UNIT # 9

INTRODUCTION TO COORDINATE GEOMETRY

Ex # 9.1

Introduction

The relationship between algebra and geometry was given by a French Philosopher and Mathematician Rene Descartes in 1637 when his book *La Geometrie* was published.

Coordinate Geometry

The study of geometric properties of figures by the study of their equations is called coordinated or analytic geometry.

Coordinates

Coordinates are a set of values which helps to show the exact position of a point in the coordinate plane.

Coordinate plane

A coordinate plane is formed by intersection of two perpendicular lines known as x – axis and y – axis at origin. These two perpendicular lines is divided into four quadrants.

Distance between points on real line

Suppose we are given two distinct points a & b on the real line then

The directed distance from a to b is $b-a$

The directed distance from b to a is $a-b$

Note:

The distance between two points on the real line can never be negative.

The distance between a and b is $|a-b|$ or $|b-a|$

Or

The distance d between points x_1 & x_2 on the real line is given by $d=|x_2-x_1|=(x_2-x_1)^2$

Note:

The order of subtraction with x_1 & x_2 does not matter in finding the distance between them since

$|x_1-x_2|=|x_2-x_1|$ and $(x_1-x_2)^2=(x_2-x_1)^2$

Example # 1

Determine the distance between -3 and 4 on the real line. What is the directed distance from -3 *to* 4 and from 4 *to* -3 ?

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Solution:

The distance between -3 *and* 4 is given by:

$|4-(-3)|=|4+3|=|7|=7$

Or

$|-3-4|=|-7|=7$

The directed distance form -3 *to* 4 is

$4-(-3)=4+3=7$

The directed distance from 4 *to* -3 is:

$-3-4=-7=7$

As distance can never be negative

Distance between two points in a plane

Suppose two points on the same horizontal line or the same vertical line in the plane, then the distance between them is given by:

Distance of $x-$ coordinates

Let the two points on $x-coordinates$ are $P(x_1,y_1)$ *and* $Q(x_2,\ y_2)$, the distance of $x-coordinates$ is $|x_2-x_1|$

Distance of $y-$ coordinates

Let the two points on $y-coordinates$ are $P(x_1,y_1)$ *and* $R(x_1,\ y_2)$, the distance of $y-coordinates$ is $|y_2-y_1|$

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Example # 2

Find the distance between (2, 3) & (7, 3) and the distance between (4, 6) & $(4, -9)$

Solution:

In the figure:

Let $P(2, 3)$ & $Q(7, 3)$ lie on the same horizantal line so the distance is:

$L_1 = |7 - 2|$

$L_1 = |5|$

$L_1 = 5$

And also

Let $R(4, 6)$ & $S(4, -9)$ lie on the same vertical line so the distance is:

$L_2 = |6 - (-9)|$

$L_2 = |6 + 9|$

$L_2 = |15|$

$L_2 = 15$

Distance formula

the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Derivation

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In the given figure

The two points are $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

Let $\overline{OM_1} = x_1$ and $\overline{OM_2} = x_2$

Now

$\overline{M_1M_2} = \overline{OM_2} - \overline{OM_1}$

$\overline{M_1M_2} = x_2 - x_1$

$\overline{M_1M_2} = \overline{P_1N}$

$\overline{P_1N} = x_2 - x_1$

Let $\overline{M_1P_1} = \overline{M_2N} = y_1$ and $\overline{M_2P_2} = y_2$

Now

$\overline{NP_2} = \overline{M_2P_2} - \overline{M_2N}$

$\overline{NP_2} = y_2 - y_1$

As P_1NP_2 is a right-angled triangle,

So, by Pythagoras theorem

$|P_1P_2|^2 = |P_1N|^2 + |NP_2|^2$

$|P_1P_2|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Taking square root on B.S

$\sqrt{|P_1P_2|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example # 3

Find the distance between points $A(-5, 1)$ and $B(3, 1)$.

Solution:

$A(-5, 1)$ and $B(3, 1)$

Let $x_1 = -5,\ y_1 = 1$

And $x_2 = 3,\ y_2 = 1$

As distance formula is:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left(3 - (-5)\right)^2 + (1 - 1)^2}$

$|AB| = \sqrt{(3 + 5)^2 + (0)^2}$

$|AB| = \sqrt{(8)^2 + 0}$

$|AB| = \sqrt{64}$

$|AB| = 8$

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Example # 4

Plot the points $(-3, 7)$ & $(5, 9)$ and find the distance between them.

Solution:

$(-3, 7)$ & $(5, 9)$

Let $x_1 = -3,\ y_1 = 7$

And $x_2 = 5,\ y_2 = 9$

As distance formula is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{\left(5 - (-3)\right)^2 + (9 - 7)^2}$

$d = \sqrt{(5 + 3)^2 + (2)^2}$

$d = \sqrt{(8)^2 + 4}$

$d = \sqrt{64 + 4}$

$d = \sqrt{68}$

$d = \sqrt{4 \times 17}$

$d = \sqrt{4} \times \sqrt{17}$

$d = 2\sqrt{17}$

Example # 5

A helicopter pilot located 1 mile west and 3 miles north of the command centre must respond to an emergency located 7 miles west and 11 miles north of the centre. How far must the helicopter travel to get to the emergency site?

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Solution:

$As\ west\ direction\ represents\ x - axis$

$and\ nort\ direction\ represents\ y - axix$

$Let\ coordinates\ of\ command\ centre = O(0, 0)$

$Coordinates\ of\ Helicopter = A(1, 3)$

$Coordinates\ of\ energency\ site = B(7, 11)$

Let $x_1 = 1,\ y_1 = 3$

And $x_2 = 7,\ y_2 = 11$

As distance formula is:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(7 - 1)^2 + (11 - 3)^2}$

$|AB| = \sqrt{(6)^2 + (8)^2}$

$|AB| = \sqrt{36 + 64}$

$|AB| = \sqrt{100}$

$|AB| = 10$

Thus, the helicopter must travel 10 miles to get the emergency site.

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Q1: Find the length of AB in the following figures.

(i)

Solution:

$|AB| = |4 - 0|$

$|AB| = |4|$

$|AB| = 4$

(ii)

Solution:

$|AB| = |0 - (-2)|$

$|AB| = |0 + 2|$

$|AB| = |2|$

$|AB| = 2$

(iii)

Solution:

$|AB| = |5 - 2|$

$|AB| = |3|$

$|AB| = 3$

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(iv)

Solution:
$|AB| = |-2-(-7)|$
$|AB| = |-2+7|$
$|AB| = |5|$
$|AB| = 5$

(v)

Solution:
$|AB| = |2-(-3)|$
$|AB| = |2+3|$
$|AB| = |5|$
$|AB| = 5$

(vi)

Solution:
$|AB| = |1-(-1)|$
$|AB| = |1+1|$
$|AB| = |2|$
$|AB| = 2$

Q2: **Find distance between each pair of points.**

(i) $(\mathbf{1},\mathbf{1}),(\mathbf{3},\mathbf{3})$

Solution:
$(1,1),(3,3)$
Let $x_1 = 1,\ y_1 = 1$
And $x_2 = 3,\ y_2 = 3$
As distance formula is:
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$d = \sqrt{(3-1)^2+(3-1)^2}$
$d = \sqrt{(2)^2+(2)^2}$
$d = \sqrt{4+4}$
$d = \sqrt{8}$
$d = \sqrt{4\times 2}$
$d = \sqrt{4}\times\sqrt{2}$
$d = 2\sqrt{2}$

(ii) $(\mathbf{1},\mathbf{2}),(\mathbf{4},\mathbf{5})$

Solution:
$(1,2),(4,5)$
Let $x_1 = 1,\ y_1 = 2$
And $x_2 = 4,\ y_2 = 5$

Ex # 9.1

As distance formula is:
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$d = \sqrt{(4-1)^2+(5-2)^2}$
$d = \sqrt{(3)^2+(3)^2}$
$d = \sqrt{9+9}$
$d = \sqrt{18}$
$d = \sqrt{9\times 2}$
$d = \sqrt{9}\times\sqrt{2}$
$d = 3\sqrt{2}$

(iii) $(\mathbf{2},-\mathbf{2}),(\mathbf{2},-\mathbf{3})$

Solution:
$(2,-2),(2,-3)$
Let $x_1 = 2,\ y_1 = -2$
And $x_2 = 2,\ y_2 = -3$
As distance formula is:
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$d = \sqrt{(2-2)^2+\left(-3-(-2)\right)^2}$
$d = \sqrt{(0)^2+(-3+2)^2}$
$d = \sqrt{0+(-1)^2}$
$d = \sqrt{1}$
$d = 1$

(iv) $(\mathbf{3},-\mathbf{5}),(\mathbf{5},-\mathbf{7})$

Solution:
$(3,-5),(5,-7)$
Let $x_1 = 3,\ y_1 = -5$
And $x_2 = 5,\ y_2 = -7$
As distance formula is:
$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$d = \sqrt{(5-3)^2+\left(-5-(-7)\right)^2}$
$d = \sqrt{(2)^2+(-5+7)^2}$
$d = \sqrt{4+(2)^2}$
$d = \sqrt{4+4}$
$d = \sqrt{8}$
$d = \sqrt{4\times 2}$
$d = \sqrt{4}\times\sqrt{2}$
$d = 2\sqrt{2}$

Ex # 9.1

Q3: **Given points $O(0,0), A(3,4), B(-5,12), C(15,-8), D(11,-3), E(-9,-4)$. Determine length of the following segments.**

(i) $\overline{OA}$

Solution:

$\overline{OA}$

$O(0,0), A(3,4)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = 3,\ y_2 = 4$

As distance formula is:

$|OA| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|OA| = \sqrt{(3-0)^2 + (4-0)^2}$

$|OA| = \sqrt{(3)^2 + (4)^2}$

$|OA| = \sqrt{9 + 16}$

$|OA| = \sqrt{25}$

$|OA| = 5$

(ii) $\overline{OB}$

Solution:

$\overline{OB}$

$O(0,0), B(-5,12)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = -5,\ y_2 = 12$

As distance formula is:

$|OB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|OB| = \sqrt{(-5-0)^2 + (12-0)^2}$

