(x^2+2) + \frac{Cx+D}{x^2+2} \times (x^2-3)(x^2+2)$$

$4x^2-28 = (Ax+B)(x^2+2) + (Cx+D)(x^2-3) \quad \ldots\ldots \text{equ(ii)}$

equ (ii) $\Rightarrow$

$4x^2-28 = Ax^3+2Ax+Bx^2+2B+Cx^3-3Cx+Dx^2-3D$

$4x^2-28 = Ax^3+Cx^3+Bx^2+Dx^2+2Ax-3Cx+2B-3D$

$4x^2-28 = (A+C)x^3+(B+D)x^2+(2A-3C)x+(2B-3D)$

Compare the coefficients of x^3, x^2, x *and constant* we get

$A+C=0 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$

$B+D=4 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$

$2A-3C=0 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$

$2B-3D=-28 \quad \ldots\ldots \text{equ}(\boldsymbol{d})$

From equ$(\boldsymbol{a})$

$A=-C \quad \ldots\ldots \text{equ}(\boldsymbol{e})$

Put $\boldsymbol{A}=-\boldsymbol{C}$ in equ $(\boldsymbol{c})$

$2(-C)-3C=0$

$-2C-3C=0$

$-5C=0$

$C=\frac{0}{-5}$

$C=0$

Put $C=0$ in equ $(\boldsymbol{e})$

$A=-(0)$

$A=0$

Exercise # 4.2

From $equ(\boldsymbol{b})$

$B = 4 - D \quad \ldots..equ(\boldsymbol{f})$

Put $B = 4 - D$ in equ $(\boldsymbol{d})$

$2(4 - D) - 3D = -28$

$8 - 2D - 3D = -28$

$-5D = -28 - 8$

$-5D = -36$

$5D = 36$

$D = \frac{36}{5}$

Put $D = \frac{36}{5}$ in equ $(\boldsymbol{f})$

$B = 4 - \frac{36}{5}$

$B = \frac{20 - 36}{5}$

$B = \frac{-16}{5}$

Put the values of A, B , C and D in equ (i)

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{(0)x + \left(\frac{-16}{5}\right)}{x^2 - 3} + \frac{(0)x + \frac{36}{5}}{x^2 + 2}$$

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{\frac{-16}{5}}{x^2 - 3} + \frac{\frac{36}{5}}{x^2 + 2}$$

$$\frac{4x^2 - 28}{(x^2 - 3)(x^2 + 2)} = \frac{-16}{5(x^2 - 3)} + \frac{36}{5(x^2 + 2)}$$

Example 7: Find partial fraction of $\boldsymbol{\frac{1}{(x - 1)(x^2 + 1)^2}}$

Solution:

$$\frac{1}{(x - 1)(x^2 + 1)^2}$$

Let

$$\frac{1}{(x - 1)(x^2 + 1)^2} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \quad \ldots..equ(i)$$

Multiply equ (i) by $(x - 1)(x^2 + 1)^2$

$$\frac{1}{(x - 1)(x^2 + 1)^2} \times (x - 1)(x^2 + 1)^2 = \frac{A}{x - 1} \times (x - 1)(x^2 + 1)^2 + \frac{Bx + C}{x^2 + 1} \times (x - 1)(x^2 + 1)^2 + \frac{Dx + E}{(x^2 + 1)^2} \times (x - 1)(x^2 + 1)^2$$

$1 = A(x^2 + 1)^2 + (Bx + C)(x - 1)(x^2 + 1) + (Dx + E)(x - 1) \quad \ldots..equ(ii)$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 = A((1)^2 + 1)^2 + (Bx + C)(0)(x^2 + 1) + (Dx + E)(0)$

$1 = A(1 + 1)^2 + 0 + 0$

$1 = A(2)^2$

$1 = A(4)$

Exercise # 4.2

$\frac{1}{4} = A$

$A = \frac{1}{4}$

equ (ii) $\Rightarrow$

$1 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)$

$1 = A(x^4+2x^2+1) + (Bx+C)(x^3+x-x^2-1) + Dx^2 - Dx + Ex - E$

$1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 - Bx^3 - Bx + Cx^3 + Cx - Cx^2 - C + Dx^2 - Dx + Ex - E$

$1 = Ax^4 + Bx^4 - Bx^3 + Cx^3 + 2Ax^2 + Bx^2 - Cx^2 + Dx^2 - Bx + Cx - Dx + Ex + A - C - E$

$1 = (A+B)x^4 + (-B+C)x^3 + (2A+B-C+D)x^2 + (-B+C-D+E)x + (A-C-E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A + B = 0$ ......equ($\boldsymbol{a}$)

$-\mathrm{B} + \mathrm{C} = 0$ ......equ($\boldsymbol{b}$)

$2\mathrm{A} + \mathrm{B} - \mathrm{C} + \mathrm{D} = 0$ ......equ($\boldsymbol{c}$)

$-\mathrm{B} + \mathrm{C} - \mathrm{D} + \mathrm{E} = 0$ ......equ($\boldsymbol{d}$)

$\mathrm{A} - \mathrm{C} - \mathrm{E} = 1$ ......equ($\boldsymbol{e}$)

Put $A = \frac{1}{4}$ in equ ($\boldsymbol{a}$)

$\frac{1}{4} + B = 0$

$B = -\frac{1}{4}$

Put $B = -\frac{1}{4}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{1}{4}\right) + \mathrm{C} = 0$

$\frac{1}{4} + C = 0$

$C = -\frac{1}{4}$

Put the values of A, B *and* C *in equ* (c)

$2\left(\frac{1}{4}\right) + \left(-\frac{1}{4}\right) - \left(-\frac{1}{4}\right) + \mathrm{D} = 0$

$\frac{2}{4} - \frac{1}{4} + \frac{1}{4} + \mathrm{D} = 0$

$\frac{1}{2} + \mathrm{D} = 0$

$\mathrm{D} = -\frac{1}{2}$

Put the values of A *and* C *in equ* ($\boldsymbol{e}$)

$\mathrm{A} - \mathrm{C} - \mathrm{E} = 1$

$\frac{1}{4} - \left(-\frac{1}{4}\right) - \mathrm{E} = 1$

$\frac{1}{4} + \frac{1}{4} - \mathrm{E} = 1$

$\frac{1+1}{4} = 1 + \mathrm{E}$

Exercise # 4.2

$\frac{2}{4} - 1 = \text{E}$

$\frac{1}{2} - 1 = \text{E}$

$\frac{1-2}{2} = \text{E}$

$\frac{-1}{2} = \text{E}$

$\text{E} = \frac{-1}{2}$

Put the values of A, B , C , D and E in equ (i)

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{\frac{1}{4}}{x-1} + \frac{-\frac{1}{4}x + \left(-\frac{1}{4}\right)}{x^2+1} + \frac{-\frac{1}{2}x + \left(-\frac{1}{2}\right)}{(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} + \frac{\frac{-x-1}{4}}{x^2+1} + \frac{\frac{-x-1}{2}}{(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} + \frac{-(x+1)}{4(x^2+1)} + \frac{-(x+1)}{2(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2} = \frac{1}{4(x-1)} - \frac{x+1}{4(x^2+1)} - \frac{x+1}{2(x^2+1)^2}$$

Exercise # 4.2
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Resolve the following fractions into partial fraction.