$|OB| = \sqrt{(-5)^2 + (12)^2}$

$|OB| = \sqrt{25 + 144}$

$|OB| = \sqrt{169}$

$|OB| = 13$

(iii) $\overline{OC}$

Solution:

$\overline{OC}$

$O(0,0), C(15,-8)$

Let $x_1 = 0,\ y_1 = 0$

And $x_2 = 15,\ y_2 = -8$

As distance formula is:

$|OC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|OC| = \sqrt{(15-0)^2 + (-8-0)^2}$

$|OC| = \sqrt{(15)^2 + (-8)^2}$

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$|OC| = \sqrt{225 + 64}$

$|OC| = \sqrt{289}$

$|OC| = 17$

(iv) $\overline{AD}$

Solution:

$\overline{AD}$

$A(3,4), D(11,-3)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = 11,\ y_2 = -3$

As distance formula is:

$|AD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AD| = \sqrt{(11-3)^2 + (-3-4)^2}$

$|AD| = \sqrt{(8)^2 + (-7)^2}$

$|AD| = \sqrt{64 + 49}$

$|AD| = \sqrt{113}$

(v) $\overline{AB}$

Solution:

$\overline{AB}$

$A(3,4), B(-5,12)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = -5,\ y_2 = 12$

As distance formula is:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(-5-3)^2 + (12-4)^2}$

$|AB| = \sqrt{(-8)^2 + (8)^2}$

$|AB| = \sqrt{64 + 64}$

$|AB| = \sqrt{128}$

$|AB| = \sqrt{64 \times 2}$

$|AB| = \sqrt{64} \times \sqrt{2}$

$|AB| = 8\sqrt{2}$

(vi) $\overline{AC}$

Solution:

$\overline{AC}$

$A(3,4), C(15,-8)$

Let $x_1 = 3,\ y_1 = 4$

And $x_2 = 15,\ y_2 = -8$

As distance formula is:

$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

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$|AC| = \sqrt{(15-3)^2 + (-8-4)^2}$

$|AC| = \sqrt{(12)^2 + (-12)^2}$

$|AC| = \sqrt{144 + 144}$

$|AC| = \sqrt{288}$

$|AC| = \sqrt{144 \times 2}$

$|AC| = \sqrt{144} \times \sqrt{2}$

$|AC| = 12\sqrt{2}$

(vii) $\overline{BE}$

Solution:

$\overline{BE}$

$B(-5, 12), E(-9, -4)$

Let $x_1 = -5,\ y_1 = 12$

And $x_2 = -9,\ y_2 = -4$

As distance formula is:

$|BE| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|BE| = \sqrt{(-9-(-5))^2 + (-4-12)^2}$

$|BE| = \sqrt{(-9+5)^2 + (-16)^2}$

$|BE| = \sqrt{(-4)^2 + 256}$

$|BE| = \sqrt{16 + 256}$

$|BE| = \sqrt{272}$

$|BE| = \sqrt{4 \times 68}$

$|BE| = \sqrt{4} \times \sqrt{68}$

$|BE| = 2\sqrt{68}$

Ex # 9.2

Collinear points

Three or more points which lie on the same straight line are called collinear points.

Non – collinear points

The set of points that are not lie on the same straight line is called non – collinear points.

Ex # 9.2

Example # 6

Prove that the points $A(5,-2), B(1,2), C(-2,5)$ are collinear.

Solution:

$A(5,-2), B(1,2), C(-2,5)$

Let $x_1 = 5,\ y_1 = -2$

And $x_2 = 1,\ y_2 = 2$

Also $x_3 = -2,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(1-5)^2 + (2-(-2))^2}$

$|AB| = \sqrt{(-4)^2 + (2+2)^2}$

$|AB| = \sqrt{16 + (4)^2}$

$|AB| = \sqrt{16 + 16}$

$|AB| = \sqrt{32}$

$|AB| = \sqrt{16 \times 2}$

$|AB| = \sqrt{16} \times \sqrt{2}$

$|AB| = 4\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(-2-1)^2 + (5-2)^2}$

$|BC| = \sqrt{(-3)^2 + (3)^2}$

$|BC| = \sqrt{9 + 9}$

$|BC| = \sqrt{18}$

$|BC| = \sqrt{9 \times 2}$

$|BC| = \sqrt{9} \times \sqrt{2}$

$|BC| = 3\sqrt{2}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(-2-5)^2 + (5-(-2))^2}$

$|AC| = \sqrt{(-7)^2 + (5+2)^2}$

$|AC| = \sqrt{49 + (7)^2}$

$|AC| = \sqrt{49 + 49}$

$|AC| = \sqrt{98}$

$|AC| = \sqrt{49 \times 2}$

$|AC| = \sqrt{49} \times \sqrt{2}$

$|AC| = 7\sqrt{2}$

Ex # 9.2

For Colinear Points

$$|AC| = |AB| + |BC|$$
$$7\sqrt{2} = 4\sqrt{2} + 3\sqrt{2}$$

Thus the points are colinear points.

Equilateral Triangle

A triangle in which all the three sides and angles are equal is called equilateral triangle. In equilateral triangle measure of each angle is 60^0.

Example # 7

Prove that the points $A(-2,0), B(2,0)$, $C(0,\sqrt{12})$ is an equilateral triangle.

Solution:

$A(-2,0), B(2,0), C(0,\sqrt{12})$

Let $x_1 = -2,\ y_1 = 0$

And $x_2 = 2,\ y_2 = 0$

Also $x_3 = 0,\ y_3 = \sqrt{12}$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(2 - (-2))^2 + (0 - 0)^2}$

$|AB| = \sqrt{(2 + 2)^2 + (0)^2}$

$|AB| = \sqrt{(4)^2 + 0}$

$|AB| = \sqrt{16}$

$|AB| = 4$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(0 - 2)^2 + \left(\sqrt{12} - 0\right)^2}$

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$|BC| = \sqrt{(-2)^2 + \left(\sqrt{12}\right)^2}$

$|BC| = \sqrt{4 + 12}$

$|BC| = \sqrt{16}$

$|BC| = 4$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(0 - (-2))^2 + \left(\sqrt{12} - 0\right)^2}$

$|AC| = \sqrt{(0 + 2)^2 + \left(\sqrt{12}\right)^2}$

$|AC| = \sqrt{(2)^2 + 12}$

$|AC| = \sqrt{4 + 12}$

$|AC| = \sqrt{16}$

$|AC| = 4$

For Equilateral Triangle

All three sides of a triangle are equal

$|AB| = |BC| = |AC| = 4$

Thus the points A, B and C are the vertices of an equilateral triangle.

Isosceles Triangle

A triangle in which two sides and two angles are equal is called isosceles triangle.

Note:

In isosceles triangle, two equal angles are opposite to the equal sides.

Example # 8

Show that points $A(3,2), B(9,10), C(1,16)$ are vertices of an isosceles triangle.

Solution:

$A(3,2), B(9,10), C(1,16)$

Let $x_1 = 3,\ y_1 = 2$

And $x_2 = 9,\ y_2 = 10$

Also $x_3 = 1,\ y_3 = 16$

Ex # 9.2

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(9 - 3)^2 + (10 - 2)^2}$

$|AB| = \sqrt{(6)^2 + (8)^2}$

$|AB| = \sqrt{36 + 64}$

$|AB| = \sqrt{100}$

$|AB| = 10$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(1 - 9)^2 + (16 - 10)^2}$

$|BC| = \sqrt{(-8)^2 + (6)^2}$

$|BC| = \sqrt{64 + 36}$

$|BC| = \sqrt{100}$

$|BC| = 10$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(1 - 3)^2 + (16 - 2)^2}$

$|AC| = \sqrt{(-2)^2 + (14)^2}$

$|AC| = \sqrt{4 + 196}$

$|AC| = \sqrt{200}$

$|AC| = \sqrt{100 \times 2}$

$|AC| = \sqrt{100} \times \sqrt{2}$

$|AC| = 10\sqrt{2}$

For Isosceles Triangle

Two sides of a triangle are equal.

$|AB| = |BC| = 10$

Thus, the points A, B and C are the vertices of isosceles triangle.

Ex # 9.2

Scalene Triangle

A triangle in which all three sides and angles are different is called scalene triangle.

Example # 9: Show that the points $A(1, 2)$, $B(0, 4)$, $C(3, 5)$ are vertices of scalene triangle.

Solution:

$A(1, 2), B(0, 4), C(3, 5)$

Let $x_1 = 1,\ y_1 = 2$

And $x_2 = 0,\ y_2 = 4$

Also $x_3 = 3,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(0 - 1)^2 + (4 - 2)^2}$

$|AB| = \sqrt{(-1)^2 + (2)^2}$

$|AB| = \sqrt{1 + 4}$

$|AB| = \sqrt{5}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(3 - 0)^2 + (5 - 4)^2}$

$|BC| = \sqrt{(3)^2 + (1)^2}$

$|BC| = \sqrt{9 + 1}$

$|BC| = \sqrt{10}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(3 - 1)^2 + (5 - 2)^2}$

$|AC| = \sqrt{(2)^2 + (-3)^2}$

$|AC| = \sqrt{4 + 9}$

$|AC| = \sqrt{13}$

For Scalene Triangle

All the three sides of a triangle are different.

$|AB| \neq |BC| \neq |AC|$

$\sqrt{5} \neq \sqrt{10} \neq \sqrt{13}$

Thus, the points A, B and C are the vertices of scalene triangle.

Ex # 9.2

Right angled triangle

A right-angled triangle in which one angle is equal to 90^0 i.e. right angle

Pythagoras theorem

$(Base)^2 + (Prep)^2 = (Hyp)^2$

Note:

The side opposite to the 90^0 is called hypotenuse.

Hypotenuse is always greater than the other two sides.

Example # 10

Construct the triangle ABC with the help of the points $A(1,-2), B(5,1), C(2,5)$, and prove that the triangle is a right – angled triangle.