(1) $\frac{1}{x(x^2+1)}$

Solution:

Let

$\frac{1}{x(x^2+1)} = \frac{\text{A}}{x} + \frac{\text{B}x + \text{C}}{x^2+1}$ .....equ(i)

Multiply equ (i) by $x(x^2+1)$

$\frac{1}{x(x^2+1)} \times x(x^2+1) = \frac{\text{A}}{x} \times x(x^2+1) + \frac{Bx + \text{C}}{x^2+1} \times x(x^2+1)$

$1 = A(x^2+1) + (Bx+C)x$ .....equ(ii)

Put $x = 0$ in equ (ii)

$1 = A((0)^2+1) + (B(0)+C)(0)$

$1 = A(0+1) + 0$

$1 = A(1)$

$1 = A$

$\text{A} = 1$

equ (ii) ⇒

$1 = A(x^2+1) + (Bx+C)x$

$1 = Ax^2 + A + Bx^2 + Cx$

$1 = Ax^2 + Bx^2 + Cx + A$

$1 = (A+B)x^2 + Cx + A$

By comparing the coefficients of x^2, x *and constant* we get

Exercise # 4.2

$A + B = 0$ .....equ($\boldsymbol{a}$)
$C = 0$ .....equ($\boldsymbol{b}$)
$A = 1$ .....equ($\boldsymbol{c}$)
Put $A = 1$ in equ ($\boldsymbol{a}$)
$1 + B = 0$
$B = -1$
Put the values of A, B and C in equ (i)

$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-1x+0}{x^2+1}$$
$$\frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$
$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

(2) $\boldsymbol{\dfrac{x^2+3x+1}{(x-1)(x^2+3)}}$

Solution:

$$\frac{x^2+3x+1}{(x-1)(x^2+3)}$$

Let

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3} \quad \text{.....equ(i)}$$

Multiply equ (i) by $(x-1)(x^2+3)$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} \times (x-1)(x^2+3) = \frac{A}{x-1} \times (x-1)(x^2+3) + \frac{Bx+C}{x^2+3} \times (x-1)(x^2+3)$$

$x^2 + 3x + 1 = A(x^2+3) + (Bx+C)(x-1)$ .....equ(ii)
Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)
$(1)^2 + 3(1) + 1 = A((1)^2+3) + (B(1)+C)(0)$
$1 + 3 + 1 = A(1+3) + 0$
$5 = A(4)$
$5 = 4A$
$\frac{5}{4} = A$
$A = \frac{5}{4}$
equ (ii) $\Rightarrow$
$x^2 + 3x + 1 = A(x^2+3) + (Bx+C)(x-1)$
$x^2 + 3x + 1 = Ax^2 + 3A + Bx^2 - Bx + Cx - C$
$x^2 + 3x + 1 = Ax^2 + Bx^2 - Bx + Cx + 3A - C$
$x^2 + 3x + 1 = (A+B)x^2 + (-B+C)x + (3A-C)$
Compare the coefficients of x^2, x *and constant* we get
$A + B = 1$ .....equ($\boldsymbol{a}$)
$-B + C = 3$ .....equ($\boldsymbol{b}$)
$3A - C = 1$ .....equ($\boldsymbol{c}$)

Exercise # 4.2

Put $A = \frac{5}{4}$ in equ $(\boldsymbol{a})$

$\frac{5}{4} + B = 1$

$B = 1 - \frac{5}{4}$

$B = \frac{4-5}{4}$

$B = \frac{-1}{4}$

Put $B = \frac{-1}{4}$ in equ $(\boldsymbol{b})$

$-\left(\frac{-1}{4}\right) + \mathrm{C} = 3$

$\frac{1}{4} + \mathrm{C} = 3$

$C = 3 + \frac{-1}{4}$

$C = 3 - \frac{1}{4}$

$C = \frac{12-1}{4}$

$C = \frac{11}{4}$

Put the values of A, B and C in equ (i)

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{\frac{5}{4}}{x-1} + \frac{\frac{-1}{4}x + \frac{11}{4}}{x^2+3}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{\frac{5}{4}}{x-1} + \frac{\frac{-1x+11}{4}}{x^2+3}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} + \frac{-1x+11}{4(x^2+3)}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} + \frac{-(1x-11)}{4(x^2+3)}$$

$$\frac{x^2+3x+1}{(x-1)(x^2+3)} = \frac{5}{4(x-1)} - \frac{x-11}{4(x^2+3)}$$

(3) $\boldsymbol{\frac{2x+1}{(x^2+1)(x-1)}}$

<u>Solution</u>:

$\frac{2x+1}{(x^2+1)(x-1)}$

Let

$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{\mathrm{Ax+B}}{x^2+1} + \frac{\mathrm{C}}{x-1} \quad \ldots\ldots \mathrm{equ(i)}$$

Multiply equ (i) by $(x^2+1)(x-1)$

Exercise # 4.2

$$\frac{2x+1}{(x^2+1)(x-1)} \times (x^2+1)(x-1) = \frac{\text{Ax}+\text{B}}{x^2+1} \times (x^2+1)(x-1) + \frac{C}{x-1} \times (x^2+1)(x-1)$$

$2x+1 = (Ax+B)(x-1) + C(x^2+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$2x+1 = (Ax+B)(x-1) + C(x^2+1)$

$2(1)+1 = (A(1)+B)(0) + C((1)^2+1)$

$2+1 = 0 + C(1+1)$

$3 = C(2)$

$3 = 2C$

$\frac{3}{2} = C$

$\text{C} = \frac{3}{2}$

equ (ii) $\Rightarrow$

$2x+1 = (Ax+B)(x-1) + C(x^2+1)$

$2x+1 = Ax^2 - Ax + Bx - B + Cx^2 + C$

$2x+1 = Ax^2 + Cx^2 - Ax + Bx - B + C$

$2x+1 = (A+C)x^2 + (-A+B)x + (-B+C)$

Compare the coefficients of $x^2, x \text{ and constant}$ we get

$A + C = 0 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$

$-\text{A} + \text{B} = 2 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$

$-\text{B} + \text{C} = 1 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$

Put $\text{C} = \frac{3}{2}$ in equ $(\boldsymbol{a})$

$A + \frac{3}{2} = 0$

$A = -\frac{3}{2}$

Put $A = -\frac{3}{2}$ in equ $(\boldsymbol{b})$

$-\left(-\frac{3}{2}\right) + \text{B} = 2$

$\frac{3}{2} + \text{B} = 2$

$B = 2 - \frac{3}{2}$

$B = \frac{4-3}{2}$

$B = \frac{1}{2}$

Put the values of A, B and C in equ (i)

$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{\frac{-3}{2}x + \frac{1}{2}}{x^2+1} + \frac{\frac{3}{2}}{x-1}$$

$$\frac{2x+1}{(x^2+1)(x-1)} = \frac{\frac{-3x+1}{2}}{x^2+1} + \frac{\frac{3}{2}}{x-1}$$

Exercise # 4.2

$$\frac{2x+1}{(x^2+1)(x-1)}=\frac{-3x+1}{2(x^2+1)}+\frac{3}{2(x-1)}$$
$$\frac{2x+1}{(x^2+1)(x-1)}=\frac{-(3x-1)}{2(x^2+1)}+\frac{3}{2(x-1)}$$
$$\frac{2x+1}{(x^2+1)(x-1)}=-\frac{3x-1}{2(x^2+1)}+\frac{3}{2(x-1)}$$

(4) $\frac{-3}{x^2(x^2+5)}$

Solution:

$$\frac{-3}{x^2(x^2+5)}$$

Let

$$\frac{-3}{x^2(x^2+5)}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{x^2+5}\quad .....\text{equ(i)}$$

Multiply equ (i) by $x^2(x^2+5)$

$$\frac{-3}{x^2(x^2+5)}\times x^2(x^2+5)=\frac{A}{x}\times x^2(x^2+5)+\frac{B}{x^2}\times x^2(x^2+5)+\frac{Cx+D}{x^2+5}\times x^2(x^2+5)$$

$-3=Ax(x^2+5)+B(x^2+5)+(Cx+D)x^2\quad .....\text{equ(ii)}$

Put $x=0$ in equ (ii)

$-3=A(0)(x^2+5)+B((0)^2+5)+(Cx+D)(0)^2$

$-3=A(0)+B(0+5)+(Cx+D)(0)$

$-3=0+B(5)+0$

$-3=5B$

$\frac{-3}{5}=B$

$B=\frac{-3}{5}$

equ (ii) $\Rightarrow$

$-3=Ax(x^2+5)+B(x^2+5)+(Cx+D)x^2$

$-3=Ax^3+5Ax+Bx^2+5B+Cx^3+Dx^2$

$-3=Ax^3+Cx^3+Bx^2+Dx^2+5Ax+5B$

$-3=(A+C)x^3+(B+D)x^2+5Ax+5B$

By comparing the coefficients of x^3, x^2, x and constant we get

$A+C=0\quad .....\text{equ}(a)$

$B+D=0\quad .....\text{equ}(b)$

$5A=0\quad .....\text{equ}(c)$

$5B=-3\quad .....\text{equ}(d)$

From equ(c)

$A=\frac{0}{5}$

$A=0$

$Put\ A=0\ in\ equ\ (a)$

$0+C=0$

$C=0$

Exercise # 4.2

$Put\ B = \frac{-3}{5}\ in\ equ\ (b)$

$\frac{-3}{5} + D = 0$

$D = \frac{3}{5}$

Put the values of A, B , C and D in equ (i)

$$\frac{-3}{x^2(x^2+5)} = \frac{0}{x} + \frac{\frac{-3}{5}}{x^2} + \frac{0x + \frac{3}{5}}{x^2+5}$$

$$\frac{-3}{x^2(x^2+5)} = 0 + \frac{-3}{5x^2} + \frac{3}{5(x^2+5)}$$

$$\frac{-3}{x^2(x^2+5)} = \frac{-3}{5x^2} + \frac{3}{5(x^2+5)}$$

(5) $\frac{\mathbf{3x-2}}{\mathbf{(x+4)(3x^2+1)}}$

Solution:

$$\frac{3x-2}{(x+4)(3x^2+1)}$$

Let

$$\frac{3x-2}{(x+4)(3x^2+1)} = \frac{A}{x+4} + \frac{Bx+C}{3x^2+1} \quad .....equ(i)$$

Multiply equ (i) by $(x+4)(3x^2+1)$

$$\frac{3x-2}{(x+4)(3x^2+1)} \times (x+4)(3x^2+1) = \frac{A}{x+4} \times (x+4)(3x^2+1) + \frac{Bx+C}{3x^2+1} \times (x+4)(3x^2+1)$$

$3x - 2 = A(3x^2+1) + (Bx+C)(x+4) \quad .....equ(ii)$

Put $x + 4 = 0 \Rightarrow x = -4$ in equ (ii)

$3(-4) - 2 = A(3(-4)^2+1) + (B(-4)+C)(0)$

$-12 - 2 = A(3(16)+1) + 0$

$-14 = A(48+1)$

$-14 = A(49)$

$\frac{-14}{49} = A$

$\frac{-2}{7} = A$

$A = \frac{-2}{7}$

equ (ii) $\Rightarrow$

$3x - 2 = A(3x^2+1) + (Bx+C)(x+4)$

$3x - 2 = 3Ax^2 + A + Bx^2 + 4Bx + Cx + 4C$

$3x - 2 = 3Ax^2 + Bx^2 + 4Bx + Cx + A + 4C$

$3x - 2 = (3A+B)x^2 + (4B+C)x + (A+4C)$

Compare the coefficients of $x^2, x\ and\ constant$ we get

$3A + B = 0 \quad .....equ(\boldsymbol{a})$

$4B + C = 3 \quad .....equ(\boldsymbol{b})$

$A + 4C = -2 \quad .....equ(\boldsymbol{c})$

Exercise # 4.2

Put $A = \frac{-2}{7}$ in equ $(\boldsymbol{a})$

$$3\left(\frac{-2}{7}\right) + B = 0$$

$$\frac{-6}{7} + B = 0$$

$$B = \frac{6}{7}$$

Put $B = \frac{6}{7}$ in equ $(\boldsymbol{b})$

$$4\left(\frac{6}{7}\right) + \mathrm{C} = 3$$

$$\frac{24}{7} + \mathrm{C} = 3$$

$$C = 3 - \frac{24}{7}$$

$$C = \frac{21 - 24}{7}$$

$$C = \frac{-3}{7}$$

Put the values of A, B and C in equ (i)

$$\frac{3x-2}{(x+4)(3x^2+1)} = \frac{\frac{-2}{7}}{x+4} + \frac{\frac{6}{7}x + \frac{-3}{7}}{3x^2+1}$$

$$\frac{3x-2}{(x+4)(3x^2+1)} = \frac{\frac{-2}{7}}{x+4} + \frac{\frac{6x-3}{7}}{3x^2+1}$$

$$\frac{3x-2}{(x+4)(3x^2+1)} = \frac{-2}{7(x+4)} + \frac{6x-3}{7(3x^2+1)}$$

(6) $\boldsymbol{\frac{5x}{(x+1)(x^2-2)^2}}$

Solution:

$$\frac{5x}{(x+1)(x^2-2)^2}$$

Let

$$\frac{5x}{(x+1)(x^2-2)^2} = \frac{\mathrm{A}}{x+1} + \frac{\mathrm{B}x + \mathrm{C}}{x^2-2} + \frac{Dx + \mathrm{E}}{(x^2-2)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x^2-2)^2$

$$\frac{5x}{(x+1)(x^2-2)^2} \times (x+1)(x^2-2)^2 = \frac{\mathrm{A}}{x+1} \times (x+1)(x^2-2)^2 + \frac{\mathrm{B}x + \mathrm{C}}{x^2-2} \times (x+1)(x^2-2)^2 + \frac{Dx + \mathrm{E}}{(x^2-2)^2} \times (x+1)(x^2-2)^2$$

$$5x = A(x^2-2)^2 + (Bx + C)(x+1)(x^2-2) + (Dx + E)(x+1) \quad \ldots\ldots \text{equ(ii)}$$

Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

$$5(-1) = A((-1)^2 - 2)^2 + (B(-1) + C)(0)(x^2-2) + (Dx + E)(0)$$

$$-5 = A(1-2)^2 + 0 + 0$$

$$-5 = A(-1)^2$$

Exercise # 4.2

$-5 = A(1)$
$-5 = A$
$A = -5$
equ (ii) $\Rightarrow$
$5x = A(x^2-2)^2 + (Bx+C)(x+1)(x^2-2) + (Dx+E)(x+1)$
$5x = A(x^4-4x^2+4) + (Bx+C)(x^3-2x+x^2-2) + Dx^2 + Dx + Ex + E$
$5x = Ax^4 - 4Ax^2 + 4A + Bx^4 - 2Bx^2 + Bx^3 - 2Bx + Cx^3 - 2Cx + Cx^2 - 2C + Dx^2 + Dx + Ex + E$
$5x = Ax^4 + Bx^4 + Bx^3 + Cx^3 - 4Ax^2 - 2Bx^2 + Cx^2 + Dx^2 - 2Bx - 2Cx + Dx + Ex + 4A - 2C + E$
$5x = (A+B)x^4 + (B+C)x^3 + (-4A-2B+C+D)x^2 + (-2B-2C+D+E)x + (4A-2C+E)$
Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get
$A + B = 0$ .....equ(***a***)
$B + C = 0$ .....equ(***b***)
$-4A - 2B + C + D = 0$ .....equ(***c***)
$-2B - 2C + D + E = 5$ .....equ(***d***)
$4A - 2C + E = 0$ .....equ(***e***)
Put $A = -5$ in equ (***a***)
$-5 + B = 0$
$B = 5$
Put $B = 5$ in equ (***b***)
$5 + C = 0$
$C = -5$
Put the values of A, B and C in equ (c)
$-4(-5) - 2(5) + (-5) + D = 0$
$20 - 10 - 5 + D = 0$
$10 - 5 + D = 0$
$5 + D = 0$
$D = -5$
Put the values of A and C in equ (e)
$4(-5) - 2(-5) + E = 0$
$-20 + 10 + E = 0$
$-10 + E = 0$
$E = 10$
Put the values of A, B , C , D and E in equ (i)

$$\frac{5x}{(x+1)(x^2-2)^2} = \frac{-5}{x+1} + \frac{5x+(-5)}{x^2-2} + \frac{-5x+10}{(x^2-2)^2}$$

$$\frac{5x}{(x+1)(x^2-2)^2} = \frac{-5}{x+1} + \frac{5x-5}{x^2-2} + \frac{-5x+10}{(x^2-2)^2}$$

(7) $\dfrac{\mathbf{5x^2 - 4x + 8}}{\mathbf{(x^2+1)^2(x-2)}}$

Solution:

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}$$

Let

Exercise # 4.2

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{Ax+B}{x^2+1}+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}+\frac{\mathrm{E}}{x-2}\quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(x-2)$

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}\times(x^2+1)^2(x-2)=\frac{Ax+B}{x^2+1}\times(x^2+1)^2(x-2)+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}\times(x^2+1)^2(x-2)+\frac{\mathrm{E}}{x-2}\times(x^2+1)^2(x-2)$$

$5x^2-4x+8=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)+E(x^2+1)^2\quad \ldots..\text{equ(ii)}$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$5(2)^2-4(2)+8=(A(2)+B)(x^2+1)(0)+(C(2)+D)(0)+E((2)^2+1)^2$