Solution:

$A(1,-2), B(5,1), C(2,5)$

Let $x_1 = 1,\ y_1 = -2$

And $x_2 = 5,\ y_2 = 1$

Also $x_3 = 2,\ y_3 = 5$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(5-1)^2 + (1-(-2))^2}$

$|AB| = \sqrt{(4)^2 + (1+2)^2}$

$|AB| = \sqrt{16 + (3)^2}$

$|AB| = \sqrt{16 + 9}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(2-5)^2 + (5-1)^2}$

$|BC| = \sqrt{(-3)^2 + (4)^2}$

$|BC| = \sqrt{9 + 6}$

$|BC| = \sqrt{25}$

$|BC| = 5$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(2-1)^2 + (5-(-2))^2}$

$|AC| = \sqrt{(1)^2 + (5+2)^2}$

$|AC| = \sqrt{1 + (7)^2}$

Ex # 9.2

$|AC| = \sqrt{1 + 49}$

$|AC| = \sqrt{50}$

$|AC| = \sqrt{25 \times 2}$

$|AC| = \sqrt{25} \times \sqrt{2}$

$|AC| = 5\sqrt{2}$

For Right angled Triangle

$(Base)^2 + (Prep)^2 = (Hyp)^2$

So

$|AB|^2 + |BC|^2 = |AC|^2$

$(5)^2 + (5)^2 = (5\sqrt{2})^2$

$25 + 25 = (5)^2(\sqrt{2})^2$

$50 = 25(2)$

$50 = 50$

Thus, the points A, B and C are the vertices of right – angled triangle.

Square

A closed figure formed by four non – collinear points (vertices) in which the length of all sides are equal and measure of each angle is 90^0

The diagonals of square are equal in length.

Example # 11

By means of distance formula, show that the points $A(-1,4), B(1,2), C(3,4), D(1,6)$ form a square and verify that the diagonals have equal lengths

Solution:

$A(-1,4), B(1,2), C(3,4), D(1,6)$

Let $x_1 = -1,\ y_1 = 4$

And $x_2 = 1,\ y_2 = 2$

Also $x_3 = 3,\ y_3 = 4$

Also $x_4 = 1,\ y_4 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Ex # 9.2

$|AB| = \sqrt{(1-(-1))^2 + (2-4)^2}$

$|AB| = \sqrt{(1+1)^2 + (-2)^2}$

$|AB| = \sqrt{(2)^2 + 4}$

$|AB| = \sqrt{4+4}$

$|AB| = \sqrt{8}$

$|AB| = \sqrt{4 \times 2}$

$|AB| = \sqrt{4} \times \sqrt{2}$

$|AB| = 2\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(3-1)^2 + (4-2)^2}$

$|BC| = \sqrt{(2)^2 + (2)^2}$

$|BC| = \sqrt{4+4}$

$|BC| = \sqrt{8}$

$|BC| = \sqrt{4 \times 2}$

$|BC| = \sqrt{4} \times \sqrt{2}$

$|BC| = 2\sqrt{2}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(1-3)^2 + (6-4)^2}$

$|CD| = \sqrt{(-2)^2 + (2)^2}$

$|CD| = \sqrt{(2)^2 + (2)^2}$

$|CD| = \sqrt{4+4}$

$|CD| = \sqrt{8}$

$|CD| = \sqrt{4 \times 2}$

$|CD| = \sqrt{4} \times \sqrt{2}$

$|CD| = 2\sqrt{2}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(1-(-1))^2 + (6-4)^2}$

$|AD| = \sqrt{(1+1)^2 + (2)^2}$

$|AD| = \sqrt{(2)^2 + (2)^2}$

$|AD| = \sqrt{4+4}$

$|AD| = \sqrt{8}$

$|AD| = \sqrt{4 \times 2}$

$|AD| = \sqrt{4} \times \sqrt{2}$

Ex # 9.2

$|AD| = 2\sqrt{2}$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_2 - y_1)^2}$

$|AC| = \sqrt{(3-(-1))^2 + (4-4)^2}$

$|AC| = \sqrt{(3+1)^2 + (0)^2}$

$|AC| = \sqrt{(4)^2 + 0}$

$|AC| = \sqrt{16}$

$|AC| = 4$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(1-1)^2 + (6-2)^2}$

$|BD| = \sqrt{(0)^2 + (4)^2}$

$|BD| = \sqrt{0+16}$

$|BD| = \sqrt{16}$

$|BD| = 4$

For Square

All the sides are equal.

$|AB| = |BC| = |CD| = |AD| = 2\sqrt{2}$

And also, diagonals are equal

$|AC| = |BD| = 4$

Thus, the points A, B, C and D are the vertices of Square.

Rectangle

A rectangle is a geometric shape that has four sides, four vertices and four angles.
The opposite sides of a rectangle are equal in length and measure of each angle is 90^0.
The diagonals of a rectangle are equal in length.

Ex # 9.2

Example # 12 **Show that the points $\boldsymbol{A(2,4)}$, $\boldsymbol{B(4,2)}$, $\boldsymbol{C(8,6)}$, $\boldsymbol{D(6,8)}$ are the vertices of a rectangle. Also plot the points.**

Solution:

$A(2,4), B(4,2), C(8,6), D(6,8)$

Let $x_1 = 2,\ y_1 = 4$ And $x_2 = 4,\ y_2 = 2$

Also $x_3 = 8,\ y_3 = 6$ Also $x_4 = 6,\ y_4 = 8$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(4-2)^2 + (2-4)^2}$

$|AB| = \sqrt{(2)^2 + (-2)^2}$

$|AB| = \sqrt{4+4}$

$|AB| = \sqrt{8}$

$|AB| = \sqrt{4 \times 2}$

$|AB| = \sqrt{4} \times \sqrt{2}$

$|AB| = 2\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(8-4)^2 + (6-2)^2}$

$|BC| = \sqrt{(4)^2 + (4)^2}$

$|BC| = \sqrt{16+16}$

$|BC| = \sqrt{32}$

$|BC| = \sqrt{16 \times 2}$

$|BC| = \sqrt{16} \times \sqrt{2}$

$|BC| = 4\sqrt{2}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(6-8)^2 + (8-6)^2}$

$|CD| = \sqrt{(-2)^2 + (2)^2}$

$|CD| = \sqrt{(2)^2 + (2)^2}$

$|CD| = \sqrt{4+4}$

$|CD| = \sqrt{8}$

$|CD| = \sqrt{4 \times 2}$

$|CD| = \sqrt{4} \times \sqrt{2}$

$|CD| = 2\sqrt{2}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(6-2)^2 + (8-4)^2}$

Ex # 9.2

$|AD| = \sqrt{(4)^2 + (4)^2}$

$|AD| = \sqrt{16+16}$

$|AD| = \sqrt{32}$

$|AD| = \sqrt{16 \times 2}$

$|AD| = \sqrt{16} \times \sqrt{2}$

$|AD| = 4\sqrt{2}$

Now to find its Diagonal

Diagonal $\boldsymbol{\overline{AC}}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_2 - y_1)^2}$

$|AC| = \sqrt{(8-2)^2 + (6-4)^2}$

$|AC| = \sqrt{(6)^2 + (2)^2}$

$|AC| = \sqrt{36+4}$

$|AC| = \sqrt{40}$

$|AC| = \sqrt{4 \times 10}$

$|AC| = \sqrt{4} \times \sqrt{10}$

$|AC| = 2\sqrt{10}$

And Diagonal $\boldsymbol{\overline{BD}}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(6-4)^2 + (8-2)^2}$

$|BD| = \sqrt{(2)^2 + (6)^2}$

$|BD| = \sqrt{4+36}$

$|BD| = \sqrt{40}$

$|BD| = \sqrt{4 \times 10}$

$|BD| = \sqrt{4} \times \sqrt{10}$

$|BD| = 2\sqrt{10}$

For Rectangle

Opposite sides are equal.

$|AB| = |CD| = 2\sqrt{2}$ ***and*** $|BC| = |AD| = 4\sqrt{2}$

And also, diagonals are equal

$|AC| = |BD| = 2\sqrt{10}$

Thus, the points A, B, C and D are the vertices of Rectangle.

Ex # 9.2

Parallelogram

In a parallelogram the opposite sides are congruent and the diagonal bisect each other.

Example # 13

Show that the points $F(-1,5), G(3,3), H(6,-4)$ and $J(2,-2)$ are the vertices of a parallelogram. Also plot the points.

Solution:

$F(-1,5), G(3,3), H(6,-4)\ and\ J(2,-2)$

Let $x_1 = -1,\ y_1 = 5$

And $x_2 = 3,\ y_2 = 3$

Also $x_3 = 6,\ y_3 = -4$

Also $x_4 = 2,\ y_4 = -2$

As distance of $\overline{FG}$:

$|FG| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|FG| = \sqrt{\left(3 - (-1)\right)^2 + (3 - 5)^2}$

$|FG| = \sqrt{(3 + 1)^2 + (-2)^2}$

$|FG| = \sqrt{(4)^2 + 4}$

$|FG| = \sqrt{16 + 4}$

$|FG| = \sqrt{20}$

$|FG| = \sqrt{4 \times 5}$

$|FG| = \sqrt{4} \times \sqrt{5}$

$|FG| = 2\sqrt{5}$

Now distance of $\overline{GH}$:

$|GH| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|GH| = \sqrt{(ss6 - 3)^2 + (-4 - 3)^2}$

$|GH| = \sqrt{(3)^2 + (-7)^2}$

$|GH| = \sqrt{9 + 49}$

$|GH| = \sqrt{58}$

Also distance of $\overline{HJ}$:

$|HJ| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|HJ| = \sqrt{(2 - 6)^2 + \left(-2 - (-4)\right)^2}$

$|HJ| = \sqrt{(-4)^2 + (-2 + 4)^2}$

$|HJ| = \sqrt{16 + (2)^2}$

$|HJ| = \sqrt{16 + 4}$

$|HJ| = \sqrt{20}$

$|HJ| = \sqrt{4 \times 5}$

Ex # 9.2

$|HJ| = \sqrt{4} \times \sqrt{5}$

$|HJ| = 2\sqrt{5}$

Also distance of $\overline{JF}$:

$|JF| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|JF| = \sqrt{\left(2 - (-1)\right)^2 + (-2 - 5)^2}$

$|JF| = \sqrt{(2 + 1)^2 + (-7)^2}$

$|JF| = \sqrt{(3)^2 + 49}$

$|JF| = \sqrt{9 + 49}$

$|JF| = \sqrt{58}$

Now to find its Diagonal

Diagonal $\overline{FH}$:

$|FH| = \sqrt{(x_3 - x_1)^2 + (y_2 - y_1)^2}$

$|FH| = \sqrt{\left(6 - (-1)\right)^2 + (-4 - 5)^2}$

$|FH| = \sqrt{(6 + 1)^2 + (-9)^2}$

$|FH| = \sqrt{(7)^2 + 81}$

$|FH| = \sqrt{49 + 81}$

$|FH| = \sqrt{130}$

And Diagonal $\overline{GJ}$:

$|GJ| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|GJ| = \sqrt{(2 - 3)^2 + (-2 - 3)^2}$

$|GJ| = \sqrt{(-1)^2 + (-5)^2}$

$|GJ| = \sqrt{1 + 25}$

$|GJ| = \sqrt{26}$

For Parallelogram

Opposite sides are equal.

$|FG| = |HJ| = 2\sqrt{5}\ \boldsymbol{and}\ |GH| = |JF| = \sqrt{58}$

$|FH| \neq |GJ|$

$\sqrt{130} \neq \sqrt{26}$

As diagonals are not equal

Thus, the points A, B, C and D are the vertices of Parallelogram.

Ex # 9.2

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Q1: **Prove that $A(-4,-3), B(1,4), C(6,11)$ are collinear.**

Solution:

$A(-4,-3), B(1,4), C(6,11)$

Let $x_1 = -4,\ y_1 = -3$

And $x_2 = 1,\ y_2 = 4$

Also $x_3 = 6,\ y_3 = 11$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(1-(-4))^2 + (4-(-3))^2}$

$|AB| = \sqrt{(1+4)^2 + (4+3)^2}$

$|AB| = \sqrt{(5)^2 + (7)^2}$

$|AB| = \sqrt{25 + 49}$

$|AB| = \sqrt{74}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(6-1)^2 + (11-4)^2}$

$|BC| = \sqrt{(5)^2 + (7)^2}$

$|BC| = \sqrt{25 + 49}$

$|BC| = \sqrt{74}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(6-(-4))^2 + (11-(-3))^2}$

$|AC| = \sqrt{(6+4)^2 + (11+3)^2}$

$|AC| = \sqrt{(10)^2 + (14)^2}$

$|AC| = \sqrt{100 + 196}$

$|AC| = \sqrt{296}$

$|AC| = \sqrt{4 \times 74}$

$|AC| = \sqrt{4} \times \sqrt{74}$

$|AC| = 2\sqrt{74}$

For Colinear Points

$|AC| = |AB| + |BC|$

$2\sqrt{74} = \sqrt{74} + \sqrt{74}$

Thus, the points are colinear points.

Ex # 9.2

Q2: **Prove that $A(-1,3), B(-4,7), C(0,4)$ is an isosceles triangle.**

Solution:

$A(-1,3), B(-4,7), C(0,4)$

Let $x_1 = -1,\ y_1 = 3$

And $x_2 = -4,\ y_2 = 7$

Also $x_3 = 0,\ y_3 = 4$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(-4-(-1))^2 + (7-3)^2}$

$|AB| = \sqrt{(-4+1)^2 + (4)^2}$

$|AB| = \sqrt{(-3)^2 + 16}$

$|AB| = \sqrt{9 + 16}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{\left(0-(-4)\right)^2 + (4-7)^2}$

$|BC| = \sqrt{(0+4)^2 + (-3)^2}$

$|BC| = \sqrt{(4)^2 + 9}$

$|BC| = \sqrt{16 + 9}$

$|BC| = \sqrt{25}$

$|BC| = 5$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(0-(-1))^2 + (4-3)^2}$

$|AC| = \sqrt{(0+1)^2 + (1)^2}$

$|AC| = \sqrt{(1)^2 + 1}$

$|AC| = \sqrt{1 + 1}$

$|AC| = \sqrt{2}$

For Isosceles Triangle

Two sides of a triangle are equal.

$|AB| = |BC| = 5$

Thus, the points A, B and C are the vertices of isosceles triangle.

Ex # 9.2

Q3: **Show that points $A(2,3), B(8,11), C(0,17)$ are vertices of an isosceles triangle.**

Solution:

$A(2,3), B(8,11), C(0,17)$

Let $x_1 = 2,\ y_1 = 3$

And $x_2 = 8,\ y_2 = 11$

Also $x_3 = 0,\ y_3 = 17$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(8-2)^2 + (11-3)^2}$

$|AB| = \sqrt{(6)^2 + (8)^2}$

$|AB| = \sqrt{36 + 64}$

$|AB| = \sqrt{100}$

$|AB| = 10$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(0-8)^2 + (17-11)^2}$

$|BC| = \sqrt{(-8)^2 + (6)^2}$

$|BC| = \sqrt{64 + 36}$

$|BC| = \sqrt{100}$

$|BC| = 10$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(0-2)^2 + (17-3)^2}$

$|AC| = \sqrt{(-2)^2 + (14)^2}$

$|AC| = \sqrt{4 + 196}$

$|AC| = \sqrt{200}$

$|AC| = \sqrt{100 \times 2}$

$|AC| = \sqrt{100} \times \sqrt{2}$

$|AC| = 10\sqrt{2}$

For Isosceles Triangle

Two sides of a triangle are equal.

$|AB| = |BC| = 10$

Thus, the points A, B and C are the vertices of isosceles triangle.

Ex # 9.2

Q4: (i) **Show that points $A(1,2), B(3,4), C(0,-1)$ are vertices of scalene triangle.**

Solution:

$A(1,2), B(3,4), C(0,-1)$

Let $x_1 = 1,\ y_1 = 2$

And $x_2 = 3,\ y_2 = 4$

Also $x_3 = 0,\ y_3 = -1$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(3-1)^2 + (4-2)^2}$

$|AB| = \sqrt{(2)^2 + (2)^2}$

$|AB| = \sqrt{4 + 4}$

$|AB| = \sqrt{8}$

$|AB| = \sqrt{4 \times 2}$

$|AB| = \sqrt{4} \times \sqrt{2}$

$|AB| = 2\sqrt{2}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(0-3)^2 + (-1-4)^2}$

$|BC| = \sqrt{(-3)^2 + (-5)^2}$

$|BC| = \sqrt{9 + 25}$

$|BC| = \sqrt{34}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(0-1)^2 + (-1-2)^2}$

$|AC| = \sqrt{(-1)^2 + (-3)^2}$

$|AC| = \sqrt{1 + 9}$

$|AC| = \sqrt{10}$

For Scalene Triangle

All the three sides of a triangle are different.

$|AB| \neq |BC| \neq |AC|$

$2\sqrt{2} \neq \sqrt{34} \neq \sqrt{10}$

Thus, the points A, B and C are the vertices of scalene triangle.

Ex # 9.2

Q4: (ii) **Show that points $A(-4,-1), B(1,0), C(7,-3)$ are vertices of Scalene triangle.**

Solution:

$A(-4,-1), B(1,0), C(7,-3)$

Let $x_1 = -4,\ y_1 = -1$

And $x_2 = 1,\ y_2 = 0$

Also $x_3 = 7,\ y_3 = -3$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left(1-(-4)\right)^2 + \left(0-(-1)\right)^2}$

$|AB| = \sqrt{(1+4)^2 + (0+1)^2}$

$|AB| = \sqrt{(5)^2 + (1)^2}$

$|AB| = \sqrt{25+1}$

$|AB| = \sqrt{26}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(7-1)^2 + (-3-0)^2}$

$|BC| = \sqrt{(6)^2 + (-3)^2}$

$|BC| = \sqrt{36+9}$

$|BC| = \sqrt{45}$

$|BC| = \sqrt{9 \times 5}$

$|BC| = \sqrt{9} \times \sqrt{5}$

$|BC| = 3\sqrt{5}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(7-(-4)\right)^2 + \left(-3-(-1)\right)^2}$

$|AC| = \sqrt{(7+4)^2 + (-3+1)^2}$

$|AC| = \sqrt{(11)^2 + (-2)^2}$

$|AC| = \sqrt{121+4}$

$|AC| = \sqrt{125}$

$|AC| = \sqrt{25 \times 5}$

$|AC| = \sqrt{25} \times \sqrt{5}$

$|AC| = 5\sqrt{5}$

For Scalene Triangle

All the three sides of a triangle are different.

$|AB| \neq |BC| \neq |AC|$

$\sqrt{26} \neq 3\sqrt{5} \neq 5\sqrt{5}$

Thus, the points A, B and C are the vertices of scalene triangle.

Ex # 9.2

Q5: **Prove that $A(-2,-2), B(4,-2), C(4,6)$ are vertices of right – angled triangle.**

Solution:

$A(-2,-2), B(4,-2), C(4,6)$

Let $x_1 = -2,\ y_1 = -2$

And $x_2 = 4,\ y_2 = -2$

Also $x_3 = 4,\ y_3 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left(4-(-2)\right)^2 + \left(-2-(-2)\right)^2}$

$|AB| = \sqrt{(4+2)^2 + (-2+2)^2}$

$|AB| = \sqrt{(6)^2 + (0)^2}$

$|AB| = \sqrt{36+0}$

$|AB| = \sqrt{36}$

$|AB| = 6$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(4-4)^2 + \left(6-(-2)\right)^2}$

$|BC| = \sqrt{(0)^2 + (6+2)^2}$

$|BC| = \sqrt{0 + (8)^2}$

$|BC| = \sqrt{64}$

$|BC| = 8$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(4-(-2)\right)^2 + \left(6-(-2)\right)^2}$

$|AC| = \sqrt{(4+2)^2 + (6+2)^2}$

$|AC| = \sqrt{(6)^2 + (8)^2}$

$|AC| = \sqrt{36+64}$

$|AC| = \sqrt{100}$

$|AC| = 10$

For Right angled Triangle

$(Base)^2 + (Prep)^2 = (Hyp)^2$

So

$|AB|^2 + |BC|^2 = |AC|^2$

$(6\,)^2 + (8\,)^2 = (10\,)^2$

$36 + 64 = 100$

$100 = 100$

Thus, the points A, B and C are the vertices of right – angled triangle.