$5(4)-8+8=0+0+E(4+1)^2$

$20=E(5)^2$

$20=E(25)$

$\frac{20}{25}=E$

$\frac{4}{5}=E$

$E=\frac{4}{5}$

equ (ii) $\Rightarrow$

$5x^2-4x+8=(Ax+B)(x^2+1)(x-2)+(Cx+D)(x-2)+E(x^2+1)^2$

$5x^2-4x+8=(Ax+B)(x^3-2x^2+x-2)+Cx^2-2Cx+Dx-2D+E(x^4+2x^2+1)$

$5x^2-4x+8=Ax^4-2Ax^3+Ax^2-2Ax+Bx^3-2Bx^2+Bx-2B+Cx^2-2Cx+Dx-2D+Ex^4+2Ex^2+E$

$5x^2-4x+8=Ax^4+Ex^4-2Ax^3+Bx^3+Ax^2-2Bx^2+Cx^2+2Ex^2-2Ax+Bx-2Cx+Dx-2B-2D+E$

$5x^2-4x+8=(A+E)x^4+(-2A+B)x^3+(A-2B+C+2E)x^2+(-2A+B-2C+D)x+(-2B-2D+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A+E=0\quad \ldots..\text{equ}(\boldsymbol{a})$

$-2\mathrm{A}+\mathrm{B}=0\quad \ldots..\text{equ}(\boldsymbol{b})$

$\mathrm{A}-2\mathrm{B}+\mathrm{C}+2\mathrm{E}=5\quad \ldots..\text{equ}(\boldsymbol{c})$

$-2\mathrm{A}+\mathrm{B}-2\mathrm{C}+\mathrm{D}=-4\quad \ldots..\text{equ}(\boldsymbol{d})$

$-2\mathrm{B}-2\mathrm{D}+\mathrm{E}=8\quad \ldots..\text{equ}(\boldsymbol{e})$

Put $E=\frac{4}{5}$ in equ $(\boldsymbol{a})$

$A+\frac{4}{5}=0$

$A=-\frac{4}{5}$

Put $A=-\frac{4}{5}$ in equ $(\boldsymbol{b})$

$-2\left(-\frac{4}{5}\right)+\mathrm{B}=0$

$\frac{8}{5}+B=0$

$B=-\frac{8}{5}$

Put the values of A, B *and* C *in equ* (c)

Exercise # 4.2

$$-\frac{4}{5}-2\left(-\frac{8}{5}\right)+\mathrm{C}+2\left(\frac{4}{5}\right)=5$$
$$-\frac{4}{5}+\frac{16}{5}+\mathrm{C}+\frac{8}{5}=5$$
$$-\frac{4}{5}+\frac{16}{5}+\frac{8}{5}+\mathrm{C}=5$$
$$\frac{-4+16+8}{5}+\mathrm{C}=5$$
$$\frac{-4+16+8}{5}+\mathrm{C}=5$$
$$\frac{20}{5}+\mathrm{C}=5$$
$$4+\mathrm{C}=5$$
$$\mathrm{C}=5-4$$
$$\mathrm{C}=1$$

Put the values of A, B and E in equ (d)

$$-2\left(-\frac{4}{5}\right)+\left(-\frac{8}{5}\right)-2(1)+D=-4$$
$$\frac{8}{5}-\frac{8}{5}-2+D=-4$$
$$-2+D=-4$$
$$D=-4+2$$
$$D=-2$$

Put the values of A, B , C , D and E in equ (i)

$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{-\frac{4}{5}x+\frac{-8}{5}}{x^2+1}+\frac{1x+(-2)}{(x^2+1)^2}+\frac{\frac{4}{5}}{x-2}$$
$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{\frac{-4x-8}{5}}{x^2+1}+\frac{x-2}{(x^2+1)^2}+\frac{\frac{4}{5}}{x-2}$$
$$\frac{5x^2-4x+8}{(x^2+1)^2(x-2)}=\frac{-4x-8}{5(x^2+1)}+\frac{x-2}{(x^2+1)^2}+\frac{4}{5(x-2)}$$

Important

(8) $\frac{\mathbf{4x-5}}{\mathbf{(x^2+4)^2}}$

Solution:

$$\frac{4x-5}{(x^2+4)^2}$$

Let

$$\frac{4x-5}{(x^2+4)^2}=\frac{Ax+B}{x^2+4}+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+4)^2}\quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x^2+4)^2$

$$\frac{4x-5}{(x^2+4)^2}\times(x^2+4)^2=\frac{Ax+B}{x^2+4}\times(x^2+4)^2+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+4)^2}\times(x^2+4)^2$$
$$4x-5=(Ax+B)(x^2+4)+Cx+D\quad \ldots\ldots \text{equ(ii)}$$

equ (ii) ⇒

Exercise # 4.2

$4x-5 = Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$

$4x-5 = Ax^3 + Bx^2 + 4Ax + Cx + 4B + D$

$4x-5 = Ax^3 + Bx^2 + (4A + C)x + (4B + D)$

Compare the coefficients of x^3 , x^2 , x *and constant* we get

$A = 0$ …..equ($\boldsymbol{a}$)

$B = 0$ …..equ($\boldsymbol{b}$)

$4A + C = 4$ …..equ($\boldsymbol{c}$)

$4B + D = -5$ …..equ($\boldsymbol{d}$)

Put $A = 0$ in equ ($\boldsymbol{c}$)

$4(0) + C = 4$

$C = 4$

Put $B = 0$ in equ ($\boldsymbol{d}$)

$4(0) + D = -5$

$D = -5$

Put the values of A, B , C and D in equ (i)

$$\frac{4x-5}{(x^2+4)^2} = \frac{(0)x+0}{x^2+4} + \frac{4x+(-5)}{(x^2+4)^2}$$

$$\frac{4x-5}{(x^2+4)^2} = 0 + \frac{4x-5}{(x^2+4)^2}$$

$$\frac{4x-5}{(x^2+4)^2} = \frac{4x-5}{(x^2+4)^2}$$

(9) $\boldsymbol{\dfrac{8x^2}{(x^2+1)(1-x^4)}}$

<u>Solution</u>:

$$\frac{8x^2}{(x^2+1)(1-x^4)} = \frac{8x^2}{(x^2+1)(1+x^2)(1-x^2)}$$

$$\frac{8x^2}{(x^2+1)(1-x^4)} = \frac{8x^2}{(x^2+1)^2(1+x)(1-x)}$$

Let

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{1+x} + \frac{F}{1-x} \quad \text{…..equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(1+x)(1-x)$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)} \times (x^2+1)^2(1+x)(1-x) = \frac{Ax+B}{x^2+1} \times (x^2+1)^2(1+x)(1-x) + \frac{Cx+D}{(x^2+1)^2} \times (x^2+1)^2(1+x)(1-x) + \frac{E}{1+x} \times (x^2+1)^2(1+x)(1-x) + \frac{F}{1-x} \times (x^2+1)^2(1+x)(1-x)$$

$8x^2 = (Ax+B)(x^2+1)(1+x)(1-x) + (Cx+D)(1+x)(1-x) + E(x^2+1)^2(1-x) + F(x^2+1)^2(1+x)$

Put $1 + x = 0 \Rightarrow x = -1$ in above equation

$8(-1)^2 = (Ax+B)(x^2+1)(0)(1-x) + (Cx+D)(0)(1-x) + E((-1)^2+1)^2(1-(-1)) + F(x^2+1)^2(0)$