Ex # 9.2

Q6: **Prove that $A(-2,0), B(6,0), C(6,6), D(-2,6)$ are vertices of a rectangle.**

Solution:

$A(-2,0), B(6,0), C(6,6), D(-2,6)$

Let $x_1 = -2,\ y_1 = 0$

And $x_2 = 6,\ y_2 = 0$

Also $x_3 = 6,\ y_3 = 6$

Also $x_4 = -2,\ y_4 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(6-(-2))^2 + (0-0)^2}$

$|AB| = \sqrt{(6+2)^2 + (0)^2}$

$|AB| = \sqrt{(8)^2 + 0}$

$|AB| = \sqrt{64}$

$|AB| = 8$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(6-6)^2 + (6-0)^2}$

$|BC| = \sqrt{(0)^2 + (6)^2}$

$|BC| = \sqrt{0 + (6)^2}$

$|BC| = \sqrt{36}$

$|BC| = 6$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(-2-6)^2 + (6-6)^2}$

$|CD| = \sqrt{(-8)^2 + (0)^2}$

$|CD| = \sqrt{64 + 0}$

$|CD| = \sqrt{64}$

$|CD| = 8$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(-2-(-2))^2 + (6-0)^2}$

$|AD| = \sqrt{(-2+2)^2 + (6)^2}$

$|AD| = \sqrt{(0)^2 + 36}$

$|AD| = \sqrt{0 + 36}$

$|AD| = \sqrt{36}$

$|AD| = 6$

Ex # 9.2

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(6-(-2))^2 + (6-0)^2}$

$|AC| = \sqrt{(6+2)^2 + (6)^2}$

$|AC| = \sqrt{(8)^2 + 36}$

$|AC| = \sqrt{64 + 36}$

$|AC| = \sqrt{100}$

$|AC| = 10$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(-2-6)^2 + (6-0)^2}$

$|BD| = \sqrt{(-8)^2 + (6)^2}$

$|BD| = \sqrt{64 + 36}$

$|BD| = \sqrt{100}$

$|BD| = 10$

For Rectangle

Opposite sides are equal.

$|AB| = |CD| = 8$ ***and*** $|BC| = |AD| = 6$

And also, diagonals are equal

$|AC| = |BD| = 10$

Thus, the points A, B, C and D are the vertices of Rectangle.

Q7: **The Vertices of the rectangle ABCD are $A(2,0), B(5,0), C(5,4), D(2,4)$. How long is the diagonal AC?**

Solution:

$A(2,0), B(5,0), C(5,4), D(2,4)$

As to find diagonal AC, so take vertex A and C.

Diagonal $\overline{AC}$:

$A(2,0), C(5,4)$

Let $x_1 = 2,\ y_1 = 0$

And $x_2 = 5,\ y_2 = 4$

As distance of $\overline{AC}$:

$|AC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AC| = \sqrt{(5-2)^2 + (4-0)^2}$

$|AC| = \sqrt{(3)^2 + (4)^2}$

$|AC| = \sqrt{9 + 16}$

$|AC| = \sqrt{25}$

$|AC| = 5$

Thus, diagonal AC = 5

Ex # 9.2

Q8: **Prove that $A(-4,-1), B(1,0), C(7,-3), D(2,-4)$ are vertices of a parallelogram.**

Solution:

$A(-4,-1), B(1,0), C(7,-3), D(2,-4)$

Let $x_1 = -4,\ y_1 = -1$

And $x_2 = 1,\ y_2 = 0$

Also $x_3 = 7,\ y_3 = -3$

Also $x_4 = 2,\ y_4 = -4$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left(1-(-4)\right)^2 + (0-(-1))^2}$

$|AB| = \sqrt{(1+4)^2 + (0+1)^2}$

$|AB| = \sqrt{(5)^2 + (1)^2}$

$|AB| = \sqrt{25+1}$

$|AB| = \sqrt{26}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(7-1)^2 + (-3-0)^2}$

$|BC| = \sqrt{(6)^2 + (-3)^2}$

$|BC| = \sqrt{36+9}$

$|BC| = \sqrt{45}$

$|BC| = \sqrt{9 \times 5}$

$|BC| = \sqrt{9} \times \sqrt{5}$

$|BC| = 3\sqrt{5}$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(2-7)^2 + (-4-(-3))^2}$

$|CD| = \sqrt{(-5)^2 + (-4+3)^2}$

$|CD| = \sqrt{25 + (-1)^2}$

$|CD| = \sqrt{25+1}$

$|CD| = \sqrt{26}$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{\left(2-(-4)\right)^2 + (-4-(-1))^2}$

$|AD| = \sqrt{(2+4)^2 + (-4+1)^2}$

$|AD| = \sqrt{(6)^2 + (-3)^2}$

Ex # 9.2

$|AD| = \sqrt{36+9}$

$|AD| = \sqrt{0+36}$

$|AD| = \sqrt{45}$

$|AD| = \sqrt{9 \times 5}$

$|AD| = \sqrt{9} \times \sqrt{5}$

$|AD| = 3\sqrt{5}$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(7-(-4)\right)^2 + (-3-(-1))^2}$

$|AC| = \sqrt{(7+4)^2 + (-3+1)^2}$

$|AC| = \sqrt{(11)^2 + (-2)^2}$

$|AC| = \sqrt{121+4}$

$|AC| = \sqrt{125}$

$|AC| = \sqrt{25 \times 5}$

$|AC| = \sqrt{25} \times \sqrt{5}$

$|AC| = 5\sqrt{5}$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(2-1)^2 + (-4-0)^2}$

$|BD| = \sqrt{(1)^2 + (-4)^2}$

$|BD| = \sqrt{1+16}$

$|BD| = \sqrt{17}$

For Parallelogram

Opposite sides are equal.

$|AB| = |CD| = \sqrt{26}$ ***and*** $|BC| = |AD| = 3\sqrt{5}$

And also, diagonals are equal

$|AC| \neq |BD|$

$5\sqrt{5} \neq \sqrt{17}$

As diagonals are not equal

Thus, the points A, B, C and D are the vertices of Parallelogram.

Ex # 9.2

Q9: **Find b such that the points $A(2,b), B(5,5)$, $C(-6,0)$ are vertices of a right-angled triangle with $\angle BAC = 90^0$**

Solution:

$A(2,b), B(5,5), C(-6,0)$

Let $x_1 = 2,\ y_1 = b$

And $x_2 = 5,\ y_2 = 5$

Also $x_3 = -6,\ y_3 = 0$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(5-2)^2 + (5-b)^2}$

$|AB| = \sqrt{(3)^2 + (5)^2 + (b)^2 - 2(5)(b)}$

$|AB| = \sqrt{9 + 25 + b^2 - 10b}$

$|AB| = \sqrt{34 + b^2 - 10b}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(-6-5)^2 + (0-5)^2}$

$|BC| = \sqrt{(-11)^2 + (-5)^2}$

$|BC| = \sqrt{121 + 25}$

$|BC| = \sqrt{146}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(-6-2)^2 + (0-b)^2}$

$|AC| = \sqrt{(-8)^2 + (-b)^2}$

$|AC| = \sqrt{64 + b^2}$

As $\angle BAC = 90^0$

Now by Pythagoras theorem

$(Base)^2 + (Prep)^2 = (Hyp)^2$

$|AB|^2 + |AC|^2 = |BC|^2$

By putting values

$\left(\sqrt{34 + b^2 - 10b}\right)^2 + \left(\sqrt{64 + b^2}\right)^2 = \left(\sqrt{146}\right)^2$

$34 + b^2 - 10b + 64 + b^2 = 146$

$b^2 + b^2 - 10b + 34 + 64 = 146$

$2b^2 - 10b + 98 = 146$

$2b^2 - 10b + 98 - 146 = 0$

$2b^2 - 10b - 48 = 0$

$2(b^2 - 5b - 24) = 0$

Divided B.S by 2, we get

$b^2 - 5b - 24 = 0$

Ex # 9.2

$b^2 + 3b - 8b - 24 = 0$

$b(b+3) - 8(b+3) = 0$

$(b+3)(b-8) = 0$

$b + 3 = 0 \quad or \quad b - 8 = 0$

$b = -3 \quad or \quad b = 8$

Q10: **Given $A(-4,-2), B(1,-3), C(3,1)$, find the coordinate of D in the 2nd quadrant such that quadrilateral ABCD is a parallelogram.**

Solution:

Let the coordinate D is (x, y)

Thus the vertices of a parallelogram are

$A(-4,-2), B(1,-3), C(3,1), D(x,y)$

As AC and BD are the diagonals

Now

$\text{Mid – point of AC} = \left(\frac{-4+3}{2}, \frac{-2+1}{2}\right)$

$\text{Mid – point of AC} = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Also

$\text{Mid – point of BD} = \left(\frac{1+x}{2}, \frac{-3+y}{2}\right)$

As diagonals of a parallelogram bisect each other

So

$\frac{1+x}{2} = \frac{-1}{2} \quad and \quad \frac{-3+y}{2} = \frac{-1}{2}$

$1 + x = -1 \quad and \quad -3 + y = -1$

$x = -1 - 1 \quad and \quad y = -1 + 3$

$x = -2 \quad and \quad y = 2$

Thus

Thus the coordinate D is $(-2, 2)$

Mid – Point Formula:

The mid – point of the line segment obtained by joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ and $C(x, y)$ is mid – point of AB, then

$C(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Ex # 9.3

Example # 14

Find the coordinates of mid – point of the segment joining the points A(4, 6) and B(2, 1)

Solution:

Let $C(x\,,y)$ *is the mid − point of* AB, *then*

$$C(x\,,y) = \left(\frac{x_1 + x_2}{2}\,,\frac{y_1 + y_2}{2}\right)$$

Put the values

$$C(x\,,y) = \left(\frac{4 + 2}{2}\,,\frac{6 + 1}{2}\right)$$

$$C(x\,,y) = \left(\frac{6}{2}\,,\frac{7}{2}\right)$$

$$C(x\,,y) = \left(3\,,\frac{7}{2}\right)$$

Example # 15

The coordinates of the mid – point of a line segment $\overline{AB}$ are $(2, 5)$ and that of A are $(-4, -6)$. Find the coordinates of point B.

Solution:

Let the midpoint is $C(2, 5)$

As one end of a line segment $= A(x_1, y_1) = A(-4, -6)$

And other end of a line segment $= B(x_2, y_2) = ?$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}\,,\frac{y_1 + y_2}{2}\right)$$

Put the values

$$C(2, 5) = \left(\frac{-4 + x_2}{2}\,,\frac{-6 + y_2}{2}\right)$$

Now by comparing

$$2 = \frac{-4 + x_2}{2} \quad \& \quad 5 = \frac{-6 + y_2}{2}$$

$$2 \times 2 = -4 + x_2 \quad \& \quad 5 \times 2 = -6 + y_2$$

$$4 = -4 + x_2 \quad \& \quad 10 = -6 + y_2$$

$$4 + 4 = x_2 \quad \& \quad 10 + 6 = y_2$$

$$8 = x_2 \quad \& \quad 16 = y_2$$

$$x_2 = 8 \quad \& \quad y_2 = 16$$

Thus the other end of a line segment $= B(8, 16)$

Ex # 9.3

Result # 1

Prove that the line segment joining the mid – points of two sides of a triangle is equal to half of the length of the third side.

Proof:

$A(0, 0), B(a, 0), C(b, c)$

As D is the midpoint of AC

$$\text{D} = \left(\frac{0 + b}{2}\,,\frac{0 + c}{2}\right)$$

$$\text{D} = \left(\frac{b}{2}\,,\frac{c}{2}\right)$$

Also E is the midpoint of BC

$$\text{E} = \left(\frac{a + b}{2}\,,\frac{0 + c}{2}\right)$$

$$\text{E} = \left(\frac{a + b}{2}\,,\frac{c}{2}\right)$$

Now to find the distance of $\overline{AB}$

$$|AB| = \sqrt{(a - 0)^2 + (0 - 0)^2}$$

$$|AB| = \sqrt{(a)^2 + (0)^2}$$

$$|AB| = \sqrt{a^2}$$

$$|AB| = a$$

Now to find the distance of $\overline{DE}$

$$|DE| = \sqrt{\left(\frac{a + b}{2} - \frac{b}{2}\right)^2 + \left(\frac{c}{2} - \frac{c}{2}\right)^2}$$

$$|DE| = \sqrt{\left(\frac{a + b - b}{2}\right)^2 + (0)^2}$$

$$|DE| = \sqrt{\left(\frac{a}{2}\right)^2}$$

$$|DE| = \frac{a}{2}$$

But $a = |AB|$

$$|DE| = \frac{|AB|}{2}$$

$$|DE| = \frac{1}{2}|AB|$$

Result # 2

The mid – points of the hypotenuse of a right angled triangle is equidistance from the vertices.

Proof:

In right angled triangle ABC $\overline{BC}$ is the hypotenouse and D is the mid – point

The vertices are $A(0,0), B(a,0), C(0,b)$

As D is the midpoint of BC

$$D = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$$

$$D = \left(\frac{a}{2}, \frac{b}{2}\right)$$

To Prove:

As mid – point D is equidistant from the vertices.

Thus $|AD| = |BD| = |CD|$

Now to find the distance of $\overline{AD}$

$$|AD| = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2}$$

$$|AD| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$|AD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \quad \ldots\ldots equ(i)$$

Now to find the distance of $\overline{BD}$

$$|BD| = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2}$$

$$|BD| = \sqrt{\left(\frac{a-2a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$|BD| = \sqrt{\left(\frac{-a}{2}\right)^2 + \frac{b^2}{4}}$$

$$|BD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$|BD| = \sqrt{\frac{a^2+b^2}{4}} \quad \ldots\ldots equ(ii)$$

Now to find the distance of $\overline{CD}$

$$|CD| = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-b\right)^2}$$

$$|CD| = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b-2b}{2}\right)^2}$$

$$|CD| = \sqrt{\frac{a^2}{4} + \left(\frac{-b}{2}\right)^2}$$

$$|CD| = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$|CD| = \sqrt{\frac{a^2+b^2}{4}} \quad \ldots\ldots equ(iii)$$

From equ(i), (ii) & (iii)

$$|AD| = |BD| = |CD|$$

Result # 3

Verify the diagonals of any rectangle are equal in length.

Proof:

In rectangle ABCD $\overline{AC}$ *and* $\overline{BD}$ are the diagonals

The vertices are $A(0,0), B(a,0), C(a,b), D(0,b)$

To Prove:

As diagonals are equal in length

Thus $|AC| = |BD|$

Now to find the distance of $\overline{AC}$

$$|AC| = \sqrt{(a-0)^2 + (b-0)^2}$$

$$|AC| = \sqrt{(a)^2 + (b)^2}$$

$$|AC| = \sqrt{a^2 + b^2} \quad \ldots\ldots equ(i)$$

Now to find the distance of $\overline{BD}$

$$|BD| = \sqrt{(0-a)^2 + (b-0)^2}$$

$$|BD| = \sqrt{(-a)^2 + (b)^2}$$

$$|BD| = \sqrt{a^2 + b^2} \quad \ldots\ldots equ(ii)$$

From equ(i), (ii)

$|AC| = |BD|$

Result # 4

Show that diagonals of a parallelogram bisect each other.

Proof:

In parallelogram ABCD $\overline{AC}$ and $\overline{BD}$ are the diagonals

The vertices are $A(0,0), B(a,0), C(b,c), D(b-a,c)$

Let E is the midpoint of AC

$$E = \left(\frac{0+b}{2}, \frac{0+c}{2}\right)$$

$$E = \left(\frac{b}{2}, \frac{c}{2}\right)$$

Also F is the midpoint of BD

$$F = \left(\frac{a+b-a}{2}, \frac{0+c}{2}\right)$$

$$F = \left(\frac{b}{2}, \frac{c}{2}\right)$$

As the mid – points E and F are same.

Thus

$|AE| = |EC|$ and $|BF| = |FD|$

Result # 5 :Prove that is a right angled triangle square of the length of the hypotenuse is equal to the sum of the square of the length of two legs.

Proof:

In right – angled triangle $\overline{BC}$ *hypotenuse*

The vertices are $A(0,0), B(a,0), C(0,b)$

To prove:

$$(Hyp)^2 = (one\ leg)^2 + (other\ leg)^2$$

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(a-0)^2 + (0-0)^2}$$

$$|AB| = \sqrt{(a)^2 + (0)^2}$$

$$|AB| = \sqrt{a^2}$$

$$|AB| = a$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(0-a)^2 + (b-0)^2}$$

$$|BC| = \sqrt{(-a)^2 + (b)^2}$$

$$|BC| = \sqrt{a^2 + b^2}$$

Also distance of $\overline{AC}$:

$$|AC| = \sqrt{(0-0)^2 + (b-0)^2}$$

$$|AC| = \sqrt{(0)^2 + (b)^2}$$

$$|AC| = \sqrt{b^2}$$

$$|AC| = b$$

Let $\angle BAC = 90^0$

$$(Hyp)^2 = (one\ leg)^2 + (other\ leg)^2$$

$$|BC|^2 = |AB|^2 + |AC|^2$$

$$\left(\sqrt{a^2+b^2}\right)^2 = (a)^2 + (b)^2$$

$$a^2 + b^2 = a^2 + b^2$$

Hence the result is proved.

Ex # 9.3

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Q1: Find the coordinates of the midpoint of the segment with the given end points.

(i) $(\mathbf{8,-5})$ **and** $(\mathbf{-2,9})$

Solution:

$(8,-5)$ and $(-2,9)$

Let $x_1 = 8,\ y_1 = -5$

And $x_2 = -2,\ y_2 = 9$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{8+(-2)}{2}, \frac{9+(-5)}{2}\right)$$

$$\text{Midpoint} = \left(\frac{8-2}{2}, \frac{9-5}{2}\right)$$

$$\text{Midpoint} = \left(\frac{6}{2}, \frac{4}{2}\right)$$

$$\text{Midpoint} = (3, 2)$$

(ii) $(\mathbf{7,6})$ **and** $(\mathbf{3,2})$

Solution:

$(7,6)$ and $(3,2)$

Let $x_1 = 7,\ y_1 = 6$

And $x_2 = 3,\ y_2 = 2$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{3+7}{2}, \frac{2+6}{2}\right)$$

$$\text{Midpoint} = \left(\frac{10}{2}, \frac{8}{2}\right)$$

$$\text{Midpoint} = (5, 4)$$

Ex # 9.3

(iii) $(-2,3)$ **and** $(-9,-6)$

Solution:

$(-2,3)$and $(-9,-6)$

Let $x_1 = -2,\ y_1 = -3$

And $x_2 = -9,\ y_2 = -6$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{-2 + (-9)}{2}, \frac{-3 + (-6)}{2}\right)$$

$$\text{Midpoint} = \left(\frac{-2 - 9}{2}, \frac{-3 - 6}{2}\right)$$

$$\text{Midpoint} = \left(\frac{-11}{2}, \frac{-9}{2}\right)$$

(iv) $(a + b, a - b)$ **and** $(-a, b)$

Solution:

$(a + b, a - b)$ and $(-a, b)$

Let $x_1 = a + b,\ y_1 = a - b$

And $x_2 = -a,\ y_2 = b$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Put the values

$$\text{Midpoint} = \left(\frac{a + b + (-a)}{2}, \frac{a - b + b}{2}\right)$$

$$\text{Midpoint} = \left(\frac{a + b - a}{2}, \frac{a}{2}\right)$$

$$\text{Midpoint} = \left(\frac{a - a + b}{2}, \frac{a}{2}\right)$$

$$\text{Midpoint} = \left(\frac{b}{2}, \frac{a}{2}\right)$$

Q2: The mid-point and one end of a line segment are $(3, 7)$ and $(4, 2)$ respectively. Find the other end point.