Exercise # 4.2

$8(1) = 0 + 0 + E(1+1)^2(1+1) + 0$
$8 = E(2)^2(2)$
$8 = E(4)(2)$
$8 = E(8)$
$\frac{8}{8} = E$
$1 = E$
$E = 1$
Put $1 - x = 0 \Rightarrow -x = -1 \;\Rightarrow x = 1$ in equ (ii)
$8(1)^2 = (Ax+B)(x^2+1)(1+x)(0) + (Cx+D)(1+x)(0) + +E(x^2+1)^2(0) + F((1)^2+1)^2(1+1)$
$8(1) = 0 + 0 + 0 + F(1+1)^2(1+1)$
$8 = F(2)^2(2)$
$8 = F(4)(2)$
$8 = F(8)$
$\frac{8}{8} = F$
$1 = F$
$F = 1$
equ (ii) $\Rightarrow$
$8x^2 = (Ax+B)(x^2+1)(1+x)(1-x) + (Cx+D)(1+x)(1-x) + E(x^2+1)^2(1-x) + F(x^2+1)^2(1+x)$
$8x^2 = (Ax+B)(x^2+1)(1-x^2) + (Cx+D)(1-x^2) + E(x^4+2x^2+1)(1-x) + F(x^4+2x^2+1)(1+x)$
$8x^2 = (Ax+B)(1-x^4) + Cx - Cx^3 + D - Dx^2 + E(x^4 - x^5 + 2x^2 - 2x^3 + 1 - x) + F(x^4 + x^5 + 2x^2 + 2x^3 + 1 + x)$
$8x^2 = Ax - Ax^5 + B - Bx^4 + Cx - Cx^3 + D - Dx^2 + Ex^4 - Ex^5 + 2Ex^2 - 2Ex^3 + E - Ex + Fx^4 + Fx^5 + 2Fx^2 + 2Fx^3 + F + Fx$
$8x^2 = -Ax^5 - Ex^5 + Fx^5 - Bx^4 + Ex^4 + Fx^4 - Cx^3 - 2Ex^3 + 2Fx^3 - Dx^2 + 2Ex^2 + 2Fx^2 + Ax + Cx - Ex + Fx + B + D + E + F$
$8x^2 = (-A-E+F)x^5 + (-B+E+F)x^4 + (-C-2E+2F)x^3 + (-D+2E+2F)x^2 + (A+C-E+F)x + (B+D+E+F)$
Compare the coefficients of $x^5\ ,x^4\ ,x^3\ ,x^2\ ,x$ *and constant* we get
$-A - E + F = 0$ …..equ($\boldsymbol{a}$)
$-B + E + F = 0$ …..equ($\boldsymbol{b}$)
$-C - 2E + 2F = 0$ …..equ($\boldsymbol{c}$)
$-D + 2E + 2F = 8$ …..equ($\boldsymbol{d}$)
$A + C - E + F = 0$ …..equ($\boldsymbol{e}$)
$B + D + E + F = 0$ …..equ($\boldsymbol{f}$)
Put the values of E *and* F in equ ($\boldsymbol{a}$)
$-A - 1 + 1 = 0$
$-A = 0$
$A = 0$
Put the values of E *and* F in equ ($\boldsymbol{b}$)
$-B + 1 + 1 = 0$
$-B + 2 = 0$
$-B = -2$
$B = 2$
Put the values of E *and* F in equ ($\boldsymbol{c}$)
$-C - 2(1) + 2(1) = 0$
$-C - 2 + 2 = 0$
$-C = 0$
$C = 0$

Exercise # 4.2

Put the values of E and F in equ $(\boldsymbol{d})$

$-\mathrm{D}+2(1)+2(1)=8$

$-\mathrm{D}+2+2=8$

$-\mathrm{D}+4=8$

$-\mathrm{D}=8-4$

$-\mathrm{D}=4$

$\mathrm{D}=-4$

Put the values of A, B , C , D , E and F in equ (i)

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{0x+2}{x^2+1}+\frac{0x+(-4)}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{2}{x^2+1}+\frac{-4}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

$$\frac{8x^2}{(x^2+1)^2(1+x)(1-x)}=\frac{2}{x^2+1}-\frac{4}{(x^2+1)^2}+\frac{1}{1+x}+\frac{1}{1-x}$$

(10) $\boldsymbol{\dfrac{2x^2+4}{(x^2+1)^2(x-1)}}$

<u>Solution</u>:

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}$$

Let

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}+\frac{\mathrm{E}}{x-1} \quad \ldots..\text{equ(i)}$$

Multiply equ (i) by $(x^2+1)^2(x-1)$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}\times(x^2+1)^2(x-1)=\frac{Ax+B}{x^2+1}\times(x^2+1)^2(x-1)+\frac{\mathrm{C}x+\mathrm{D}}{(x^2+1)^2}\times(x^2+1)^2(x-1)+\frac{\mathrm{E}}{x-1}\times(x^2+1)^2(x-1)$$

$2x^2+4=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2 \quad \ldots..\text{equ(ii)}$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$2(1)^2+4=(A(1)+B)(x^2+1)(0)+(C(1)+D)(0)+E((1)^2+1)^2$

$2(1)+4=0+0+E(1+1)^2$

$2+4=E(2)^2$

$6=E(4)$

$\frac{6}{4}=E$

$\frac{3}{2}=E$

$E=\frac{3}{2}$

equ (ii) $\Rightarrow$

$2x^2+4=(Ax+B)(x^2+1)(x-1)+(Cx+D)(x-1)+E(x^2+1)^2$

$2x^2+4=(Ax+B)(x^3-x^2+x-1)+Cx^2-Cx+Dx-D+E(x^4+2x^2+1)$

$2x^2+4=Ax^4-Ax^3+Ax^2-Ax+Bx^3-Bx^2+Bx-B+Cx^2-Cx+Dx-D+Ex^4+2Ex^2+E$

Exercise # 4.2

$2x^2+4=Ax^4+Ex^4-Ax^3+Bx^3+Ax^2-Bx^2+Cx^2+2Ex^2-Ax+Bx-Cx+Dx-B-D+E$

$2x^2+4=(A+E)x^4+(-A+B)x^3+(A-B+C+2E)x^2+(-A+B-C+D)x+(-B-D+E)$

Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get

$A+E=0$ ......equ($\boldsymbol{a}$)

$-A+B=0$ ......equ($\boldsymbol{b}$)

$A-B+C+2E=2$ ......equ($\boldsymbol{c}$)

$-A+B-C+D=0$ ......equ($\boldsymbol{d}$)

$-B-D+E=8$ ......equ($\boldsymbol{e}$)

Put $E=\frac{3}{2}$ in equ ($\boldsymbol{a}$)

$A+\frac{3}{2}=0$

$A=-\frac{3}{2}$

Put $A=-\frac{3}{2}$ in equ ($\boldsymbol{b}$)

$-\left(-\frac{3}{2}\right)+B=0$

$\frac{3}{2}+B=0$

$B=-\frac{3}{2}$

Put the values of A,B *and* E *in equ* ($\boldsymbol{c}$)

$-\frac{3}{2}-\left(-\frac{3}{2}\right)+C+2\left(\frac{3}{2}\right)=2$

$-\frac{3}{2}+\frac{3}{2}+C+3=2$

$0+C=2-3$

$C=-1$

Put the values of A,B *and* C *in equ* ($\boldsymbol{d}$)

$-A+B-C+D=0$

$-\left(-\frac{3}{2}\right)+\left(-\frac{3}{2}\right)-(-1)+D=0$

$\frac{3}{2}-\frac{3}{2}+1+D=0$

$0+1+D=0$

$1+D=0$

$D=-1$

Put the values of A, B , C , D and E in equ (i)

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{E}{x-1}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{-\frac{3}{2}x+\left(-\frac{3}{2}\right)}{x^2+1}+\frac{-1x+(-1)}{(x^2+1)^2}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{\frac{-3x-3}{2}}{x^2+1}+\frac{-x-1}{(x^2+1)^2}+\frac{\frac{3}{2}}{x-1}$$

Exercise # 4.2

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{-3x-3}{2(x^2+1)}+\frac{-(x+1)}{(x^2+1)^2}+\frac{3}{2(x-1)}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=\frac{-(3x+3)}{2(x^2+1)}-\frac{x+1}{(x^2+1)^2}+\frac{3}{2(x-1)}$$

$$\frac{2x^2+4}{(x^2+1)^2(x-1)}=-\frac{3x+3}{2(x^2+1)}-\frac{x+1}{(x^2+1)^2}+\frac{3}{2(x-1)}$$

Review Exercise # 4

Q2: Resolve the following fractions into partial fraction.

(1) $\frac{2x^2}{(x+1)(x-1)}$

Solution:

$$\frac{2x^2}{(x+1)(x-1)}$$

As $\frac{2x^2}{(x+1)(x-1)}$ is improper

$$\frac{2x^2}{(x+1)(x-1)} = \frac{2x^2}{x^2-1}$$

So

$$\begin{array}{r|l} & 2 \\ \hline x^2-1 & 2x^2 \\ & \pm 2x^2 \mp 2 \\ \hline & 2 \end{array}$$

$$\frac{2x^2}{x^2-1} = 2 + \frac{2}{x^2-1}$$

$$\frac{2x^2}{x^2-1} = 2 + \frac{2}{(x+1)(x-1)} \quad \ldots\ldots \text{equ}(\mathbf{A})$$

Now

Let

$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)$

$$\frac{2}{(x+1)(x-1)} \times (x+1)(x-1) = \frac{A}{x+1} \times (x+1)(x-1) + \frac{B}{x-1} \times (x+1)(x-1)$$

$2 = A(x-1) + B(x+1) \quad \ldots\ldots \text{equ(ii)}$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$2 = A(-1-1) + B(0)$

$2 = A(-2) + 0$

$2 = -2A$

$\frac{2}{-2} = A$

$-1 = A$

$A = -1$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$2 = A(0) + B(1+1)$

$2 = 0 + B(2)$

$2 = 2B$

$\frac{2}{2} = B$

$1 = B$

$B = 1$

Put the values of A and B in equ (i)

$$\frac{2}{(x+1)(x-1)}=\frac{-1}{x+1}+\frac{1}{x-1}$$

Put the above in equ (**A**)

$$\frac{2x^2}{(x+1)(x-1)}=2+\frac{-1}{x+1}+\frac{1}{x-1}$$

$$\frac{2x^2}{(x+1)(x-1)}=2-\frac{1}{x+1}+\frac{1}{x-1}$$

(2) $\boldsymbol{\dfrac{2x^3-3x^2+9x+8}{x^2-3x+2}}$

<u>Solution</u>:

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2}$$

As $\dfrac{2x^3-3x^2+9x+8}{x^2-3x+2}$ is improper

So

$$\begin{array}{r|l} & 2x+3 \\ \hline x^2-3x+2 & 2x^3-3x^2+9x+8 \\ & \pm 2x^3 \mp 6x^2 \pm 4x \\ \hline & 3x^2+5x+8 \\ & \pm 3x^2 \mp 9x \pm 6 \\ \hline & 14x+2 \end{array}$$

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2}=2x+3+\frac{14x+2}{x^2-3x+2}$$

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2}=2x+3+\frac{14x+2}{(x-2)(x-1)} \quad \ldots\ldots \text{equ}(\mathbf{A})$$

R.W

$x^2-3x+2=x^2-2x-1x+2$

$x^2-3x+2=x(x-2)-1(x-2)$

$x^2-3x+2=(x-2)(x-2)$

Now

Let

$$\frac{14x+2}{(x-2)(x-1)}=\frac{A}{x-2}+\frac{B}{x-1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-2)(x-1)$

$$\frac{14x+2}{(x-2)(x-1)}\times(x-2)(x-1)=\frac{A}{x-2}\times(x-2)(x-1)+\frac{B}{x-1}\times(x-2)(x-1)$$

$14x+2=A(x-1)+B(x-2) \quad \ldots\ldots \text{equ(ii)}$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$14(2)+2=A(2-1)+B(0)$

$28+2=A(1)+0$

$30=A$

$A=30$

Put $x-1=0 \Rightarrow x=1$ in equ (ii)

$14(1)+2=A(0)+B(1-2)$

$14+2=0+B(-1)$

$16 = -\text{B}$

$-16 = \text{B}$

$\text{B} = -16$

Put the values of A and B in equ (i)

$$\frac{14x+2}{(x-2)(x-1)} = \frac{30}{x-2} + \frac{-16}{x-1}$$

$$\frac{14x+2}{(x-2)(x-1)} = \frac{30}{x-2} - \frac{16}{x-1}$$

Put the above in equ (**A**)

$$\frac{2x^3-3x^2+9x+8}{x^2-3x+2} = 2x+3 + \frac{30}{x-2} - \frac{16}{x-1}$$

(3) $\boldsymbol{\frac{3x-1}{x^3-2x^2+x}}$

Solution:

$$\frac{3x-1}{x^3-2x^2+x}$$

$$\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x^2-2x+1)}$$

$$\frac{3x-1}{x^3-2x^2+x} = \frac{3x-1}{x(x-1)^2}$$

Let

$$\frac{3x-1}{x(x-1)^2} = \frac{\text{A}}{x} + \frac{\text{B}}{x-1} + \frac{\text{C}}{(x-1)^2} \quad \text{.....equ(i)}$$

Multiply equ (i) by $x(x-1)^2$

$$\frac{3x-1}{x(x-1)^2} \times x(x-1)^2 = \frac{\text{A}}{x} \times x(x-1)^2 + \frac{\text{B}}{x-1} \times x(x-1)^2 + \frac{\text{C}}{(x-1)^2} \times x(x-1)^2$$

$3x-1 = A(x-1)^2 + Bx(x-1) + Cx \quad \text{.....equ(ii)}$

Put $x = 0$ in equ (ii)

$3(0) - 1 = A(0-1)^2 + B(0)(0-1) + C(0)$

$0 - 1 = A(-1)^2 + 0 + 0$

$-1 = A(1)$

$-1 = A$

$A = -1$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$3(1) - 1 = A(0)^2 + B(1)(0) + C(1)$

$3 - 1 = 0 + 0 + C$

$2 = C$

$C = 2$

equ (ii) $\Rightarrow$

$3x-1 = A(x-1)^2 + Bx(x-1) + Cx$

$3x-1 = A(x^2-2x+1) + Bx^2 - Bx + Cx$

$3x-1 = Ax^2 - 2Ax + A + Bx^2 - Bx + Cx$

$3x-1 = Ax^2 + Bx^2 - 2Ax - Bx + Cx + A$

$3x-1 = (A+B)x^2 + (-2A-B+C)x + A$

Review Exercise # 4

By comparing the coefficients of x^2, we get

$A + B = 0$

Put $A = -1$

$-1 + B = 0$

$B = 1$

Put the values of A, B and C in equu (i)

$$\frac{3x-1}{x(x-1)^2} = \frac{-1}{x} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

Important

(4) $\frac{\boldsymbol{x+1}}{\boldsymbol{(x-1)^2}}$

Solution:

$$\frac{x+1}{(x-1)^2}$$

Let

$$\frac{x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-1)^2$

$$\frac{x+1}{(x-1)^2} \times (x-1)^2 = \frac{A}{x-1} \times (x-1)^2 + \frac{B}{(x-1)^2} \times (x-1)^2$$

$x + 1 = A(x-1) + B \quad \ldots\ldots \text{equ(ii)}$

Put $x - 1 = 0 \Rightarrow x = 1$ in equ (ii)

$1 + 1 = A(0) + B$

$2 = B$

$B = 2$

equ (ii) $\Rightarrow$

$x + 1 = A(x-1) + B$

$x + 1 = Ax - A + B$

$x + 1 = Ax - A + B$

$x + 1 = Ax + (-A + B)$

By comparing the coefficients of x, we get

$A = 1$

Put the values of A and B in equ (i)

$$\frac{x+1}{(x-1)^2} = \frac{1}{x-1} + \frac{2}{(x-1)^2}$$

(5) $\frac{\boldsymbol{2x^2}}{\boldsymbol{x^4-4}}$

Solution:

$$\frac{2x^2}{x^4-4} = \frac{2x^2}{(x^2)^2-(2)^2}$$

Review Exercise # 4

$$\frac{2x^2}{x^4-4}=\frac{2x^2}{(x^2+2)(x^2-2)}$$

Let

$$\frac{2x^2}{(x^2+2)(x^2-2)}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2-2}\quad .....\text{equ(i)}$$

Multiply equ (i) by $(x^2+2)(x^2-2)$

$$\frac{2x^2}{(x^2+2)(x^2-2)}\times(x^2+2)(x^2-2)=\frac{Ax+B}{x^2+2}\times(x^2+2)(x^2-2)+\frac{Cx+D}{x^2-2}\times(x^2+2)(x^2-2)$$