Solution:

Let the midpoint is $C(3, 7)$

As one end of a line segment $= A(x_1, y_1) = A(4, 2)$

And other end of a line segment $= B(x_2, y_2) = ?$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex # 9.3

Put the values

$$C(3, 7) = \left(\frac{3 + x_2}{2}, \frac{7 + y_2}{2}\right)$$

Now by comparing

$$3 = \frac{4 + x_2}{2} \quad \& \quad 7 = \frac{2 + y_2}{2}$$

$$3 \times 2 = 4 + x_2 \quad \& \quad 7 \times 2 = 2 + y_2$$

$$6 = 4 + x_2 \quad \& \quad 14 = 2 + y_2$$

$$6 - 4 = x_2 \quad \& \quad 14 - 2 = y_2$$

$$2 = x_2 \quad \& \quad 12 = y_2$$

$$x_2 = 2 \quad \& \quad y_2 = 12$$

Thus the other end of a line segment $= B(2, 12)$

Q3: The midpoints of the sides of a triangle are (2, 5), (4, 2), (1, 1). Find the coordinates of the three vertices.

Solution:

As the midpoints are (2, 5), (4, 2), (1, 1)

Let the coordinates of the vertices are

$A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$

Let $(2, 5)$ be the midpoint of AB

$$(2, 5) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Now by comparing

$$2 = \frac{x_1 + x_2}{2} \quad \& \quad 5 = \frac{y_1 + y_2}{2}$$

$$2 \times 2 = x_1 + x_2 \quad \& \quad 5 \times 2 = y_1 + y_2$$

$$4 = x_1 + x_2 \quad \& \quad 10 = y_1 + y_2$$

$$x_1 + x_2 = 4 \quad \& \quad y_1 + y_2 = 10$$

Let $(4, 2)$ be the midpoint of BC

$$(4, 2) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

Now by comparing

$$4 = \frac{x_2 + x_3}{2} \quad \& \quad 2 = \frac{y_2 + y_3}{2}$$

$$4 \times 2 = x_2 + x_3 \quad \& \quad 2 \times 2 = y_2 + y_3$$

$$8 = x_2 + x_3 \quad \& \quad 4 = y_2 + y_3$$

$$x_2 + x_3 = 8 \quad \& \quad y_2 + y_3 = 4$$

Let $(1, 1)$ be the midpoint of AC

$$(1, 1) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

Now by comparing

$$1 = \frac{x_1 + x_3}{2} \quad \& \quad 1 = \frac{y_1 + y_3}{2}$$

$$1 \times 2 = x_1 + x_3 \quad \& \quad 1 \times 2 = y_1 + y_3$$

$$2 = x_1 + x_3 \quad \& \quad 2 = y_1 + y_3$$

Ex # 9.3

$x_1 + x_3 = 2$ & $y_1 + y_3 = 2$

Let

$x_1 + x_2 = 4$ ...... equ(i)

$x_2 + x_3 = 8$ ...... equ(ii)

$x_1 + x_3 = 2$ ...... equ(iii)

$y_1 + y_2 = 10$ ...... equ(a)

$y_2 + y_3 = 4$ ...... equ(b)

$y_1 + y_3 = 2$ ...... equ(c)

Now equ(i) − equ(ii)

$(x_1 + x_2) - (x_2 + x_3) = 4 - 8$

$x_1 + x_2 - x_2 - x_3 = -4$

$x_1 - x_3 = -4$ ...... equ(iv)

Now equ(iii) + equ(iv)

$x_1 + x_3 + x_1 - x_3 = 2 + (-4)$

$x_1 + x_1 = 2 - 4$

$2x_1 = -2$

$\frac{2x_1}{2} = \frac{-2}{2}$

$x_1 = -1$

Put $x_1 = -1$ in equ(i)

$-1 + x_2 = 4$

$x_2 = 4 + 1$

$x_2 = 5$

Put $x_2 = 5$ in equ(ii)

$5 + x_3 = 8$

$x_3 = 8 - 5$

$x_3 = 3$

Now equ(a) − equ(b)

$(y_1 + y_2) - (y_2 + y_3) = 10 - 4$

$y_1 + y_2 - y_2 - y_3 = 6$

$y_1 - y_3 = 6$ ...... equ(d)

Now equ(c) + equ(d)

$y_1 + y_3 + y_1 - y_3 = 2 + 6$

$y_1 + y_1 = 8$

$2y_1 = 8$

$\frac{2y_1}{2} = \frac{8}{2}$

$y_1 = 4$

Put $y_1 = 4$ in equ(a)

$4 + y_2 = 10$

$y_2 = 10 - 4$

$y_2 = 6$

Ex # 9.3

Put $y_2 = 6$ in equ(b)

$6 + y_3 = 4$

$y_3 = 4 - 6$

$y_3 = -2$

Let the coordinates of the vertices are

$A(-1, 4)$, $B(5, 6)$ & $C(3, -2)$

Q4: The distance between two points with coordinates (1, 1) and (4, y) is 5.

Solution:

As the coordinates are $(1, 1), (4, y)$

And distance $= d = 5$

Let $x_1 = 1,\ y_1 = 1$

And $x_2 = 4,\ y_2 = y$

As distance formula is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Put the values

$5 = \sqrt{(4-1)^2 + (y-1)^2}$

$5 = \sqrt{(3)^2 + (y)^2 - 2(y)(1) + (1)^2}$

$5 = \sqrt{9 + y^2 - 2y + 1}$

$5 = \sqrt{y^2 - 2y + 1 + 9}$

$5 = \sqrt{y^2 - 2y + 10}$

$\sqrt{y^2 - 2y + 10} = 5$

Taking square on B.S

$\left(\sqrt{y^2 - 2y + 10}\right)^2 = (5)^2$

$y^2 - 2y + 10 = 25$

$y^2 - 2y + 10 - 25 = 0$

$y^2 - 2y - 15 = 0$

$y^2 + 3y - 5y - 15 = 0$

$y(y + 3) - 5(y + 3) = 0$

$(y + 3)(y - 5) = 0$

$y + 3 = 0$ or $y - 5 = 0$

$y = -3$ or $y = 5$

Thus $y = -3$ or $y = 5$

Review Ex #9

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Q2: **Find the distance between A and B on the number line below.**

Solution:

$|AB| = |6 - (-4)|$

$|AB| = |6 + 4|$

$|AB| = |10|$

$|AB| = 10$

Q3: **What is the distance between two points with coordinates of $(\mathbf{1}, -\mathbf{5})$and $(-\mathbf{5}, \mathbf{7})$?**

Solution:

$(1, -5)$ and $(-5, 7)$

Let $x_1 = 1,\ y_1 = -5$

And $x_2 = -5,\ y_2 = 7$

As distance formula is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-5 - 1)^2 + \left(7 - (-5)\right)^2}$

$d = \sqrt{(-6)^2 + (7 + 5)^2}$

$d = \sqrt{36 + (12)^2}$

$d = \sqrt{36 + 144}$

$d = \sqrt{180}$

$d = \sqrt{36 \times 5}$

$d = \sqrt{36} \times \sqrt{5}$

$d = 6\sqrt{5}$

Q4: **Using distance formula, show that the points $(\mathbf{4}, -\mathbf{3}), \boldsymbol{B}(\mathbf{2}, \mathbf{0}), \boldsymbol{C}(-\mathbf{2}, \mathbf{6})$ are collinear.**

Solution:

$(4, -3), B(2, 0), C(-2, 6)$

Let $x_1 = 4,\ y_1 = -3$

And $x_2 = 2,\ y_2 = 0$

Also $x_3 = -2,\ y_3 = 6$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(2 - 4))^2 + (0 - (-3))^2}$

$|AB| = \sqrt{(-2)^2 + (0 + 3)^2}$

$|AB| = \sqrt{4 + (3)^2}$

$|AB| = \sqrt{4 + 9}$

$|AB| = \sqrt{13}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(-2 - 2)^2 + (6 - 0)^2}$

$|BC| = \sqrt{(-4)^2 + (6)^2}$

$|BC| = \sqrt{16 + 36}$

$|BC| = \sqrt{52}$

$|BC| = \sqrt{4 \times 13}$

$|BC| = \sqrt{4} \times \sqrt{13}$

$|BC| = 2\sqrt{13}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(-2 - 4)^2 + (6 - (-3))^2}$

$|AC| = \sqrt{(-6)^2 + (6 + 3)^2}$

$|AC| = \sqrt{36 + (9)^2}$

$|AC| = \sqrt{36 + 81}$

$|AC| = \sqrt{117}$

$|AC| = \sqrt{9 \times 13}$

$|AC| = \sqrt{9} \times \sqrt{13}$

$|AC| = 3\sqrt{13}$

For Colinear Points

$|AC| = |AB| + |BC|$

$2\sqrt{74} = \sqrt{74} + \sqrt{74}$

Thus, the points are colinear points.

Q5: **Find the point on the $\boldsymbol{x}-\ \boldsymbol{axis}$ which is equidistant from $(\mathbf{0}, \mathbf{1})\ \boldsymbol{and}\ (\mathbf{3}, \mathbf{3})$.**

Solution:

As the given points are $A(0, 1)\ and\ B(3, 3)$

$Let\ P\ be\ the\ point\ on\ x - axis$

So $P(x, 0)$

As point P is equidistant from A and B

$|AP| = |BP|$

$\sqrt{(x - 0)^2 + (0 - 1)^2} = \sqrt{(x - 3)^2 + (0 - 3)^2}$

$\sqrt{(x)^2 + (-1)^2} = \sqrt{x^2 - 2(x)(3) + (3)^2 + (-3)^2}$

$\sqrt{x^2 + 1} = \sqrt{x^2 - 6x + 9 + 9}$

$\sqrt{x^2 + 1} = \sqrt{x^2 - 6x + 18}$

Review # 9

Taking square on B.S

$$\left(\sqrt{x^2+1}\right)^2 = \left(\sqrt{x^2-6x+18}\right)^2$$
$$x^2+1 = x^2-6x+18$$
$$x^2-x^2+6x = 18-1$$
$$6x = 17$$
$$x = \frac{17}{6}$$
$$Thus, the\ point\ on\ x-axis\ is\ \left(\frac{17}{6}, 0\right)$$

Q6: A segment has one endpoint at (15, 22) and a midpoint at (5, 18), what are the coordinates of the other endpoint?