$2x^2=(Ax+B)(x^2-2)+(Cx+D)(x^2+2)\quad .....\text{equ(ii)}$

equ (ii) $\Rightarrow$

$2x^2=Ax^3-2Ax+Bx^2-2B+Cx^3+2Cx+Dx^2+2D$

$2x^2=Ax^3+Cx^3+Bx^2+Dx^2-2Ax+2Cx-2B+2D$

$2x^2=(A+C)x^3+(B+D)x^2+(-2A+2C)x+(-2B+2D)$

Compare the coefficients of x^3 , x^2 , x *and constant* we get

$A+C=0\quad .....\text{equ}(\boldsymbol{a})$

$B+D=2\quad .....\text{equ}(\boldsymbol{b})$

$-2A+2C=0\quad .....\text{equ}(\boldsymbol{c})$

$-2B+2D=0\quad .....\text{equ}(\boldsymbol{d})$

equ (c) $\Rightarrow$

$-2(A-C)=0$

$A-C=0$

$A=C\quad .....\text{equ}(\boldsymbol{e})$

Put $A=C$ in equ $(\boldsymbol{a})$

$C+C=0$

$2C=0$

$C=\frac{0}{2}$

$C=0$

Now Put $C=0$ in equ $(\boldsymbol{e})$

$A=0$

equ (d) $\Rightarrow$

$-2(B-D)=0$

$B-D=0$

$B=D\quad .....\text{equ}(\boldsymbol{f})$

Put $B=D$ in equ (b)

$D+D=2$

$2D=2$

$D=\frac{2}{2}$

$D=1$

Now Put $D=1$ in equ $(\boldsymbol{f})$

$B=1$

Put the values of A, B , C and D in equ (i)

Review Exercise # 4

$$\frac{2x^2}{(x^2+2)(x^2-2)} = \frac{0x+1}{x^2+2} + \frac{0x+1}{x^2-2}$$
$$\frac{2x^2}{(x^2+2)(x^2-2)} = \frac{1}{x^2+2} + \frac{1}{x^2-2}$$

(6) $\mathbf{\dfrac{3x^2+3x+2}{x^4-1}}$

Solution:

$$\frac{3x^2+3x+2}{x^4-1} = \frac{3x^2+3x+2}{(x^2)^2-(1)^2}$$
$$\frac{3x^2+3x+2}{x^4-1} = \frac{3x^2+3x+2}{(x^2-1)(x^2+1)}$$
$$\frac{3x^2+3x+2}{x^4-1} = \frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}$$

Let

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x+1)(x-1)(x^2+1)$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)} \times (x+1)(x-1)(x^2+1) = \frac{A}{x+1} \times (x+1)(x-1)(x^2+1) + \frac{B}{x-1} \times (x+1)(x-1)(x^2+1) + \frac{Cx+D}{x^2+1} \times (x+1)(x-1)(x^2+1)$$

$$3x^2+3x+2 = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x+1)(x-1) \quad \ldots\ldots \text{equ(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ in equ (ii)

$$3(-1)^2+3(-1)+2 = A(-1-1)((-1)^2+1) + B(0)(x^2+1) + (Cx+D)(0)(x-1)$$
$$3(1)-3+2 = A(-2)(1+1)+0+0$$
$$3-3+2 = A(-2)(2)$$
$$2 = -4A$$
$$\frac{2}{-4} = A$$
$$\frac{1}{-2} = A$$
$$-\frac{1}{2} = A$$
$$A = -\frac{1}{2}$$

Put $x-1=0 \;\Rightarrow x=1$ in equ (ii)

$$3(1)^2+3(1)+2 = A(0)(x^2+1) + B(1+1)((1)^2+1) + (Cx+D)(x+1)(0)$$
$$3(1)+3+2 = 0 + B(2)(1+1) + 0$$
$$3+3+2 = B(2)(2)$$
$$8 = 4B$$
$$\frac{8}{4} = B$$
$$2 = B$$
$$B = 2$$

Review Exercise # 4

equ (**ii**) ⇒

$3x^2+3x+2=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x+1)(x-1)$

$3x^2+3x+2=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+(Cx+D)(x^2-1)$

$3x^2+3x+2=Ax^3+Ax-Ax^2-A+Bx^3+Bx+Bx^2+B+Cx^3-Cx+Dx^2-D$

$3x^2+3x+2=Ax^3+Bx^3+Cx^3-Ax^2+Bx^2+Dx^2+Ax+Bx-Cx-A+B-D$

$3x^2+3x+2=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$

Compare the coefficients of x^3 , x^2 , x $and\ constant$ we get

$A+B+C=0$ …..equ($\boldsymbol{a}$)

$-A+B+D=3$ …..equ($\boldsymbol{b}$)

$A+B-C=3$ …..equ($\boldsymbol{c}$)

$-A+B-D$ …..equ($\boldsymbol{d}$)

Put the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ in equ ($\boldsymbol{a}$)

$$-\frac{1}{2}+2+C=0$$

$$\frac{-1+4}{2}+C=0$$

$$\frac{3}{2}+C=0$$

$$C=-\frac{3}{2}$$

Put the values of $\boldsymbol{A}$ and $\boldsymbol{B}$ in equ ($\boldsymbol{b}$)

$$-\left(-\frac{1}{2}\right)+2+D=3$$

$$\frac{1}{2}+2-3+D=0$$

$$\frac{1+4-6}{2}+D=0$$

$$\frac{1+4-6}{2}+D=0$$

$$\frac{-1}{2}+D=0$$

$$D=\frac{1}{2}$$

Put the values of A, B , C and D in equ (i)

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}=\frac{-\frac{1}{2}}{x+1}+\frac{2}{x-1}+\frac{-\frac{3}{2}x+\frac{1}{2}}{x^2+1}$$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}=\frac{-1}{2(x+1)}+\frac{2}{x-1}+\frac{\frac{-3x+1}{2}}{x^2+1}$$

$$\frac{3x^2+3x+2}{(x+1)(x-1)(x^2+1)}=\frac{-1}{2(x+1)}+\frac{2}{x-1}+\frac{-3x+1}{2(x^2+1)}$$

(7) $\boldsymbol{\frac{x^3+3x^2+1}{(x^2+1)^2}}$

Solution:

$$\frac{x^3+3x^2+1}{(x^2+1)^2}$$

Let

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2} \quad \ldots\ldots\text{equ(i)}$$

Multiply equ (i) by $(x^2+4)^2$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}\times(x^2+1)^2=\frac{Ax+B}{x^2+1}\times(x^2+1)^2+\frac{Cx+D}{(x^2+1)^2}\times(x^2+1)^2$$

$$x^3+3x^2+1=(Ax+B)(x^2+1)+Cx+D \quad \ldots\ldots\text{equ(ii)}$$

equ (ii) ⇒

$$x^3+3x^2+1=Ax^3+Ax+Bx^2+B+Cx+D$$

$$x^3+3x^2+1=Ax^3+Bx^2+Ax+Cx+B+D$$

$$x^3+3x^2+1=Ax^3+Bx^2+(A+C)x+(B+D)$$

Compare the coefficients of x^3 , x^2 , x *and constant* we get

$A=1 \quad \ldots\ldots\text{equ}(\boldsymbol{a})$

$B=3 \quad \ldots\ldots\text{equ}(\boldsymbol{b})$

$A+C=0 \quad \ldots\ldots\text{equ}(\boldsymbol{c})$

$B+D=1 \ldots\ldots\text{equ}(\boldsymbol{d})$

Put $A=1$ in equ $(\boldsymbol{c})$

$1+C=0$

$C=-1$

Put $B=3$ in equ $(\boldsymbol{d})$

$3+D=1$

$D=1-3$

$D=-2$

Put the values of A, B , C and D in equ (i)

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{1x+3}{x^2+1}+\frac{-1x-2}{(x^2+1)^2}$$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{x+3}{x^2+1}+\frac{-(x+2)}{(x^2+1)^2}$$

$$\frac{x^3+3x^2+1}{(x^2+1)^2}=\frac{x+3}{x^2+1}-\frac{x+2}{(x^2+1)^2}$$

(8) $\boldsymbol{\dfrac{2x^3-1}{x^3+x^2}}$

Solution:

$$\frac{2x^3-1}{x^3+x^2}$$

As $\dfrac{2x^3-1}{x^3+x^2}$ is improper

So

$$x^2+x^2\ \overline{\big)\ 2x^3-1\qquad}^{\,2}$$

$$\underline{\pm 2x^3 \qquad \pm 2x^2}$$

Review Exercise # 4

$$\overline{\quad\quad\quad\quad}$$
$$-2x^2-1$$

$$\frac{2x^3-1}{x^3+x^2}=2+\frac{-2x^2-1}{x^3+x^2}$$
$$\frac{2x^3-1}{x^3+x^2}=2+\frac{-(2x^2+1)}{x^2(x+1)}$$
$$\frac{2x^3-1}{x^3+x^2}=2-\frac{2x^2+1}{x^2(x+1)} \quad \ldots\ldots \text{equ}(\mathbf{A})$$

Now

Let

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{\text{A}}{x}+\frac{\text{B}}{x^2}+\frac{\text{C}}{x+1} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $x^2(x^2+5)$

$$\frac{-2x^2-1}{x^2(x+1)}\times x^2(x+1)=\frac{\text{A}}{x}\times x^2(x+1)+\frac{\text{B}}{x^2}\times x^2(x+1)+\frac{\text{C}}{x+1}\times x^2(x^2+1)$$

$-2x^2-1=Ax(x+1)+B(x+1)+Cx^2 \quad \ldots\ldots \text{equ(ii)}$

Put $x=0$ in equ (ii)

$-2(0)^2-1=A(0)(0+1)+B(0+1)+C(0)^2$

$0-1=0+B(1)+0$

$-1=B$

$B=-1$

equ (ii) $\Rightarrow$

$-2x^2-1=Ax(x+1)+B(x+1)+Cx^2$

$-2x^2-1=Ax^2+Ax+Bx+B+Cx^2$

$-2x^2-1=Ax^2+Cx^2+Ax+Bx+B$

$-2x^2-1=(A+C)x^2+(A+B)x+B$

By comparing the coefficients of x^3, x^2, x and constant we get

$A+C=-2 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$

$A+B=0 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$

$B=-1 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$

$Put\ B=-1\ \ in\ equ\ (\boldsymbol{b})$

$A+(-1)=0$

$A-1=0$

$A=1$

$Put\ A=1\ in\ equ\ (\boldsymbol{a})$

$1+C=-2$

$C=-2-1$

$C=-3$

Put the values of A, B and C in equ (i)

$$\frac{-2x^2-1}{x^2(x+1)}=\frac{1}{x}+\frac{-1}{x^2}+\frac{-3}{x+1}$$
$$\frac{-2x^2-1}{x^2(x+1)}=\frac{1}{x}-\frac{1}{x^2}-\frac{3}{x+1}$$

Review Exercise # 4

(9) $\dfrac{4x^2+3x+14}{x^3-8}$

Solution:

$$\frac{4x^2+3x+14}{x^3-8}=\frac{4x^2+3x+14}{x^3-2^3}$$

$$\frac{4x^2+3x+14}{x^3-8}=\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}$$

Let

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+4} \quad \ldots\ldots \text{equ(i)}$$

Multiply equ (i) by $(x-2)(x^2+2x+4)$

$$\frac{4x^2+3x+14}{(x-2)(x^2+2x+4)}\times(x-2)(x^2+2x+4)$$

$$=\frac{A}{x-2}\times(x-2)(x^2+2x+4)+\frac{Bx+C}{x^2+2x+4}\times(x-2)(x^2+2x+4)$$

$$4x^2+3x+14=A(x^2+2x+4)+(Bx+C)(x-2) \quad \ldots\ldots \text{equ(ii)}$$

Put $x-2=0 \Rightarrow x=2$ in equ (ii)

$$4(2)^2+3(2)+14=A[(2)^2+2(2)+4]+(Bx+C)(0)$$

$$4(4)+6+14=A(4+4+4)+0$$

$$16+20=A(12)$$

$$36=12A$$

$$\frac{36}{12}=A$$

Review Exercise # 4

$3 = A$
$A = 3$
equ (ii) $\Rightarrow$
$4x^2 + 3x + 14 = A(x^2 + 2x + 4) + (Bx + C)(x - 2)$
$4x^2 + 3x + 14 = Ax^2 + 2Ax + 4A + Bx^2 - 2Bx + Cx - 2C$
$4x^2 + 3x + 14 = Ax^2 + Bx^2 + 2Ax - 2Bx + Cx + 4A - 2C$
$4x^2 + 3x + 14 = (A + B)x^2 + (2A - 2B + C)x + (4A - 2C)$
Compare the coefficients of $x^2, x \text{ and constant}$ we get
$A + B = 4 \quad \ldots\ldots \text{equ}(\boldsymbol{a})$
$2A - 2B + C = 3 \quad \ldots\ldots \text{equ}(\boldsymbol{b})$
$4A - 2C = 14 \quad \ldots\ldots \text{equ}(\boldsymbol{c})$
Put $A = 3$ in equ $(\boldsymbol{a})$
$3 + B = 4$
$B = 4 - 3$
$B = 1$
Put $A = 3$ in equ $(\boldsymbol{c})$
$4(3) - 2C = 14$
$12 - 2C = 14$
$-2C = 14 - 12$
$-2C = 2$
$C = \frac{2}{-2}$
$C = -1$
Put the values of A, B and C in equ (i)
$$\frac{4x^2 + 3x + 14}{(x - 2)(x^2 + 2x + 4)} = \frac{3}{x - 2} + \frac{1x + (-1)}{x^2 + 2x + 4}$$
$$\frac{4x^2 + 3x + 14}{(x - 2)(x^2 + 2x + 4)} = \frac{3}{x - 2} + \frac{x - 1}{x^2 + 2x + 4}$$

Q3*: *Resolve the following fraction into partial fraction $\boldsymbol{\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}}$

Solution:
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}$$
Let
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \quad \ldots\ldots \text{equ(i)}$$
Multiply equ (i) by $(x + 1)(x^2 + 1)^2$
$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} \times (x + 1)(x^2 + 1)^2$$
$$= \frac{A}{x + 1} \times (x + 1)(x^2 + 1)^2 + \frac{Bx + C}{x^2 + 1} \times (x + 1)(x^2 + 1)^2 + \frac{Dx + E}{(x^2 + 1)^2} \times (x + 1)(x^2 + 1)^2$$
$x^4 + 3x^2 + x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \quad \ldots\ldots \text{equ(ii)}$
Put $x + 1 = 0 \Rightarrow x = -1$ in equ (ii)

Review Exercise # 4

$(-1)^4 + 3(-1)^2 + (-1) + 1 = A((-1)^2 + 1)^2 + (Bx + C)(0)(x^2 + 1) + (Dx + E)(0)$
$1 + 3(1) - 1 + 1 = A(1 + 1)^2 + 0 + 0$
$1 + 3 = A(2)^2$
$4 = A(4)$
$\frac{4}{4} = A$
$1 = A$
$A = 1$
equ (ii) $\Rightarrow$
$x^4 + 3x^2 + x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1)$
$x^4 + 3x^2 + x + 1 = A(x^4 + 2x^2 + 1) + (Bx + C)(x^3 + x + x^2 + 1) + Dx^2 + Dx + Ex + E$
$x^4 + 3x^2 + x + 1 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Bx^3 + Bx + Cx^3 + Cx + Cx^2 + C + Dx^2 + Dx + Ex + E$
$x^4 + 3x^2 + x + 1 = Ax^4 + Bx^4 + Bx^3 + Cx^3 + 2Ax^2 + Bx^2 + Cx^2 + Dx^2 + Bx + Cx + Dx + Ex + A + C + E$
$x^4 + 3x^2 + x + 1 = (A + B)x^4 + (B + C)x^3 + (2A + B + C + D)x^2 + (B + C + D + E)x + (A + C + E)$
Compare the coefficients of x^4 , x^3 , x^2 , x *and constant* we get
$A + B = 1$ …..equ($\boldsymbol{a}$)
$B + C = 0$ …..equ($\boldsymbol{b}$)
$2A + B + C + D = 3$ …..equ($\boldsymbol{c}$)
$B + C + D + E = 1$ …..equ($\boldsymbol{d}$)
$A + C + E = 1$ …..equ($\boldsymbol{e}$)
Put $A = 1$ in equ ($\boldsymbol{a}$)
$1 + B = 1$
$B = 1 - 1$
$B = 0$
Put $B = 0$ in equ ($\boldsymbol{b}$)
$0 + C = 0$
$C = 0$
Put the values of A, B *and* C *in equ* (c)
$2(1) + 0 + 0 + D = 3$
$2 + D = 3$
$D = 3 - 2$
$D = 1$
Put the values of A *and* C *in equ* (e)
$1 + 0 + E = 1$
$1 + E = 1$
$E = 1 - 1$
$E = 0$
Put the values of A, B , C , D and E in equ (i)
$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{1}{x + 1} + \frac{0x + 0}{x^2 + 1} + \frac{1x + 0}{(x^2 + 1)^2}$
$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{1}{x + 1} + \frac{x}{(x^2 + 1)^2}$

Review Exercise # 4