Solution:

Let the midpoint is $C(5, 18)$

As one end of a line segment $= A(x_1, y_1) = A(15, 22)$

And other end of a line segment $= B(x_2, y_2) = ?$

As midpoint formula is:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put the values

$$C(5, 18) = \left(\frac{15+x_2}{2}, \frac{22+y_2}{2}\right)$$

Now by comparing

$$5 = \frac{15+x_2}{2} \quad \& \quad 18 = \frac{22+y_2}{2}$$
$$5\times 2 = 15+x_2 \quad \& \quad 18\times 2 = 22+y_2$$
$$10 = 15+x_2 \quad \& \quad 36 = 22+y_2$$
$$10-15 = x_2 \quad \& \quad 36-22 = y_2$$
$$-5 = x_2 \quad \& \quad 14 = y_2$$
$$x_2 = -5 \quad \& \quad y_2 = 14$$

Thus the other end of a line segment $= B(-5, 14)$

Q7: Prove that $(2,1), (0,0), (-1,2), (1,3)$ are vertices of a rectangle.

Solution:

$Let\ A(2,1), B(0,0), C(-1,2), D(1,3)$

Let $x_1 = 2,\ y_1 = 1$

And $x_2 = 0,\ y_2 = 0$

Also $x_3 = -1,\ y_3 = 2$

Also $x_4 = 1,\ y_4 = 3$

Review # 9

As distance of $\overline{AB}$:

$$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$|AB| = \sqrt{(0-2)^2+(0-1)^2}$$
$$|AB| = \sqrt{(-2)^2+(-1)^2}$$
$$|AB| = \sqrt{4+1}$$
$$|AB| = \sqrt{5}$$

Now distance of $\overline{BC}$:

$$|BC| = \sqrt{(x_3-x_2)^2+(y_3-y_2)^2}$$
$$|BC| = \sqrt{(-1-0)^2+(2-0)^2}$$
$$|BC| = \sqrt{(-1)^2+(2)^2}$$
$$|BC| = \sqrt{1+4}$$
$$|BC| = \sqrt{5}$$

Also distance of $\overline{CD}$:

$$|CD| = \sqrt{(x_4-x_3)^2+(y_4-y_3)^2}$$
$$|CD| = \sqrt{(1-(-1))^2+(3-2)^2}$$
$$|CD| = \sqrt{(1+1)^2+(1)^2}$$
$$|CD| = \sqrt{(2)^2+1}$$
$$|CD| = \sqrt{4+1}$$
$$|CD| = \sqrt{5}$$

Also distance of $\overline{AD}$:

$$|AD| = \sqrt{(x_4-x_1)^2+(y_4-y_1)^2}$$
$$|AD| = \sqrt{(1-2)^2+(3-1)^2}$$
$$|AD| = \sqrt{(-1)^2+(2)^2}$$
$$|AD| = \sqrt{1+4}$$
$$|AD| = \sqrt{5}$$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$$|AC| = \sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$$
$$|AC| = \sqrt{(-1-2)^2+(2-1)^2}$$
$$|AC| = \sqrt{(-3)^2+(1)^2}$$
$$|AC| = \sqrt{9+1}$$
$$|AC| = \sqrt{10}$$

And Diagonal $\overline{BD}$:

$$|BD| = \sqrt{(x_4-x_2)^2+(y_4-y_2)^2}$$
$$|BD| = \sqrt{(1-0)^2+(3-0)^2}$$
$$|BD| = \sqrt{(1)^2+(3)^2}$$
$$|BD| = \sqrt{1+9}$$
$$|BD| = \sqrt{10}$$

Review # 9

For Rectangle

Opposite sides are equal.

$$|AB| = |CD| = \sqrt{5} \ \boldsymbol{and} \ |BC| = |AD| = \sqrt{5}$$

And also, diagonals are equal

$|AC| = |BD| = \sqrt{10}$

Thus, the points A, B, C and D are the vertices of Rectangle.

Also

$|AB| = |BC| = |CD| = |AD| = \sqrt{5}$

So it is also a square

Q8: Prove that $A(-1,0), B(3,3), C(6,-1), D(2,-4)$ are vertices of a square.

Solution:

$$A(-1,0), B(3,3), C(6,-1), D(2,-4)$$

Let $x_1 = -1,\ y_1 = 0$

And $x_2 = 3,\ y_2 = 3$

Also $x_3 = 6,\ y_3 = -1$

Also $x_4 = 2,\ y_4 = -4$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(3 - (-1))^2 + (3 - 0)^2}$

$|AB| = \sqrt{(3 + 1)^2 + (3)^2}$

$|AB| = \sqrt{(4)^2 + 9}$

$|AB| = \sqrt{16 + 9}$

$|AB| = \sqrt{25}$

$|AB| = 5$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(6 - 3)^2 + (-1 - 3)^2}$

$|BC| = \sqrt{(3)^2 + (-4)^2}$

$|BC| = \sqrt{9 + 16}$

$|BC| = \sqrt{25}$

$|BC| = 5$

Also distance of $\overline{CD}$:

$|CD| = \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}$

$|CD| = \sqrt{(2 - 6)^2 + (-4 - (-1))^2}$

$|CD| = \sqrt{(-4)^2 + (-4 + 1)^2}$

$|CD| = \sqrt{16 + (-3)^2}$

Review # 9

$|CD| = \sqrt{16 + 9}$

$|CD| = \sqrt{25}$

$|CD| = 5$

Also distance of $\overline{AD}$:

$|AD| = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}$

$|AD| = \sqrt{(2 - (-1))^2 + (-4 - 0)^2}$

$|AD| = \sqrt{(2 + 1)^2 + (-4)^2}$

$|AD| = \sqrt{(3)^2 + (4)^2}$

$|AD| = \sqrt{9 + 16}$

$|AD| = \sqrt{25}$

$|AD| = 5$

Now to find its Diagonal

Diagonal $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{\left(6 - (-1)\right)^2 + (-1 - 0)^2}$

$|AC| = \sqrt{(6 + 1)^2 + (-1)^2}$

$|AC| = \sqrt{(7)^2 + 1}$

$|AC| = \sqrt{49 + 1}$

$|AC| = \sqrt{50}$

$|AC| = \sqrt{25 \times 2}$

$|AC| = \sqrt{25} \times \sqrt{2}$

$|AC| = 5\sqrt{2}$

And Diagonal $\overline{BD}$:

$|BD| = \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2}$

$|BD| = \sqrt{(2 - 3)^2 + (-4 - 3)^2}$

$|BD| = \sqrt{(-1)^2 + (-7)^2}$

$|BD| = \sqrt{1 + 49}$

$|BD| = \sqrt{50}$

$|BD| = \sqrt{25 \times 2}$

$|BD| = \sqrt{25} \times \sqrt{2}$

$|BD| = 5\sqrt{2}$

For Square

All the sides are equal.

$|AB| = |BC| = |CD| = |AD| = 5$

And also, diagonals are equal

$|AC| = |BD| = 5\sqrt{2}$

Thus, the points A, B, C and D are the vertices of Square.

Review # 9

Q9: Show that $(6,5),(2,-4),$ and $(5,-1)$ is an isosceles triangle.

Solution:

Let $A(6,5), B(2,-4), C(5,-1)$

Let $x_1 = 6,\ y_1 = 5$

And $x_2 = 2,\ y_2 = -4$

Also $x_3 = 5,\ y_3 = -1$

As distance of $\overline{AB}$:

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(2-6)^2 + (-4-5)^2}$

$|AB| = \sqrt{(-4)^2 + (-9)^2}$

$|AB| = \sqrt{16 + 81}$

$|AB| = \sqrt{97}$

Now distance of $\overline{BC}$:

$|BC| = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2}$

$|BC| = \sqrt{(5-2)^2 + (-1-(-4))^2}$

$|BC| = \sqrt{(-3)^2 + (-1+4)^2}$

$|BC| = \sqrt{9 + (3)^2}$

$|BC| = \sqrt{9+9}$

$|BC| = \sqrt{18}$

$|BC| = \sqrt{9 \times 2}$

$|BC| = \sqrt{9} \times \sqrt{2}$

$|BC| = 3\sqrt{2}$

Also distance of $\overline{AC}$:

$|AC| = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

$|AC| = \sqrt{(5-6)^2 + (-1-5)^2}$

$|AC| = \sqrt{(-1)^2 + (-6)^2}$

$|AC| = \sqrt{1 + 36}$

$|AC| = \sqrt{37}$

Here

$|AB| \neq |BC| \neq |CD| \neq |AD|$

So these are not the vertices of an isosceles triangle.

Review # 9

Activity

You have a quadrilateral with vertices A(0, 0), B(9, 0), C(2, 4), D(6, 4). Find the mid – points of their diagonals. Does diagonals cut at the midpoint. Show it on graph paper.

Solution:

Let $x_1 = 0,$

$y_1 = 0$

And $x_2 = 9,$

$y_2 = 0$

Also $x_3 = 2,$

$y_3 = 4$

Also $x_4 = 6,\ y_4 = 4$

Here the diagonals are AD and BC

Now

$\text{Midpoint of AD} = \left(\frac{x_1 + x_4}{2}\ ,\frac{y_1 + y_4}{2}\right)$

Put the values

$\text{Midpoint of AD} = \left(\frac{0+6}{2}\ ,\frac{0+4}{2}\right)$

$\text{Midpoint of AD} = \left(\frac{6}{2}\ ,\frac{4}{2}\right)$

$\text{Midpoint of AD} = (3\ ,2)$

$\text{Midpoint of BC} = \left(\frac{x_2 + x_3}{2}\ ,\frac{y_2 + y_3}{2}\right)$

Put the values

$\text{Midpoint of BC} = \left(\frac{9+2}{2}\ ,\frac{0+4}{2}\right)$

$\text{Midpoint of BC} = \left(\frac{11}{2}\ ,\frac{4}{2}\right)$

$\text{Midpoint of BC} = (5.5\ ,2)$

As the mid – points of diagonals are not same.

So, the diagonals do not cut at mid – point.